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Tricritical Slowing Down of Superfluid Dynamics in ^{3}He-^{4}He Mixtures

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(Received 27 February 1975)

Measurements of the Rayleigh scattering by concentration fluctuations in the coexisting superfluid in ^{3}He-^{4}He mixtures near the tricritical temperature T_t show a single Lorentzian line of width such that $\Gamma/q^2 = D_{\rm eff} = 1.5\times10^{-4}\epsilon^{0.95\pm0.07}$ cm^2 sec^{-1} for $9\times10^{-4} \le 1 - T/T_t \equiv \epsilon \le 7\times10^{-3}$. Thus the effective diffusion coefficient $D_{\rm eff}$ is proportional to ϵ^{+1} and, since the concentration susceptibility $(\partial x/\partial\Delta)_{PT}$ is proportional to ϵ^{-1}, the effective kinetic coefficient $L_{D,\rm eff} = D_{\rm eff}/(\partial\Delta/\partial x)_{PT}$ is proportional to $\epsilon^{+1}\epsilon^{-1}\sim\epsilon^0$.

Dynamics close to conventional critical points features a phenomenon known as "critical slowing down" which is due to a singularity in the relaxation time for fluctuations of the appropriate order parameter.[1] In the case of binary fluid mixtures at a consolute critical point, slowing down of the relaxation of concentration fluctuations (typically by a factor of $\sim10^{-4}$) narrows part of the undisplaced (or Rayleigh) line in the light-scattering spectrum, providing a manifestation ideally measured with the help of lasers and optical homodyne spectroscopy.[2,3] These effects are now understood in terms of dynamical-scaling and mode-mode–coupling theories.[4-6]

Here we report that the first measurements of dynamical properties at a *tricritical* point[7-9] show critical slowing down of concentration fluctuations in the *superfluid* phase of mixtures of ^{3}He and ^{4}He. We find that the central linewidth Γ of the coexisting superfluid in the hydrodynamic regime vanishes as $(1-T/T_t)^1 = \epsilon^1$ as the tricritical temperature T_t is approached. This result should provide a fundamental test for tricritical-dynamic-scaling theory in a superfluid.

The thermodynamic properties of ^{3}He-^{4}He mixtures in the tricritical region have been rather completely characterized and static tricritical exponents have been determined.[10-12] Scattering-intensity measurements of Leiderer, Watts, and Webb,[12] for example, recently yielded tricritical exponents[13] $\beta = \gamma_\pm' = \nu_-' = \eta_-' = 1.0$ consistent with static tricritical scaling theory.[7-9] The experiments generally confirm the theory of static tricritical scaling.

Light-scattering measurements in tricritical ^{3}He-^{4}He mixtures reflect only concentration fluctuations because the concentration dependence of the refractive index overshadows its temperature and pressure dependence. Consequently the divergence of the scattered intensity on approaching the tricritical point (T_t, x_t) is due primarily to concentration fluctuations and reflects the divergence of the concentration susceptibility $(\partial x/\partial\Delta)_{PT}\sim\epsilon^{-1}$, and effects of the correlation length $\xi\sim\epsilon^{-1}$. The corresponding *spectrum* of the tricritical light scattering thus reflects the dynamics of the decay of concentration fluctuations.

The theory of the spectrum of critical light scattering in ^{3}He-^{4}He mixtures has been discussed by Gor'kov and Pitaevskii[14] and more recently by Griffin[15] on the basis of two-fluid hydrodynamics.[16] They predict a Lorentzian Rayleigh line in the superfluid phase of width $\Gamma = D_{\rm eff}q^2$ determined by an "effective" diffusion coefficient $D_{\rm eff}$ and the scattering vector $q = (4\pi/\lambda)\sin\theta/2$, where λ is the scattered-radiation wavelength and θ is the scattering angle. We can identify a corresponding symmetrical kinetic coefficient

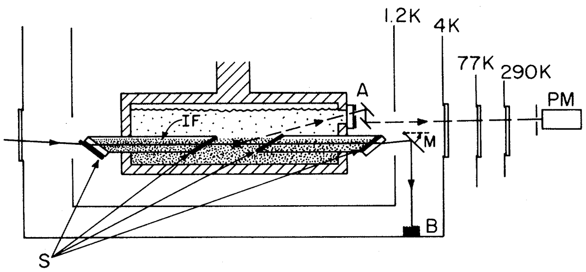

FIG. 1. Sample cell for forward-scattering experiment. Laser beam enters cell slightly below the interphase *IF* and passes through aperture slits *S* that are inclined 45° to plane of drawing. Plane of incidence is also slightly tipped from vertical. Exit apertures *A* limit the viewed area. The exit beam is reflected by mirror *M* to absorber *B* to avoid window scattering, but *M* is displaceable by magnetic coupling for beam alignment.

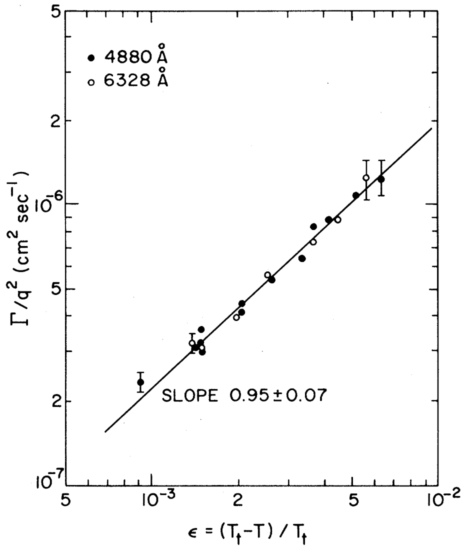

FIG. 2. Rayleigh scattering spectra at 15° scattering angle.

$L_{D,\mathrm{eff}} = D_{\mathrm{eff}}/(\partial\Delta/\partial x)_{PT}$. Recently Kawasaki and Gunton[17] and Grover and Swift[18] have analyzed the dynamical properties of the normal phases of ^{3}He-^{4}He mixtures near the tricritical point by using mode-mode-coupling theory. Grover and Swift have applied these results to the superfluid phase.[18]

Our measurements of the scattering spectrum were carried out in the same cryostat used[12] to measure scattering intensities in ^{3}He-^{4}He mixtures. Light scattering is very weak in comparison with typical binary critical mixtures.[2] Therefore multiple scattering is no problem, but optical correlation spectroscopy is difficult, especially at the limited laser power (~1 mW) necessary to avoid heating at cell windows. Hence our precision cannot compete with many previous optical-beating experiments, although it was good enough to determine dynamical behavior well within the region of static tricritical scaling. We used a photocounting technique and a digital correlator (modified Saicor model 42) to compute the autocorrelation function of the photon current. The maximum counting rates were 200 counts/sec with the viewed scattering aperture limited to 1 coherence area. Only the superfluid phase scattered strongly enough for our spectral measurements since $(\partial\Delta/\partial x)_{PT}$ in the normal fluid is an order of magnitude smaller.

Because $q\xi$ becomes large enough near T_t to enhance forward scattering, and because Γ is proportional to q^2 as anticipated in the hydrodynamic limit, the scattering spectrum was measured at $\theta = 15°$ to increase the number of photocounts per correlation time. At small θ stray light scattering must be meticulously controlled. In particular, scattering and reflections from the interface that are concentrated close to the vertical plane of incidence were avoided by tilting the plane defined by the scattered and incident beams a few degrees from vertical. In the sample cell shown in Fig. 1 a background of less than 10% over the investigated region was achieved. Therefore we obtained essentially the homodyne spectrum (of width 2Γ) with only a negligible heterodyne component. To minimize gravity effects[12] which generate a height-dependent concentration deviating from the coexistence curve, the scattering volume was placed only 0.2 mm away from the interface. The temperature was held constant to $\pm 10^{-6}$ K for the time necessary to equilibrate and to compute one datum point, typically several hours. Temperature distances from the tricritical point, $T_t - T$, were determined independently to better than 0.1 mK as described in earlier work.[12]

The resulting spectral widths of the intensity scattered by the superfluid phase are shown in Fig. 2. We have plotted Γ/q^2 versus ϵ for scattering at 15° and laser wavelengths $\lambda = 4880$ and 6328 Å, corresponding to scattering vectors q of 3.35×10^4 and 2.59×10^4 cm^{-1}, respectively. The solid line represents a fit

$$\Gamma/q^2 = 1.5\times10^{-4}\epsilon^{0.95\pm0.07}\ \mathrm{cm^2\ sec^{-1}}. \qquad (1)$$

The accessible temperature interval was limited to $9\times10^{-4} \leq \epsilon \leq 7\times10^{-3}$ by gravity effects close to

T_t and by insufficient scattered intensities and short correlation times further away. The observed correlation functions $G(t)$ could be fitted by single-exponential decay times within the experimental uncertainty, and the weight factors $G(0)$ increased in proportion to the square of the intensity of the tricritical scattering as expected of the scattering by the concentration fluctuations. Thus our tricritical Rayleigh-line spectra were clearly dominated by a single Lorentzian of width determined by the temperature and the scattering vector.

Ahlers and Greywall[19] have also noticed a divergence of the macroscopic equilibration time $\tau_e \propto \epsilon^{-1.0\pm0.1}$ for tricritical mixtures along the coexistence curve. It seems likely that this is the same concentration relaxation process that is responsible for our Rayleigh narrowing.

A theoretical expression derived from two-fluid hydrodynamics for the central linewidth Γ in the superfluid phase of ^{3}He-^{4}He mixtures[14,15] may be written in the notation of Ref. 14 as

$$\frac{\Gamma}{q^2}=\frac{(\partial\Delta/\partial x)_{PT}\,\kappa+\rho DT\{x[\partial(\sigma/x)/\partial x]_{PT}+(k_T/T)(\partial\Delta/\partial x)_{PT}\}^2}{\rho T x^2[\partial(\sigma/x)/\partial x]_{PT}^{\,2}+\rho C_{px}(\partial\Delta/\partial x)_{PT}}, \tag{2}$$

where ρ is the density, σ is the entropy, and C_{px} is the specific heat at constant pressure and concentration, which remains finite at the tricritical point.[8,10] Thus the central linewidth $\Gamma = D_{\rm eff}q^2$ involves a complicated combination of the thermal conductivity κ, the thermal diffusion ratio k_T, and the mass diffusion constant D. We note that macroscopic thermal-conduction measurements on ^{3}He-^{4}He mixtures determine an *effective* thermal conductivity[16]

$$\kappa_{\rm eff}=\kappa+\frac{\rho DT\{x[\partial(\sigma/x)/\partial x]_{PT}+(k_T/T)(\partial\Delta/\partial x)_{PT}\}^2}{(\partial\Delta/\partial x)_{PT}}. \tag{3}$$

The denominator of Eq. (2) goes to some finite limit as $T\to T_t$. Our present measurements show that $D_{\rm eff}$ is proportional to ϵ^1 and earlier measurements[12] show that $(\partial\Delta/\partial x)_{PT}$ is proportional to ϵ^1, indicating that $\kappa_{\rm eff}$ remains finite as $T\to T_t$.

Application of mode-mode–coupling theory to ^{3}He-^{4}He mixtures by Kawasaki and Gunton[17] and by Grover and Swift[18] gave tricritical exponents in the normal phase for the mass diffusion coefficient D, the thermal diffusion ratio k_T, and a thermal conductivity κ_s. Applying these results to the coexisting *superfluid* phase gives

$$D\propto\epsilon^{1/2},\quad k_T\propto\epsilon^{-1},\quad\text{and } \kappa_s\propto\epsilon^{-1/2}. \tag{4}$$

Kawasaki and Gunton[17] indicate that an *experimental* thermal conductivity κ measured in the normal phase with no diffusion current remains finite because of a cancelation of divergences. We identify this κ with the κ of Eqs. (2) and (3) and not with $\kappa_{\rm eff}$.[16] Since Eqs. (4) indicate that the quantity in curly brackets in Eq. (2) goes to a constant as $\epsilon\to0$, the Rayleigh width in the coexisting superfluid is predicted to vary according to

$$\Gamma/q^2=A\epsilon^1+B\epsilon^{1/2}, \tag{5}$$

where A and B are constants. Since we observe $\Gamma\propto\epsilon^1$ for $10^{-3}<\epsilon<10^{-2}$, consistency of our data with existing theory requires $A\approx1.2\times10^{-4}$ cm^2/sec and $B\lesssim A/10$. The thermal relaxation time observed by Ahlers and Greywall[19-21] implies $A = (1.33\pm0.03)\times10^{-4}$ cm^2/sec,[20] a value in excellent agreement with our scattering spectra.

Thus it appears that the Rayleigh linewidth, which is unequivocably a manifestation of the *concentration*-fluctuation spectra, is actually determined by the ratio of an effective *thermal* conductivity and the concentration susceptibility. This result is a consequence of the coupling of the ^{3}He flux and normal-fluid flux in the two-fluid hydrodynamics.[16,22,23] Theoretical investigations of the transport coefficients capable of determining the relative size of A and B in Eq. (5) or the existence of cancelations in Eq. (2) have not to our knowledge been attempted for the superfluid phase.

We are pleased to acknowledge helpful conversations with Professor Jack Swift, Dr. Guenter Ahlers, and Dr. Maurice Papoular. The support of the National Science Foundation, Materials Science Center at Cornell, and fellowships of the Deutsches Forschungsgemeinschaft (P.L.), the Guggenheim Foundation (W.W.W.), and the Ford Foundation (D.R.N.) are gratefully acknowledged.

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Exact Ground-State Wave Function for a One-Dimensional Plasma*

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(Received 30 January 1975)

I have determined with exact ground-state wave function and ground-state energy for a one-dimensional plasma, with density-dependent short-range force. In particular, I am able to approach the thermodynamic limit of finite density, and verify that the ground-state energy is extensive, provided that there is a uniform density of neutralizing charge.

In a previous paper,[1] I presented the necessary condition for a product wave function,

$$\Psi = \prod_{i>j} |\psi(x_i - x_j)|^{\lambda}, \tag{1}$$

to be the exact ground-state wave function of an N-body Hamiltonian with two-body potentials only. For a one-dimensional system, in terms of the logarithmic derivative φ of ψ,

$$\varphi = \psi'/\psi, \tag{2}$$

the condition is

$$\varphi(x)\varphi(y) + \varphi(y)\varphi(z) + \varphi(z)\varphi(x) = f(x) + f(y) + f(z) \tag{3a}$$

for any three numbers x, y, and z such that

$$x + y + z = 0. \tag{3b}$$

Equation (3) is a functional equation, which I have been unable to solve in general. In Ref. 1, particular solutions were discussed for the cases

$$\varphi = ax + b/x, \quad f = -ab - \tfrac{1}{2}a^2x^2, \tag{4a}$$

$$\varphi = a|x|/x, \quad f = -\tfrac{1}{3}a^2, \tag{4b}$$

$$\varphi = a\cot(x/r), \quad f = \tfrac{1}{3}a^2. \tag{4c}$$

Note that, in principle, there is a solution for case (4c) when r is imaginary. In fact (4b) might then be considered as a limiting case of (4c). Such a potential corresponds to $v(x) = (\sinh x)^{-2}$.