of ^{192}Hg a moment of inertia of about twice that of the ground-state band was necessary to reproduce the experimental energies. The success of these calculations demonstrates that decoupling is important for these negative-parity bands and is not necessarily restricted to the highest-j particles.

Calculation of the positive-parity two-quasiparticle bands are considerably more difficult because of effects associated with the inclusion of seniority-zero states. Preliminary calculations indicate a decoupled-band structure similar to that seen in the negative-parity bands. It is striking that for all the nuclei considered here, the excitation energies and apparent moments of inertia are similar for both the negative-parity decoupled bands and the positive-parity yrast bands above the backbend. This similarity comes naturally from the above decoupling picture whereas the Coriolis antipairing effect does not predict the existence of the negative-parity band.

All the above facts strongly suggest that in these nuclei the backbending is primarily due to the crossing of a decoupled, two-quasiparticle, positive-parity band and the ground-state, zero-quasiparticle band, as proposed by Stephens and Simon.[4]

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Theory of the Rainbow*

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A new theory of the rainbow is proposed and compared with the exact Mie solution. There is good agreement over a much broader range of angles and size parameters than for the Airy theory. The improvement is particularly remarkable for electric polarization. The treatment can be extended to atomic, molecular, and nuclear rainbow scattering.

The problem of high-frequency scattering by a homogeneous sphere has many important applications,[1] and it presents considerable interest in connection with optical-model, eikonal, and Regge-pole approaches to atomic, molecular, and nuclear scattering. Rainbow scattering, in particular, is a very general effect[2]; it has been observed in all these cases and it provides valuable information about the nature of the interaction.[3] For electromagnetic scattering, evaluation of the exact Mie solution[4] requires summing a number of partial waves $\gtrsim \beta = ka$ (k = wave number; a = sphere radius). Manageable and accurate approximations for $\beta \gtrsim 10^2$ are clearly desirable.

The asymptotic techniques previously developed for a scalar field[5] have been extended to the electromagnetic problem, and a detailed comparison with exact results is under way.[6] Here we report results for the rainbow region.

According to Van de Hulst,[1] Airy's classical theory of the rainbow,[7] the best approximation so far available, may be applied only for $\beta > 5000$ and $|\epsilon| \lesssim 0.5°$, where $\epsilon = \theta - \theta_R$ is the deviation from the rainbow angle θ_R ($\theta_R \approx 137.5°$ for refractive index $N = 1.33$).

Let $S_1(\beta,\theta)$ and $S_2(\beta,\theta)$ be the scattering amplitudes[1] for magnetic and electric polarization, respectively. The corresponding intensities $i_j = |S_j|^2$ ($j = 1, 2$), together with the phase difference $\delta = \arg S_1 - \arg S_2$, completely characterize the scattering.[1]

As in the scalar treatment,[5] a modified Watson transformation is applied to each term in the Debye multiple internal reflection expansion of the Mie series: $S_j = S_j^{(0)} + S_j^{(1)} + S_j^{(2)} + \dots$. The (primary) rainbow occurs in the third Debye term $S_j^{(2)}$, associated with rays that undergo a single internal reflection. Classifying different scattering-angle regions according to the number of geometrical rays emerging in the same direction, it corresponds, in this term, to a transition between a two-ray region and a zero-ray (shadow) region. In the complex angular momentum plane, this is reflected in the confluence of a pair of real saddle points[8] and their becoming complex. When the ranges of two saddle points overlap, the ordinary saddle-point method can no longer be applied. A uniform asymptotic expansion is obtained by the Chester-Friedman-Ursell (CFU) method.[9]

In the rainbow region, the dominant contribution to $S_j^{(2)}$ is given by

$$S_j^{(2)}(\beta,\theta) = -e^{i\pi/4} N(\pi \sin\theta)^{-1/2}\kappa^{3/2} F_j(\beta,\theta), \tag{1}$$

$$F_j(\beta,\theta) = \int g_j(w_1)\exp[\kappa f(w_1,\theta)]\,dw_1, \tag{2}$$

$$f(w_1,\theta) = i\{2N\cos w_2 - \cos w_1 + [2w_2 - w_1 - \tfrac{1}{2}(\pi-\theta)]\sin w_1\}, \tag{3}$$

$$g_j(w_1) = (\sin w_1)^{1/2} m \cos w_2 \cos^2 w_1 (\cos w_1 - mN\cos w_2)(\cos w_1 + mN\cos w_2)^{-3}, \tag{4}$$

where $\kappa = 2\beta$, $m = 1$ for $j = 1$, $m = N^{-2}$ for $j = 2$, and $\beta \sin w_1 = N\beta \sin w_2 = \lambda$ is the complex angular momentum. The two real saddle points in the two-ray region, $w_1 = \theta_1'$ and $w_1 = \theta_1''$, correspond to the two angles of incidence associated with geometrical rays emerging in the direction θ. The path of integration in (2) goes through these points and is the same as in the scalar case [see Ref. 5, Pt. II, Eq. (4.3)].

The CFU method leads to

$$F_j(\beta,\theta) = 2\pi i \kappa^{-1/3}\exp[\kappa A(\epsilon)]\{[p_{0j}(\epsilon) - \kappa^{-1}(q_{1j}(\epsilon) + 2\zeta(\epsilon) q_{2j}(\epsilon)) + O(\kappa^{-2})]\mathrm{Ai}(\kappa^{2/3}\zeta(\epsilon)) - \kappa^{-1/3}[q_{0j}(\epsilon) - 2\kappa^{-1}p_{2j}(\epsilon) + O(\kappa^{-2})]\mathrm{Ai}'(\kappa^{2/3}\zeta(\epsilon))\}, \tag{5}$$

where, as in the scalar case,[5]

$$\left\{\begin{matrix} A(\epsilon) \\ \tfrac{2}{3}[\zeta(\epsilon)]^{3/2} \end{matrix}\right\} = i[N(\cos\theta_2' \pm \cos\theta_2'') - \tfrac{1}{2}(\cos\theta_1' \pm \cos\theta_1'')], \tag{6}$$

correspond to half the sum and half the difference of the optical paths through the sphere, respectively. The coefficients $p_{ij}(\epsilon)$, $q_{ij}(\epsilon)$ are determined by the CFU method[9] in terms of the exactly known[5] saddle points θ_1', θ_1''. They are related, through g_j, to the Fresnel reflection and transmission coefficients for the corresponding angle of incidence and their derivatives with respect to this angle.

TABLE I. CFU coefficients[a] for $N = 1.33$, $|\epsilon| \ll \kappa^{-1/3}$.

Coefficient	$j=1$	$j=2$
$p_{0j}(\epsilon)$	$i[0.0381 - 0.031\epsilon - 0.19\epsilon^2]$	$i[0.00786 + 0.046\epsilon - 0.078\epsilon^2]$
$q_{0j}(\epsilon)$	$0.0227 - 0.15\epsilon - 0.59\epsilon^2$	$0.108 - 0.015\epsilon - 0.43\epsilon^2$
$q_{1j}(\epsilon)$	$0.40 + 3.0\epsilon$	$0.042 + 2.3\epsilon$
$p_{2j}(\epsilon)$	$-1.4i$	$-0.64i$
$q_{2j}(\epsilon)$	-4.1	-3.1

[a]In each entry, the error term is one order higher in ϵ than the last term retained.

For $|\kappa^{2/3}\zeta| \gg 1$, the above result, in contrast with Airy's theory, matches smoothly with those in the neighboring angular regions. For $|\epsilon| \ll \kappa^{-1/3}$, one can employ power series expansions of the coefficients in (5). For $N = 1.33$, one finds that $A(\epsilon) = i[1.519 + 0.431\epsilon - 0.115\epsilon^2 + O(\epsilon^3)]$, $\zeta(\epsilon) = -0.369\epsilon - 0.0745\epsilon^2 + O(\epsilon^3)$, and $p_{ij}(\epsilon)$, $q_{ij}(\epsilon)$ are given by the expressions in Table I.

The small value of $|p_{02}| \approx |p_{01}|/5$ arises from

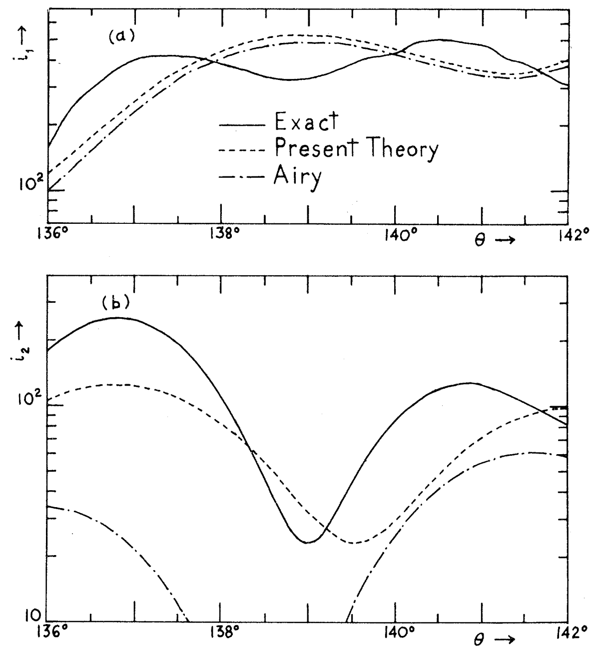

FIG. 1. Polarized intensities for $\beta = 50$: (a) i_1; (b) i_2.

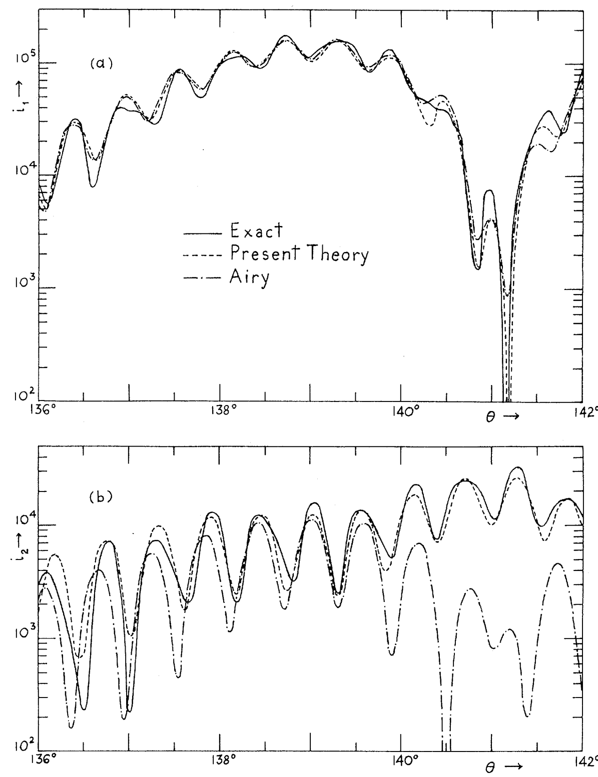

FIG. 2. Same as Fig. 1, for $\beta = 500$.

the angle of incidence for rainbow rays being close to Brewster's angle, which tends to suppress polarization 2. Airy's approximation is obtained by neglecting terms of order ϵ^2 and higher in $A(\epsilon)$ and $\zeta(\epsilon)$, taking $p_{0j}(\epsilon) = p_{0j}(0)$, and setting all other $p_{ij}(\epsilon)$, $q_{ij}(\epsilon)$ equal to zero.

The main contribution to S_j in the rainbow region, besides (1), is due to the first Debye term $S_j^{(0)}(\beta, \theta)$, that represents direct reflection at the surface. This term, including corrections of order β^{-1}, has also been computed.[6]

We have compared the exact Mie results for $N = 1.33$, $50 \leq \beta \leq 1500$, $136° \leq \theta \leq 142°$ with those obtained from (1) to (5) and with the Airy approximation, including the direct reflection term in both cases. The exact values [Eq. (6)] of $A(\epsilon)$, $\zeta(\epsilon)$ and[5,9] of the coefficients $p_{ij}(\epsilon)$, $q_{ij}(\epsilon)$, rather than their power series expansions, were employed.

The CFU expansion is rapidly convergent in the above range (cf. Table I). The main correction to the Airy theory arises from the $\mathrm{Ai}'(-x)$ term ($x = -\kappa^{2/3}\zeta$) in (5). The present theory leads to the following predictions:

(i) For polarization 1, the corrections to the Airy theory are small within the main rainbow peak ($|x| \lesssim 1$), but they become appreciable for the secondary peaks (supernumerary arcs) ($x \gg 1$). (ii) For polarization 2, the $\mathrm{Ai}'(-x)$ term is dominant in the whole range ($q_{02} \gg |p_{02}|$), giving large corrections to the Airy theory. In particular, secondary-peak maxima and minima should be interchanged for the two polarizations; this inversion has been observed at large angles,[1] where it arises from the change in sign of the reflection amplitude at Brewster's angle.

These predictions are entirely confirmed by the numerical comparisons.[6] Typical representative results are reproduced[10] in Figs. 1 to 5. The improvement over the Airy theory for polarization 2 is apparent even at $\beta = 50$ (Fig. 1), although at this low β the validity of the asymptotic approximations is being strained.

For $\beta = 500$ (Fig. 2), the main peak and part of the first secondary are covered. The superimposed oscillations with period $\Delta\epsilon \approx 300/\beta$ (in degrees) are due to interference with the direct reflection term. The out-of-phase character of the Airy approximation for polarization 2 is already noticeable.

Interference with direct reflection remains appreciable at $\beta = 1500$, even close to θ_R; to avoid the corresponding rapid oscillations, we have subtracted out the direct reflection term, plotting $|S_j - S_j^{(0)}|^2$ in Fig. 3. Several secondary peaks are covered, and the validity of predictions

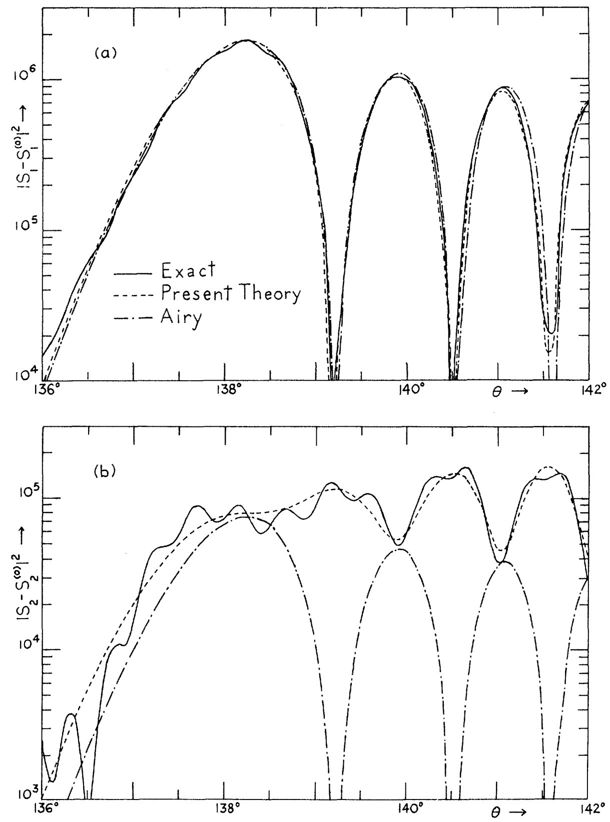

FIG. 3. Intensities after subtracting out direct reflection term, for $\beta = 1500$: (a) $|S_1 - S_1^{(0)}|^2$; (b) $|S_2 - S_2^{(0)}|^2$.

(i) and (ii) is readily apparent.

The independent quantity δ is plotted in Fig. 4 for $\beta = 1500$. Here the Airy theory fails even close to θ_R, while the present theory agrees with the exact solution remarkably well throughout. The rapid oscillations again arise from interference with direct reflection; large phase variations occur close to intensity minima.

A different measure of the overall agreement as a function of β is provided by the fractional contributions[11] from the domain (θ_1, θ_2) (here $\theta_1 = 136°$, $\theta_2 = 142°$) to the asymptotic total cross section $2\pi a^2$,

$$f_j(\beta) = \Delta\sigma_j / 2\pi a^2 = \beta^{-2} \int_{\theta_1}^{\theta_2} i_j(\beta, \theta) \sin\theta \, d\theta, \qquad (7)$$

which approach a constant for large β. These quantities are plotted in Fig. 5 for $30 \leq \beta \leq 1000$.

The oscillatory character of the deviations[12] between the present approximation and the exact solution is consistent with interpreting them as "ripple." This effect, which is present in all directions and becomes dominant in the glory, is due to incident rays near the edge of the sphere, and it is associated with Regge-pole-type contributions (surface waves) and higher-order Debye terms.[5] Thus, we believe that pure rainbow effects are completely accounted for by the present theory. A detailed discussion will be given else-

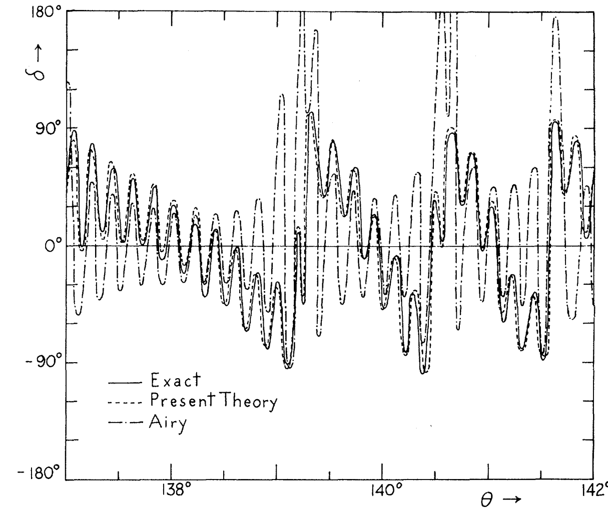

FIG. 4. Phase difference δ for $\beta = 1500$.

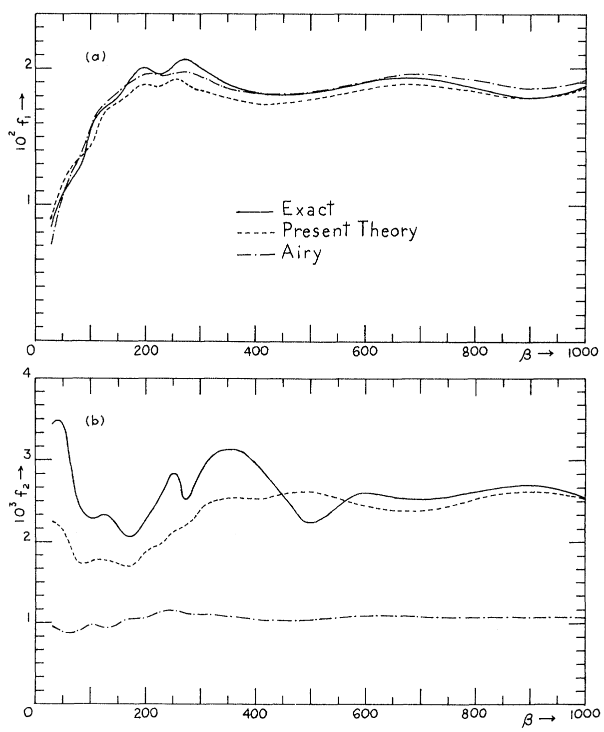

FIG. 5. Fractional contributions f_j to the total cross section [see Eq. (7)]: (a) $10^2 f_1$; (b) $10^3 f_2$.

where.[6]

Rainbow scattering in general must also be associated with a collision between two saddle points in the complex angular momentum plane, and it should therefore be expected that the CFU method also leads to improved results[13] in more general cases.

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[10]Although great care has been taken to choose computed points at sufficiently small intervals, rapidly varying portions of the curves may be subject to slight interpolation errors; however, the conclusions derived from comparison among the curves would not be affected.

[11]Actually, it is the sum f_1+f_2 that represents the corresponding contribution to the efficiency factor (see Ref. 1).

[12]In Figs. 1 to 3 and in Fig. 5, these deviations are more noticeable in polarization 2, because of its lower intensity.

[13]After submission of this work, we came across the paper by M. V. Berry, Proc. Phys. Soc., London 89, 479 (1966), where the CFU method was applied to rainbow scattering by a Lennard-Jones potential, showing substantial improvement in one numerical example.

Search for Orthopositronium Decay into Four Photons as a Test of Charge-Conjugation Invariance*

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We report the preliminary results of an experiment designed to search for the C-nonconserving decay of ground-state orthopositronium (1^3S_1) into four photons. In terms of the branching ratio $F_T^{4\gamma}=R_T^{4\gamma}(^3S_1\rightarrow 4\gamma)/R_{3\gamma}(^3S_1\rightarrow 3\gamma)$, where $R_T^{4\gamma}(^3S_1\rightarrow 4\gamma)$ is the C-nonconserving four-photon decay rate and $R_{3\gamma}(^3S_1\rightarrow 3\gamma)$ is the C-conserving three-photon decay rate of free orthopositronium in our sample, we find $F_T^{4\gamma}<8\times10^{-6}$ (68% confidence).

We report the preliminary results of a search for the C-nonconserving decay of orthopositronium (o-Ps) 1^3S_1 into four γ rays. This annihilation mode is forbidden by C invariance only.[1] Our experimental results will be discussed in terms of an upper limit on the branching ratio $F_T^{4\gamma}=R_T^{4\gamma}/R_{3\gamma}$ of the C-nonconserving 4γ decay to the C-conserving 3γ decay of o-Ps.

The branching ratio may be related to a C-nonconserving interaction Hamiltonian

$$H_i=(\lambda/m_e^8)e^4\partial_\alpha\rho_\beta F_{\alpha\delta}F_{\delta\beta}F_{\mu\nu}F_{\mu\nu} \tag{1}$$

developed by Mani and Rich.[2] This is the simplest C-nonconserving Hamiltonian that could allow the decay $^3S_1\rightarrow 4\gamma$. Here λ is the coupling constant on which we set new limits. The other symbols have their usual meaning.

Evaluating the decay rate from the above interaction we find

$$R_T^{4\gamma}=1/\tau_T^{4\gamma}=88\lambda^2\ \mathrm{sec}^{-1}, \tag{2}$$

where $R_T^{4\gamma}$ is the C-nonconserving decay rate into four λ's. The branching ratio is

$$F_T^{4\gamma}=R_T^{4\gamma}/R_{3\gamma}=1.2\times10^{-5}\lambda^2, \tag{3}$$

where we have used $R_{3\gamma}=7.25\times10^6$.[3]

Previous experiments on C nonconservation in Ps have searched for the C-nonconserving 3γ decay of the 1^1S_0 states (parapositronium). The most recent of these[4] compared the threefold coincidence count rate from Ps decay into different angular configurations. The count rates are primarily due to the decay $^3S_1\rightarrow 3\gamma$, with the process $^1S_0\rightarrow 3\gamma$ causing a small perturbation in the