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Production of Prompt Muons by the Interaction of 28-GeV Protons*

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Analyses of measurements of muons produced directly, in the forward direction, by the interaction of 28-GeV protons with uranium show that the muon/pion production ratio for negative particles is 13×10^{-5} for 11.6-GeV particles, 2.6×10^{-5} at 20.3 GeV, 1.3×10^{-5} at 23.5 GeV, and 7.0×10^{-6} at 25 GeV. The largest ratio is similar to ratios measured at higher energies and large transverse momenta and none of the ratios are easily explained in terms of conventional mechanisms.

In work reported previously,[1,2] we presented measurements of a substantial flux of prompt muons produced by the interaction of 28-GeV protons with uranium. Here we define prompt leptons as leptons produced very near the point of the hadron interaction, excluding muons derived from the decay of long-lived mesons. While production cross sections for such prompt muons were determined and discussed[2]—and attributed, tentatively, to electromagnetic production of muon pairs—the ratio of prompt-muon to pion production was not calculated for all of the measurements, and such ratios cannot be derived in any very transparent manner from the published data. In view of the considerable interest in the measurements of the ratios of prompt leptons to pions produced at large transverse momenta[3-6] by proton-nucleon interactions at very high energies, we have calculated these ratios for our data. Our measurements are complementary to the high-energy data inasmuch as our determinations were made in the forward direction, at large values of the Feynman variable x, and at the moderate proton energy of 28 GeV.

The experimental measurements made at the Brookhaven alternating-gradient synchrotron, and described in detail in Ref. 2, were conducted by determining the flux of muons which stopped in an aluminum and scintillator sandwich, where the muons were produced in a series of segmented uranium targets backed by a steel-filled magnet and steel absorbers. The energy of the muons was defined by the thickness of the steel absorber; the sign of the muon charge was determined by the magnetic deflection and the lifetime in aluminum; and the characteristics of the origin of the muons were defined by the variation of muon flux with the effective density of the target. Three targets were used: one of solid uranium with an average density $\rho(1)$, one of alternate sections of uranium and air with an average density $\rho(2)$ equal to $\frac{1}{2}\rho(1)$, and a third with an effective density $\rho(3)$ of $\frac{1}{3}\rho(1)$. The targets each had a thickness of about 5 proton interaction lengths of uranium backed by meters of steel: We can consider that the targets were effectively of infinite thickness for hadron interactions.

As a result of the competition between meson decay and meson interaction in the target, the flux of muons from meson decays is inversely proportional to the target density. We then write the muon flux $I(E_\mu, \rho)$ as a function of the target density ρ and muon energy E_μ as

$$I(E_\mu, \rho) = a(E_\mu) + b(E_\mu)/\rho,$$

where $a(E_\mu)$ is proportional to the prompt muon flux and $b(E_\mu)$ is proportional to the flux of muons from pion and K-meson decays. For convenience, we quote the results of the measurements in terms of $a(E_\mu)$ and $b(E_\mu)/\rho(1)$ which represent the prompt muon flux and the flux of muons from mesons which are produced and decay in the solid uranium target. If we assume that the prompt muons are all derived from electromagnetic production of muon pairs, the ratio R_m of prompt muons to muons from meson decay in uranium, $a\rho(1)/b$, is determined to an accuracy of about 5% by the measurements. If we relax this condition, accepting the possibility that the prompt muons might be produced singly through more exotic mechanisms which might not produce equal numbers of positive and negative muons, the accuracy of this ratio is reduced and we estimate that the error might be as large as 15%, where the main uncertainty is derived from our imprecise knowledge of the flux of muons produced through interactions of the beam protons with material upstream from the target.

Though we have then an accurate measure of the ratio of prompt muons to muons derived from meson decay at several energies E_μ, this does not tell us the ratio of prompt muons to pions at production. For this number, we need information concerning the pion spectrum for pion energies $E_\pi > E_\mu$, inasmuch as the muons from pion decay are, of course, derived from the decays of pions with energies greater than the muon energy. If we write $I_p \Delta E$ as the intensity of prompt muons stopping in the detector of length ΔE and $I_\mu \Delta E$ as the intensity of muons from pion decay which stop in the detector, we can express these intensities as

$$I_p = d\sigma_\mu / dE_\mu \tag{1}$$

and

$$I_\mu = \int_E^{E_m} I_\pi(E_\pi)(2.0\times10^{-3}/E_\pi)(0.4E_\pi)^{-1}\, dE_\pi, \tag{2}$$

where

$$I_\pi = d\sigma_\pi / dE_\pi \tag{3}$$

and E_m is the maximum energy of a pion which can decay to a muon of energy E. In Eq. (2), the first factor in the integral is the pion differential cross section, the second factor gives the probability of the pion's decaying before interaction, where E_π is measured in GeV, and the third factor is the probability that the daughter muon will have an energy E_μ.

The object of our analysis is to determine the ratio R of prompt-muon flux to pion flux,

$$R = I_p / I_\pi, \tag{4}$$

for direct comparison with recent measurements. We assume here that the production of muons from K-meson decay can be neglected, which would appear to be justified from our knowledge of K-meson–to–pion flux ratios and the relatively unfavorable kinematics of K-meson decays to muons as compared with pion decays. In all of the intensity relations we suppress a mutual proportionality constant which contains the beam intensity and the solid angle subtended by the detector. We note that the differential cross sections express cross sections for the production of particles into the solid angle subtended by the detector. Though the fluxes then represent production nominally in the forward direction, the finite size of the detector results in acceptance of particles produced at angles up to 20 mrad or transverse momenta of 500 MeV/c at the highest muon energies, and the differential cross sections represent cross sections which are an appropriate average over transverse momenta where this average is a function of muon energy. We will implicitly assume that the distribution in transverse momenta of the prompt muons is the same as for muons from pion decay.

The determination of R is made complicated by the fact that we know $I_\pi(E_\pi)$ only through the integral relation of Eq. (2) and the measurements of I_μ for positive and negative muons at four energies. Fortunately, we can draw on the extensive inclusive-cross-section measurements of pion production from nucleon-nucleon interactions to construct a model for $I_\pi(E_\pi)$ which, when constrained to fit the measurements of I_μ, should allow us to deduce the values of the ratios R in a manner which is quite insensitive to the specific form of $I_\pi(E_\pi)$. The form we choose for negative-pion production is,

$$I_\pi(E) = d\sigma_\pi / dE_\pi = A x^2 (1-x) \exp(-bx), \quad x = E_\pi / E_m, \tag{5}$$

TABLE I. Measured and calculated flux ratios. Here E_μ is the muon energy in GeV and $x_\mu \approx E_\mu/E_{max}$ is the Feynman parameter; R_m is the measured ratio of prompt-μ^- flux and the μ^- flux derived from meson decays in a thick uranium target; R_{ch} is the measured ratio of positive to negative muons from meson decay; R^- is the ratio of the prompt-negative-muon flux to the calculated flux of negative pions; R^+ is the ratio of prompt positive muons to the calculated positive-pion flux; and R_t is the ratio of all prompt muons to the calculated flux of all charged pions. The differential cross sections represent the calculated cross sections per nucleon for the direct production of positive muons (or of negative muons) deduced from the pion flux values taken from the parametrization of Sanford and Wang. The uncertainties in these cross sections are then probably as great as a factor of 2. The value of R_{ch} at 20.3 GeV was not measured but estimated from an interpolation of data taken at other energies.

E_μ	x_μ	R_m	R_{ch}	R^-	R^+	R_t	$d\sigma/dE\,d\Omega$
11.6	0.45	1.78	2.2	13×10^{-5}	7.0×10^{-5}	9.1×10^{-5}	3.0×10^{-31}
20.3	0.79	1.50	(6.0)	2.6×10^{-5}	4.8×10^{-6}	8.1×10^{-6}	2.0×10^{-33}
23.5	0.91	1.65	8.8	1.3×10^{-6}	1.6×10^{-7}	2.8×10^{-7}	1.5×10^{-34}
25.1	0.97	2.10	11.4	7.0×10^{-7}	6.2×10^{-8}	1.1×10^{-7}	1.3×10^{-35}

where E_m is the maximum pion energy kinematically allowed, A is a normalization constant, and $b=11$ for π^- mesons and 9 for π^+ mesons. When this form of I_π is inserted into Eq. (2), values of I_μ are derived which fit the measured values at energies of 11.6, 20.3, 23.5, and 25.1 GeV within an error of 10% though the values of the cross sections vary over more than 4 orders of magnitude. It is satisfying to note that the expression Eq. (5) for the inclusive differential cross sections for pion production fits the Sanford-Wang description[7] of such cross sections which is derived from an analysis of the inclusive pion production data. Here we modify the Sanford-Wang formulas slightly to ensure a constraint of the cross sections by the conservation laws by replacing the incident proton energy in their formulas by E_m, the maximum pion energy kinematically allowed in nucleon-nucleon collisions.

With this expression, Eq. (5), for I_π, the ratios R of prompt-muon production and pion production can be determined and the values are presented in Table I for positive and negative particles, together with the ratios R_t for the production of all prompt muons and all charged pions and an estimate[2] of the differential cross section for the production of prompt muons in the forward direction in nucleon-nucleon interactions. We estimate that the errors in the ratios R^+ and R^- are of the order of 20% at the lower energies but may be as great as 50% for the point at 25.1 GeV which is very near the kinematic limit: In this case the error reflects more the uncertainty in effective energy than that in the explicit value of the intensities themselves.

The importance of the comparison of prompt-muon production with pion production follows from theoretical ideas and from experimental observations. One would expect a strong connection between the processes on the basis of certain conjectures such as the presumption that the prompt muons are produced mainly through the decay of vector mesons which also decay into pion states. Also, experimental observations[3-6] indicate that the ratio of muon-to-pion intensity is large and nearly independent of transverse momentum over a large range. In these determinations, the measurements are made in kinematic regions ($x\approx 0$) where the ratio of positive-to-negative pions is near 1 while our measurements were made in kinematic regions ($x \geqslant 0.5$) where the ratio is large; indeed, far larger than could be expected if the origin of the pions was the decay of isovector (or, of course, isoscalar) vector mesons.[8] Since the charge ratio for muons of electromagnetic origin must be 1, we feel that the most apt comparison of our data with the other data is made by comparing the ratios of the charge state containing the fewer pions; in our case, the negative particles. We see then that the muon-to-pion ratios we observe for negative particles at 11.6 GeV are of the same magnitude as the values of about 10×10^{-5} observed in the high-energy, large-transverse-momentum experiments. Though our muon-to-pion ratios are much smaller at large values of x, this can be attributed wholly to kinematic effects which reduce the ratio of particles derived from two-body decays (into muon pairs) with respect to single-particle production, and these ratios are as difficult to explain in terms of conventional mechanisms as the value of 13×10^{-5} we measure at $x=0.45$ or the equally large ratios obtained by others in the high-energy experiments. While it is not possible here to examine all conceivable conventional production mechanisms, we note that the inclusive cross section for vector-meson production[8] seems too small to account for the prompt-lepton production (by about a factor of 10) if simple models which suggest equal production of ρ^+, ρ^0, ρ^-, ω^0, and φ^0 are considered.

Perhaps the independence with respect to energy of the ratio $R_m = I_p^-/I_\mu^-$, where each component varies by 4 orders of magnitude, is significant. Since both intensities measure products of two-body decays, I_p the two-body decay into a muon pair of a massive virtual photon and I_μ the two-body decay of a pion into a muon and neutrino, the kinematic factors largely cancel. Then the constant value of the ratio can be considered to suggest that the inclusive spectra of virtual photons has much the same form as the inclusive spectra for negative pions. Such a model would seem to be in agreement with large-transverse-momentum data also.

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Relativistic Oscillator Model of High-Energy Interactions*

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I present a model of hadrons composed of relativistic quarks in bound oscillator states. The model gives a good account of the multiplicity, distribution in momentum, and particle ratios for high-energy proton-proton collisions.

Recent data on inelastic lepton scattering[1,2] confirm some of the predictions based on hadrons composed of quarks. This paper takes the proposition seriously and attempts to predict how a bound system of such quarks might give rise to the observations in high-energy hadronic reactions.[3]

We want the observed linear dependence between spin J and mass M^2. Take

$$n = M^2/\Delta^2,$$

where n is the index of the excited state. Experimentally $\Delta \simeq 1$ GeV. We take relativistic quarks, and a constant wave number p_n suitable for a quasifree wave. We equate the energy of excitation ϵ_n with the momentum or wave number of the quark wave.[4] The index n is the number of nodes in the state of length r_n:

$$n = r_n p_n \propto p_n^2 = \epsilon_n^2, \qquad r_n \propto p_n = \epsilon_n. \tag{1}$$

The size of the atom, r_n, grows linearly with excitation ϵ_n.

We postulate the mode of excitation as being an exchange of valence quarks between the two colliding hadrons.[5] In the center-of-mass system