Focal-Length Dependence of Air Breakdown by a 20-psec Laser Pulse: Theoretical Interpretation through the Effective-Photon Concept

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It is shown that the use of the concept of effective photons offers a simple explanation of some recent experimental results of air breakdown by a 20-psec laser pulse, in which a quadratic dependence of breakdown laser power on lens focal length has been found. It is also demonstrated that the use of the same concept yields a different focal-length dependence in the case of gas breakdown with rectangular and Gaussian spatial beam profiles.

In a recent paper,[1] Ireland, Yi, Aaron, and Grey Morgan have clearly demonstrated that the use of large-diameter laser beams combined with lenses of short focal length introduces serious errors in the determination of the form of the focal-length dependence of laser-induced gas breakdown, because of the effects of primary spherical aberration. In addition to this, an important result which has emerged from their study is that, when spherical-aberration effects are greatly reduced by the use of small-diameter beams and long-focal-length lenses, the power P of a laser pulse with rectangular spatial profile necessary to effect gas breakdown has a quadratic dependence on the focal length f of the lens used in the experiment. Knowing that $I=4P/\pi d^2 =4P/\pi f^2\theta^2$, where I and θ are, respectively, the intensity and divergence of the laser beam and d is the focal-spot diameter, it follows from this study that the breakdown laser intensity is independent of lens focal length. This result seems to be at variance with that obtained by other investigators[2] who used Gaussian rather than rectangular spatial beam profiles and found that the breakdown laser intensity was rather strongly dependent on lens focal length.

In this Letter I attempt to show that these results do not contradict one another and that both are theoretically borne out in a simple way if the analysis of the breakdown process is done through the concept of effective photons. Recall that this concept is at the base of effective-photon theory[3,4] which is so named after the fact that a fundamental modification is introduced to the classical photon concept. In the theory the light quantum is no longer considered as a particle whose energy ϵ remains unchanged from the moment it escapes from an emitting body to the time it is absorbed, but it is assumed *ad hoc* that it can suffer a variation of energy during the course of its life. The variation depends on the density of the photons that surround the light particle, according to a relation of the form $\epsilon=h\nu/[1-\beta_\nu f(I)]$ [h is Planck's constant, ν is the light frequency, β_ν is a positive coefficient, and $f(I)$ is a function of light intensity]. The nonlinear correction term $1-\beta_\nu f(I)$ is then deemed to be equal to 1 except at very high light intensities as found in focused laser beams.

In Refs. 3 and 4, where the theory is introduced and justified, the full consequences of the above assumptions are worked out in detail in connection with the analysis of phenomena of gas ionization and breakdown by laser beams. In particular, it is affirmed that the ionization process has to follow a simple photoelectric-emission mechanism. In fact, it is said, if the photon energy goes up with light intensity I, when this has reached a sufficiently high value I_m, the photon energy attains the ionization potential W of the gas investigated. All other photons that follow in a rising-intensity laser pulse are then sufficiently energetic to ionize the gas atoms (the photons are now called "effective") and the process of ionization is a single-photon mechanism.

I would like now to apply these ideas to deduce a relation between the threshold intensity $I_{\rm th}$ of a laser pulse necessary to effect gas breakdown and the parameters of gas and radiation at breakdown.

Consider a triangular laser pulse as represented in Fig. 1 in which the intensity I_0 across any section of the beam has a uniform spatial profile, in agreement with the information supplied by Ireland *et al.*[1] concerning their experiment. The photon energy along the beam will be a function of I_0 and, when $I_0 \geqslant I_m$, where I_m is the intensity for which $\epsilon=W$, all photons will be effective and have energy sufficient to ionize the gas atoms. The number of effective photons N_p crossing unit

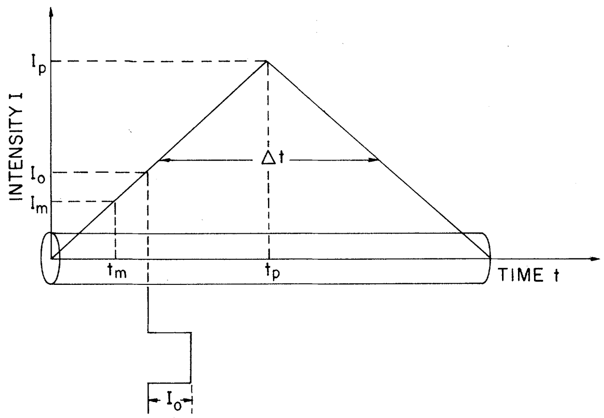

FIG. 1. Spatio-temporal profile of the laser pulse used in the experiment under analysis. I_m is the light intensity at which the photon energy $\epsilon = W$, the ionization potential of the gas investigated. t_m and t_p are, respectively, the times at which $I=I_m$ and $I=I_p$, the laser peak intensity.

area per unit time will be a function of I_0 according to the relation[5]

$$N_p(I_0) = I_0/h\nu. \tag{1}$$

The interaction of these effective photons with the gas atoms yields the following number of ions:

$$dN_i = K(N_a - N_i)N_p\,dt = (K/h\nu)(N_a - N_i)I_0\,dt, \tag{2}$$

where K is a constant of proportionality, N_i is the number density of ions already present in the gas, and N_a is the initial number density of atoms in the interaction volume. If I_p is the peak intensity of the triangular laser pulse and Δt is the half-width of the pulse in time, then $I_0 = (I_p/\Delta t)t$. Straightforward integration of (2) from $t = t_m$ to $t = t_p = \Delta t$ yields

$$\begin{aligned} \ln\left(1 - \frac{N_i}{N_a}\right) &= -\frac{K\Delta t}{2h\nu} I_p\left(1 - \frac{t_m^{\,2}}{\Delta t^2}\right) \\ &= -\frac{K\Delta t}{2h\nu} I_p\left(1 - \frac{I_m^{\,2}}{I_p^{\,2}}\right), \end{aligned} \tag{3}$$

where t_m is the time for which $I_0 = I_m$ and t_p is the time at which $I_0 = I_p$.[6]

It is generally agreed that gas breakdown induced by a laser occurs when a large constant ion density N_b has been reached in the interaction volume. The breakdown threshold intensity I_{th} is obviously much larger than I_m and the term $I_m^{\,2}/I_p^{\,2}$ in (3) can be neglected. Consequently, at breakdown

$$\ln[N_a/(N_a - N_b)] = (K\Delta t/2h\nu)I_{\text{th}}, \tag{4}$$

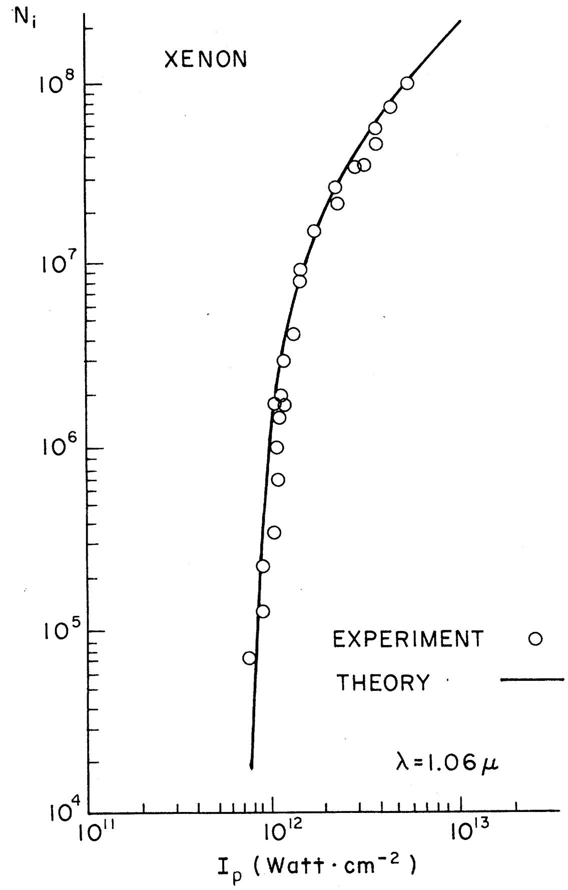

FIG. 2. Variation of total number of ions N_i as a function of laser intensity I_p.

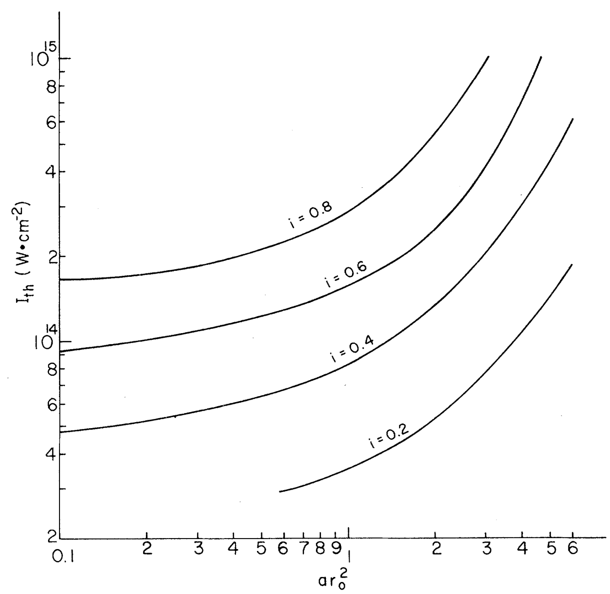

FIG. 3. Laser breakdown threshold intensity versus $ar_0^{\,2}$. The parameter $i = \overline{N}_b/N_a$ ($\overline{N}_b$ is the average breakdown ion density, N_a is the gas density) represents the degree of ionization required to obtain breakdown.

from which

$$I_{\text{th}} = (2h\nu/K\Delta t)\ln[N_a/(N_a - N_b)]. \tag{5}$$

Equation (5) affirms that in the case under analysis, breakdown with a rectangular spatial beam profile, the parameters that affect the breakdown threshold are gas density N_a, laser frequency ν, and pulse duration Δt, the focal length f being excluded, in agreement with the finding by Ireland *et al.*[1] that the threshold intensity is independent of lens focal length.

In the case of a laser pulse with a Gaussian spatial intensity profile $I = I_0\exp(-ar^2)$, the ion density in the interaction region is a function of r as well as t, and breakdown has to be redefined as the attainment of a final *average* ion density $\overline{N}_b$ within a small area of radius r_0. The analysis now proceeds as follows. First, the number of effective photons N_p crossing in unit time a unit area of a section of the beam where the intensity is I_0 ($\geq I_m$) is given by

$$N_p(r) = I_0\exp(-ar^2)/h\nu, \tag{6}$$

and their interaction with the gas atoms yields the following number of ions:

$$dN_i(r,t) = K[N_a - N_i(r,t)][I_0\exp(-ar^2)/h\nu]\,dt.$$

Integrating as before we get

$$\ln[1 - N_i(r)/N_a] = -(K/2h\nu)\exp(-ar^2)I_p\Delta t[1 - t^2(r)/\Delta t^2]. \tag{7}$$

The effective photons are located in circles of radius $r(t)$ given by[3,4]

$$r(t) = [a^{-1}\ln(I_0/I_m)]^{1/2} = [a^{-1}\ln(I_p t/\Delta t I_m)]^{1/2}, \tag{8}$$

from which

$$t(r) = (\Delta t I_m/I_p)\exp(ar^2). \tag{9}$$

Inserting (9) into (7) yields

$$\ln[1 - N_i(r)/N_a] = -(K/2h\nu)\exp(-ar^2)I_p\Delta t M_p, \tag{10}$$

where $M_p = 1 - (I_m^2/I_p^2)\exp(2ar^2)$. Hence

$$N_i(r) = N_a\{1 - \exp[-K_1 I_p M_p\exp(-ar^2)]\}, \tag{11}$$

where $K_1 = K\Delta t/2h\nu$. Expression (11) gives the final radial ion distribution within the circle of radius $r_p = [a^{-1}\ln(I_p/I_m)]^{1/2}$ occupied by the effective photons as a function of the laser peak intensity I_p. The breakdown ion density $\overline{N}_b$ is obtained by averaging the values of $N_i(r)$ within a small circle of radius r_0:

$$\overline{N}_b = (2/r_0^2)\int_0^{r_0} N_i(r)r\,dr = (2N_a/r_0^2)\int_0^{r_0}\{1 - \exp[-K_1 I_{\text{th}}M_{\text{th}}\exp(-ar^2)]\}r\,dr, \tag{12}$$

where I_{th} is the breakdown threshold laser intensity and

$$M_{\text{th}} = 1 - (I_m^2/I_{\text{th}}^2)\exp(2ar^2).$$

The Gaussian factor a appearing in (12) is expressed explicitly as

$$a = 4(\ln 2)/d^2 = 4(\ln 2)/f^2\theta^2, \tag{13}$$

as one easily deduces from the knowledge that, by definition, the diameter d of the focal spot is measured between points where the laser intensity drops to $\frac{1}{2}$ the value at $r = 0$. Hence, (12) provides an implicit relation between threshold laser intensity I_{th} and lens focal length f. We can solve (12) numerically, but have first to find the values of the constants K_1 and I_m. Clearly, they depend on the gas investigated and the wavelength of the light used. Fixing our attention on a gas, for instance xenon, for which the value of I_m is deduced from the experimental work of Agostini *et al.*[7] ($I_m = 7\times10^{11}$ W cm^{-2} at $\lambda = 1.06$ μm), we assume as the value of K the one that causes the whole set of experimental results of these authors to be correctly represented through the integral of (11):

$$N_i(\text{total}) = 2\pi N_a\int_0^{r_p}\{1 - \exp[-K_1 I_p M_p\exp(-ar^2)]\}r\,dr. \tag{14}$$

Figure 2 shows that the experimental results of Agostini *et al.*[7] for xenon are well represented by (14) when it is assumed that $K_1 = 10^{-14}$, the other parameters being $N_a = 6.06\times10^{13}$ and $a = 7.07\times10^4$ as provided by the work of these authors.

It is now a simple matter to proceed with the calculation of (12). However, in order to leave the value of r_0 undetermined, being satisfied with the knowledge that r_0 is a constant, we make a change of variable $x = ar^2$ in Eq. (12), which transforms into

$$ar_0^2(1-\overline{N}_b/N_a)=\int_0^{ar_0^2}\exp\{-K_1I_{\rm th}[1-(I_m^2/I_{\rm th}^2)\exp(2x)]\exp(-x)\}dx. \tag{15}$$

Equation (15) has been solved numerically, after insertion of various possible values of the breakdown degree of ionization $i=\overline{N}_b/N_a$, and plotted in Fig. 3. From the figure it can readily be seen that the breakdown threshold intensity $I_{\rm th}$, in the case of a Gaussian beam spatial profile, does depend on a, hence on the focal length f of the lens used in the experiment. Qualitatively, the trend of the curves is correct, in that it agrees with the results of experiments in which the threshold laser intensity increases with a. Only for small values of a (hence large f), does the dependence become weak. This result agrees with the concept that a beam with a small Gaussian factor a, having a spatial intensity profile almost constant within a large portion of its cross section, has to produce almost the same ionization effect as a beam with a rectangular spatial profile.

In conclusion, I am indebted to Dr. P. Savic and Dr. J. Lau for clarification on some mathematical points of the paper.

[1]C. L. M. Ireland, A. Yi, J. M. Aaron, and C. Grey Morgan, Appl. Phys. Lett. 24, 175 (1974).

[2]See for instance, A. F. Haught, R. G. Meyerand, and D. C. Smith, in *Physics of Quantum Electronics*, edited by P. L. Kelly, B. Lax, and P. E. Tannenwald (McGraw-Hill, New York, 1966), pp. 509–519.

[3]E. Panarella, in *Proceedings of the Eleventh International Conference on Phenomena in Ionized Gases, Prague, Czechoslovakia, 1973,* edited by I. Stoll (Czechoslovakian Academy of Sciences, Prague, Czechoslovakia, 1973), p. 256.

[4]E. Panarella, Found. Phys. 4, 227 (1974).

[5]The meaning of "light intensity" I_0 in the context of this paper is the conventional one. It is the energy flux as deduced from measurements done with a photosensitive device on a part of the laser pulse (in which $\epsilon = h\nu$) extracted from the main pulse with a calibrated beam splitter. This definition allows direct proportionality to exist between I_0 and the photon flux N_p in the main beam.

[6]The integration has been performed between t_m and t_p because breakdown usually occurs at the peak of the laser intensity.

[7]P. Agostini, G. Barjot, G. Mainfray, C. Manus, and T. Thebault, IEEE J. Quantum Electron. 6, 782 (1970).

Effect of Plasma Inhomogeneity on the Production of Energetic Electrons

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Intense radiation impinging on a plasma near the critical density may be absorbed by exciting electrostatic plasma waves. In inhomogeneous plasmas these oscillations convect out of the interaction region to lower densities where the waves have lower phase velocities. Electrons whose velocity is not much greater than the thermal velocity then absorb the wave. In this process, a very few highly energetic electrons are produced. We present theory and simulation for the absorption of these waves in an inhomogeneous plasma.

A problem of great concern in laser-fusion studies is the effect of high-energy electrons streaming into the core of the target and preheating it. As a result the target material might not be compressed sufficiently to fuse. High-energy, nonthermal electrons resulting from parametric instabilities have, in fact, been observed in experiment[1] and in homogeneous computer simulations.[2-4] In this Letter we present a theory and simulations of the production of energetic electrons in an inhomogeneous plasma and calculate the particle flux streaming into the core. The basic physical process occurring in the inhomogeneous plasma is quite different from that which occurs in the homogeneous plasma. The analysis is based upon the fact that electro-