events at high y in the antineutrino y distribution leads to a value of $B^{\bar{\nu}}$ different from the values of B^{ν} and $B(x)$, which is suggestive of an *effective* deviation from charge symmetry if the validty of scale invariance is assumed. This may arise in part from new particle production or from an anomalously large cross section for direct strange-particle production.[10] Note that new particle production tends for kinematic reasons to populate preferentially the regions of small x and large y.[11]

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Resonance Pole with $J^P = \frac{3}{2}^+$ in a Coupled-Channel Analysis of K^+p Elastic Scattering Data

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Energy-dependent partial-wave amplitudes based on a coupled-channel $(K^+p, K\Delta)$ K-matrix formalism are obtained for elastic K^+p scattering up to $P_{1ab} = 2$ GeV/c. The data used include new high-precision differential cross sections. A total of 42 parameters describing the $l \leq 4$ partial waves are used to fit 3822 data points and the best solution found has a χ^2 per degree of freedom of 1.33. This solution exhibits a T-matrix Z^* resonance pole in the P_3 partial wave with mass coordinates 1787–i100 MeV.

In recent years there has been considerable effort invested in the measurement and analysis of elastic K^+p scattering to determine whether the bump in the total cross section at $P_{1ab} = 1.2$ GeV/c can be interpreted as a Z^* resonance.[1] Most analyses to date have been basically of the single-energy type, an Argand trajectory being determined by selectively connecting single-energy solutions at adjoining energies by means of a "shortest-path" criterion. Although there have been suggestions of resonancelike behavior in these analyses [primarily in the P_3 wave (we use the partial-wave notation $\mathcal{L}_{2J}$)], the results have been inconclusive. This is due in part to the difficulty of detecting the effects of a broad inelastic resonance on the Argand plane[2] and to the uncertainties associated with the procedure of "shortest-path" detection as recently pointed out by Dean, Jensen, and Long.[3]

In the analysis described in this report, each partial-wave amplitude is parametrized as an analytic function of energy through a two-channel K-matrix formalism. It was hoped that the analytic nature of the representation would answer the criticism of Dean and Lee.[4] It would also provide an alternative to seeking a Breit-Wigner Argand trajectory by enabling a direct analytic continuation to possible resonance poles.

The data base[5-21] which includes data to to $P_{\text{lab}} = 2.0$ GeV/c is given in Table I. It incorporates for the first time the new high-precision differential cross sections of Abe *et al.*,[5] the new low-energy data of Cameron *et al.*,[11] and measured values of α from CERN.[21] Before presenting the results of the analysis, we shall briefly discuss our parametrization.

TABLE I. The data set used for this analysis. σ_{el} is the integrated elastic cross section and α is the ratio of the real to imaginary part of the forward-scattering amplitude.

Reference	Data Type	Momentum Range (GeV/c)	No. of Momenta	No. of Data
Abe(5)	$d\sigma/d\Omega$	.865-1.965	20	1322
Adams(6)	$d\sigma/d\Omega$	.432-.939	13	214
	σ_{el}	.432-.939	13	13
Albrow(7)	Pol.	.87-1.89	14	328
	$d\sigma/d\Omega$	.87-1.89	14	300
Barnett(8)	Pol.	1.37-1.89	6	161
Bland(9)	$d\sigma/d\Omega$	.864-1.207	3	60
Bowen(10)	σ_T	.366-1.160	32	32
Cameron(11)	$d\sigma/d\Omega$	.130-.755	12	198
Carrol(12)	σ_T	.417-1.066	19	19
Charles(13)	$d\sigma/d\Omega$	.909-1.907	17	797
Cool(14)	σ_T	.891-1.996	20	20
Ehrlich(15)	Pol.	1.33-1.91	7	129
Focardi(16)	$d\sigma/d\Omega$	.78	1	29
Giacomelli(17)	$d\sigma/d\Omega$	.9-1.48	9	171
Bugg, Cool(18)	σ_R	.865-1.6	12	12
Martin(19)	α	.865-1.6	12	12
CERN(21)	α	1.21-1.798	2	2

The inelastic effects in K^+p scattering are dominated by $K^*(890)$ and $\Delta(1236)$ production in the energy range of interest (the $KN\pi\pi$ channel is negligible at all but the highest energies). We shall assume that the $K\Delta$ channel dominates the inelasticities of the P, F, and G waves, but that the S wave couples more strongly to the K^*N channel (where S-wave to S-wave coupling is available). Accordingly, for each J^P partial wave, we define the 2×2 real, symmetric K matrix[20] by

$$S = (1 + i\rho^{1/2}K\rho^{1/2})(1 - i\rho^{1/2}K\rho^{1/2})^{-1},$$

where S is the usual S matrix and ρ is a 2×2, diagonal "phase-space" matrix which contains the threshold energy dependence. The elastic element of ρ is given by

$$\rho_e = [K(E, m_K, m_N)]^{2l+1},$$

while for the inelastic channel, because of the finite widths of the Δ and K^*,

$$\rho_i = (\Gamma_i \pi^{1/2})^{-1} \int_{m_K + m_\pi}^{\infty} [K(E, m_s, m)]^{2l_i+1} \exp[-(m - m_i)^2/\Gamma_i^2]\, dm,$$

where

$$[K(E, m_s, m)]^2 = [E^2 - (m + m_s)^2][E^2 - (m - m_s)^2]/4E^2(m + m_s)^2.$$

In these expressions, E is the total barycentric energy, m_i and Γ_i are the mass and width of the unstable particle in the inelastic channel (m_s refers to the stable mass), l is the orbital angular momentum of the KN channel, and l_i is the lowest orbital angular momentum of the inelastic channel coupling to l. The factor $(m + m_s)^2$ in the denominator of K^2 is introduced purely for dimensional reasons. The elements of K are parametrized with the form

$$K(j, l)_{\alpha\beta} = a(j, l)_{\alpha\beta} + b(j, l)_{\alpha\beta} T_{\text{lab}}, \qquad (1)$$

where $j = l \pm \frac{1}{2}$ and $b(j, l)_{\alpha\beta} = 0$ for $l \geq 3$. The $l \geq 5$ partial waves are not included in the analysis.

Only one acceptable solution has been obtained with this particular parametrization. This is shown in Fig. 1. Of particular interest is the resonancelike behavior in the P_3 Argand trajectory. Analytic continuation of this partial-wave amplitude reveals a pole at $E = 1787 - i100$ MeV. The pole is obtained as a zero in the determinant of the matrix $1 - i\rho^{1/2}K\rho^{1/2}$ at the stated value of

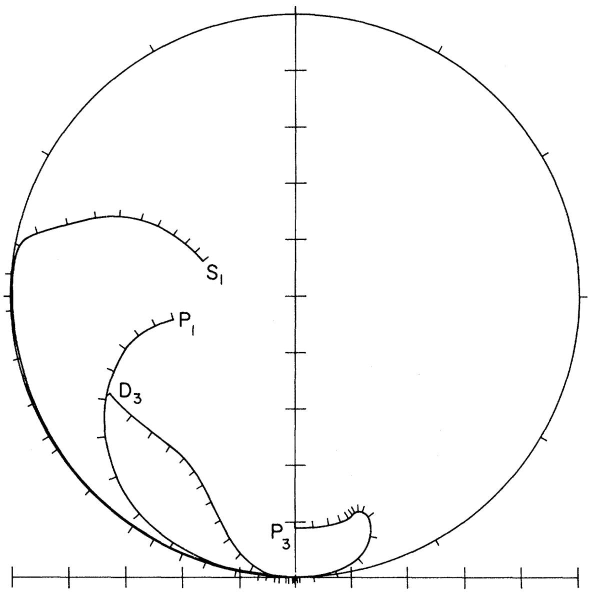

FIG. 1. The S_1, P_1, P_3, and D_3 partial-wave amplitudes obtained in this analysis. The crosshatches on the trajectories indicate increments of 100 GeV/c.

complex energy and on the physical sheet which renders it interpretable as a resonance pole. There were no other poles discovered in the representation which could be clearly identified with resonance behavior. Figure 2 presents the partial cross sections for the P_3 amplitude.

Overall, the quality of the fit to the experimental data was quite good. With no pruning of the data set, the 42-parameter solution (for the nine partial waves for which $l \leq 4$) gave $\chi^2 = 5118$ for

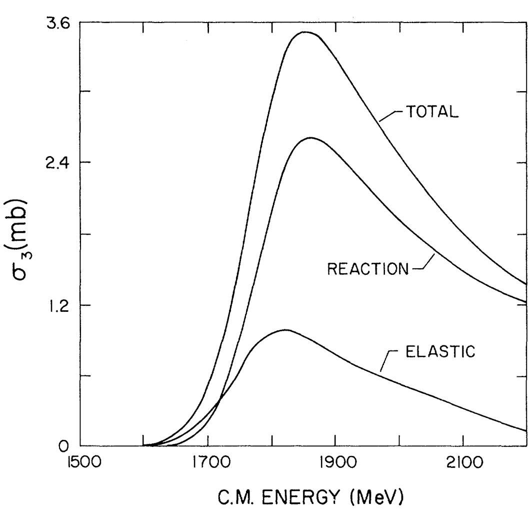

FIG. 2. The elastic, reaction, and total P_3 partial cross sections.

TABLE II. The results of pruning for various levels of χ_c, i.e., all data points whose χ^2 contribution is greater than χ_c were excluded from the analysis. At each level, the data were again searched after excluding the "bad" data points. Both χ^2/DF (χ^2 per degree of freedom) and the solution were unaffected by this search.

χ_c^2	No. of points exceeding χ_c^2	Expected No. of points exceeding χ_c^2	No. of data points	χ^2/DF
∞			3822	1.33
12	24	1.8	3803	1.26
10	39	5.4	3795	1.23
8	70	19	3762	1.16

3822 data points. In Table II, we indicate the results obtained with various levels of pruning. The quality of the fit did not deteriorate at the uppermost energies despite our simple assumptions regarding the behavior of the open channels and our complete neglect of di-pion production.

In any energy-dependent representation of scattering data, questions of form limitation must always arise. Equation (1) and the attendant assumptions represent the imposition of considerable constraints on the solution, although the quality of the fit to the data certainly supports the supposition that these are reasonable physical constraints. Nonetheless, it is quite clear that this solution depends upon the particulars of the energy dependence of our representation. In fact a number of solutions have been obtained with different parametrizations (e.g., pure Δ dominance, different parametrizations of $K_{\alpha\beta}$). However, all solutions obtained to date have two unifying features: All connect smoothly to a unique representation of the low-energy (below 600 GeV/c) data and all have a Z^* pole in the P_3 amplitude consistent with quoted results. The degree of consistency in these two respects is remarkable. To determine the extent to which the neglect of the $K^*\Delta$ channel has affected our results, we have performed an additional analysis excluding all data above 1.2 GeV/c to eliminate any possible contamination by $\pi\pi$ production. Our results and the solution were essentially unchanged. Work is currently underway to further explore the variability of our representation.

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