Hexadecapole Deformations in W and Os Nuclei from Perturbed Rotational Band Structure*

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The spectroscopic data for even-parity rotational and intrinsic states in ^{181}W and ^{187}Os are shown to be incompatible with very large static nuclear hexadecapole deformations ($\epsilon_4 \cong 0.2$) implied by conventional analysis of recent Coulomb excitation measurements of $E4$ transition moments in ^{182}W and ^{186,188}Os.

In recent months, at least two groups have reported measuring very large electric hexadecapole transition moments for nuclei in the W-Os region.[1,2] These results are based on Coulomb excitation measurements carried out with α particles. For the case of ^{182}W, the Oak Ridge National Laboratory data[1] imply static nuclear hexadecapole deformations that appear to be in serious disagreement with the nuclear charge distribution derived from the muonic x-ray data of Davidson *et al.*,[3] and in substantial disagreement with the nuclear inelastic scattering result of Hendrie.[4] The data for the osmium isotopes 186, 188, and 190 are presently only from sub-Coulomb measurements, but large $E4$ moments appear to be experimentally established in this region as well.[2] While it is possible that the discrepancy between Coulomb and nuclear measurements of the nuclear shape may be due to a real difference in the proton and neutron mass distribution, the discrepancy between Coulomb excitation and muonic x-ray data for ^{182}W is not so easily dispatched.

The implication of large values of the nuclear hexadecapole deformation for interpreting (d, t) and $(^3\mathrm{He}, \alpha)$ transfer-reaction cross sections for populating states in ^{181}W and ^{183}W has been dealt with by Casten in a recent Letter.[5] Casten finds that the unusual $l = 6$ cross-section patterns can be explained by a wide range of ϵ_4 values, and he concludes that the data are not inconsistent with hexadecapole deformations as large as $\epsilon_4 = 0.16$, well within the limits of error ($\beta_4 = -0.19 \pm 0.06$) quoted by Bemis *et al.*[1] for ^{182}W. (In this region, $\epsilon_4 \cong -\beta_4$.) However, reaction strengths into rotational bands based on the various Ω states of the relatively pure $i_{13/2}$ Nilsson manifold are dominated by transfer to spin-$\frac{13}{2}$ band members. Since other members of the bands are populated very weakly or not at all, it is not possible to set definite limits on the hexadecapole deformation from the $l = 6$ transfer data alone, because the projection quantum numbers Ω cannot be assigned.

In addition to the transfer-reaction data, there has also recently accumulated a substantial quantity of data on the so-called parity-unique $i_{13/2}$ rotational band structure in the odd-A W and Os isotopes. In this note, we show that the experimental data on even-parity rotational and intrinsic states in ^{181}W and ^{187}Os are consistent only with the smaller hexadecapole deformations ($\epsilon_4 \cong 0.06$) predicted by Nilsson *et al.*[6] and deduced by Hendrie[4] ($\epsilon_2 \cong 0.23$, $\epsilon_4 \cong 0.08$) from nuclear inelastic scattering on ^{182}W.

The primary data for this consideration are those derived from $(\alpha, 3n\gamma)$ reactions on targets of ^{180}Hf and ^{186}W. Both targets in these experiments were self-supporting metal foils prepared at the Niels Bohr Institute.[7] Details of these experiments will be published elsewhere. For the purposes of this Letter, we show in Fig. 1 the even-parity states presumed to be associated with the $i_{13/2}$ intrinsic structure in ^{181}W and ^{187}Os. Even-parity levels known in addition to those identified in the $(\alpha, 3n\gamma)$ experiments are also shown. The levels shown for ^{181}W are in agreement with similar results recently published by Lindblad, Ryde, and Kleinheinz.[12]

The procedure used to fit rotational band structure in deformed odd-A nuclei is by now standard and reasonably well understood. The core moment of inertia is assumed to be approximately the average of the neighboring even-even nuclei, and a variable moment of inertia may be introduced by including the second-order B term in the rotational energy expression.[13]

In our calculations, the diagonal quasiparticle energies are determined by selecting appropriate values for the Fermi energy λ, and for the gap parameter Δ, based on the best empirical data available for single-particle states and odd-even mass differences.[6,14] The enigmatic, but well-documented, reduction factors required for the off-diagonal matrix elements near the Fermi sur-

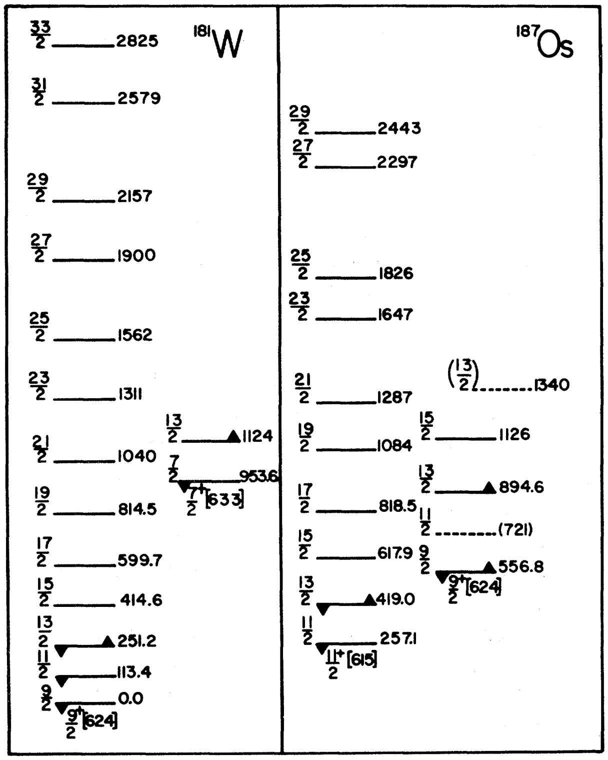

FIG. 1. Experimental even-parity states in ^{181}W and ^{187}Os known from $(\alpha, 3n\gamma)$. Inverted triangles, β-decay (Refs. 8 and 9); and triangles, transfer-reaction (Refs. 10 and 11) data.

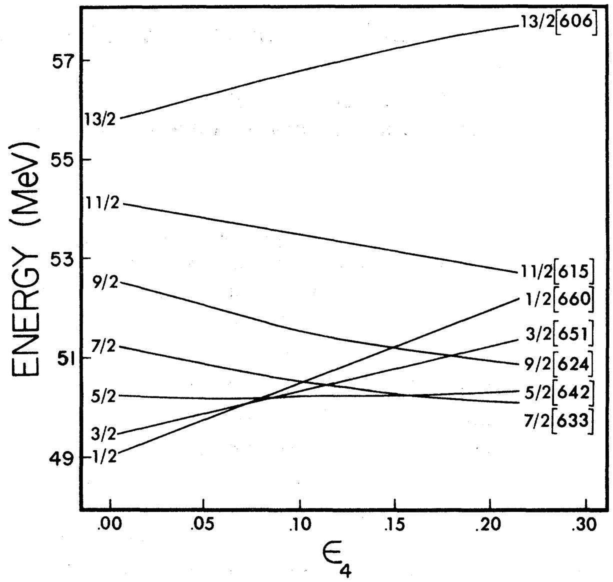

FIG. 2. The $i_{13/2}$ Nilsson orbitals as a function of hexadecapole deformation, ϵ_4. Parameters ϵ_2, μ, and κ are for ^{182}W (Ref. 6).

face occur in addition to the less significant $(U_\Omega U_{\Omega+1} + V_\Omega V_{\Omega+1})$ pairing reduction factor. The average-field single-particle energies are calculated with the code of Nilsson *et al.*[6] For orientation, Fig. 2 shows the Nilsson single-particle level energies for the $i_{13/2}$ manifold as a function of increasing positive hexadecapole deformation, ϵ_4.

The rotational and intrinsic levels based on the predominantly $i_{13/2}$ Nilsson states have been fitted in this work for three different sets of parameters: (1) the ϵ_2 and ϵ_4 values suggested by the calculations of Nilsson *et al.*[6]; (2) deformations which correspond to those implied for ^{182}W and ^{186,188}Os by conventional analysis of $E4$ moments obtained from Coulomb excitation data[1,2]; and (3) deformations which correspond approximately to the lower limit of the errors on ϵ_4 implied by the $E4$ moment analysis for ^{182}W and ^{186,188}Os. In the case of ^{182}W, these last numbers happen also to correspond rather closely to those recently suggested by Saladin[15] from a coupled-channels analysis of the Oak Ridge National Laboratory data for the Coulomb-nuclear interference region. Saladin suggests $\beta_2 = 0.27$, $\beta_4 = -0.11 \pm 0.02$ for ^{182}W and somewhat smaller hexadecapole deformations ($\beta_4 \approx -0.09$) for ^{184}W and ^{186}W, the latter two based on measurements by Lee *et al.*[15] We did not attempt fits with small values of ϵ_4, though we have shown in another paper[16] that reasonable fits to the even-parity levels in ^{179}W are possible for ϵ_4 approaching zero.

The results of the band-fitting calculations for ^{181}W and ^{187}Os are summarized in Table I. It is clear that the experimental level energies are easily explained if one assumes an input quasiparticle spectrum that corresponds to $\epsilon_4 \cong 0.06$. The only allowed deviation from the theoretical quasiparticle energies was for the state nearest the Fermi surface. This input energy was allowed to vary somewhat, a procedure which simulates small variations in the Fermi energy. The results shown represent the best of those obtained for all reasonable combinations of constrained and unconstrained parameter sets. In principle, the inability of the minimization routine to fit the data with the maximum number of free parameters would also indicate failure with a more constrained parameter set. For example, *minor* uncertainties in the input quasiparticle spectrum, uncertainties well within the accuracy that one should expect from Nilsson calculations, can easily be dealt with by varying the off-diagonal inter-

TABLE I. Energy fits to even-parity states in ^{181}W and ^{187}Os.

	Deformation ϵ_2	ϵ_4	$\sum(\Delta E)^2$ (keV^2) [a]	ΔE_{qp}^{lowest} [b] (keV)	$\hbar^2/2\mathcal{J}$ (keV)	B (eV)	$[\Omega\times(\Omega+1)]$ off-diagonal attenuation factors $\frac{1}{2}\times\frac{3}{2}$	$\frac{3}{2}\times\frac{5}{2}$	$\frac{5}{2}\times\frac{7}{2}$	$\frac{7}{2}\times\frac{9}{2}$	$\frac{9}{2}\times\frac{11}{2}$	$\frac{11}{2}\times\frac{13}{2}$
^{181}W	0.24	0.055	0.34	− 209	17.2	− 5.0	0.97	1.04	0.99	0.84	0.75	0.97
	0.24	0.11	2×10^3	− 640	25.0	− 21.0	− 0.23	1.0	1.36	1.01	0.073	1.00
	0.23	0.18	1×10^4	− 1290	26.5	− 27.0	− 0.21	0.99	1.49	1.08	− 0.31	0.60
^{187}Os	0.185	0.04	24	− 58	22.8	− 12.6	0.91	1.07	1.15	0.80	0.66	0.58
	0.19	0.10	2×10^3	+ 235	28.3	− 50.0	− 0.08	1.05	0.95	1.03	0.87	1.06
	0.18	0.20	3×10^3	− 520	32.9	− 67.0	0.09	1.11	0.99	1.22	1.29	0.67

[a] Sum of the squares of deviations between experimental and calculated level energies.

[b] Adjustment of input energy for quasiparticle nearest the Fermi surface. In the ^{187}Os case, the $\frac{27}{2}$ and $\frac{29}{2}$ members of the $\frac{11}{2}^+[615]$ band and the tentative $\frac{11}{2}^+\frac{9}{2}[624]$ (721 keV) and $\frac{13}{2}^+$ (1340 keV) states were not included in the fit. The latter state was calculated to be at 1300 keV, however. Somewhat less satisfactory fits were also obtained for $\epsilon_4=0.06$.

action (cf. the ^{187}Os fit). Even then, it is impossible to fit the experimental data with any reasonable parameter set if $\epsilon_4 \geqslant 0.08$.

The reason for the sensitivity of the band-fitting procedure to ϵ_4 is quite apparent. The even-parity bands in odd-neutron rare-earth nuclei are typically highly perturbed because of the large Coriolis mixing between the various Ω states. This perturbation is a direct consequence of the propagation of the $\Omega=\frac{1}{2}$ decoupling term through higher Ω states via the Coriolis interaction. For $\Omega=\frac{9}{2}$ and $\Omega=\frac{11}{2}$, such as encountered in ^{181}W and ^{187}Os, the observed perturbations either can be explained by adjusting the relative locations of the Nilsson quasiparticle states, or they can be fitted by adjusting the off-diagonal Coriolis matrix elements. If two or three of the $i_{13/2}$ Nilsson states can be identified and their dominant Ω quantum numbers can be characterized, the fit to the experimental levels becomes quite definitive, assuming one is willing to accept various reasonable prescriptions for attenuation of the Coriolis matrix elements near the Fermi surface. For example, in ^{181}W it is easy to fit the $\frac{9}{2}^+[624]$ ground rotational band structure *alone* for very large ϵ_4 values, but the added constraint of the $\frac{7}{2}^+[633]$ Nilsson state and its $\frac{13}{2}^+$ band member simply makes it impossible to obtain reasonable fits to the experimental level energies, even with the introduction of as many as nine independently and simultaneously variable free parameters ($E_{9/2^+[624]}$, $\hbar^2/2\mathcal{J}$, B, and all off-diagonal matrix elements.) The situation in ^{187}Os is similar, and if anything more definitive because of the $\frac{9}{2}^+[624]$ band head known from decay,[9] and the extensive $\frac{11}{2}^+[615]$ and $\frac{9}{2}^+[624]$ band structure and possible third $\frac{13}{2}^+$ state seen in the $(\alpha, 3n\gamma)$ experiment.

Two reliable model-dependent spectroscopic criteria for assigning static hexadecapole deformations *of the nuclear potential* appear to be (1) comparison with experimentally known intrinsic states and, where possible, (2) analysis of perturbed rotational band structure. The former study has been carried out in some detail by Ogle *et al.*[14] Though additional data have become available since their work was completed, nothing has, to our knowledge, come to light that would change their general conclusion that for the mass region $A=180-190$, equilibrium ϵ_4 deformations between the limits ≈ 0.02 and ≈ 0.08 are most compatible with the experimental data. The work discussed here apparently also rules out $\epsilon_4 \gtrsim 0.08$, and the interpretation of available transfer-reaction data requires only moderately large hexadecapole deformations ($\epsilon_4 \cong 0.06$).[10]

Therefore we conclude that the present γ-ray spectroscopic data in the odd-mass W and Os nuclei are incompatible with tetroidal deformations as large as those recently reported for ^{182}W and seemingly also implied for $^{186,188}Os$. It is unlikely that sharp variations in ϵ_4 would occur between neighboring odd-A and even-even nuclei, and indeed, the available experimental data on multiparticle states in ^{182}W, for example,[17] seem consistent with this view. In the osmium nuclei especially, one could argue that the prolate-oblate transition and triaxiality may influence the picture. To our knowledge, such factors have not been considered in analyzing the sub-Coulomb, Coulomb-nuclear interference, or nuclear inelastic-scattering data in this region, but again, it seems unlikely that such effects would be important for ^{182}W at least. In addition, it may be argued that neither a homogeneous charge distribution nor a deformed Fermi distribution correctly describes the situation in this region. There are, in fact, some experimental data to support the latter proposition.[18] Such local variations in charge density could have a profound influence on the analysis of sub-Coulomb scattering data.

It thus seems appropriate that further research on the problem be concentrated on reconsidering the origins and implications of the large measured $E4$ transition moments in the W and Os region of the nuclear chart.

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Experimental Test for the Charge Superselection Rule*

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An experiment is suggested to test the existence of a phase correlation for two space-separated superconductors. The phase in question is that of the eigenvalue of the operator S that creates a pair in one superconductor and removes a pair from the other. This will also provide a clear-cut test for the charge and lepton number superselection rules.

It has been argued[1,2] that, from *a priori* theoretical grounds, there is no sharp distinction between charge conservation and the conservation of other Abelian quantum numbers, such as momentum conservation. Gauge invariance renders the relative phases between different charge states arbitrary, but fixing the phase of a single suitable reference system removes this arbitrariness. In the same way, translation invariance renders the relative phases between different momentum components arbitrary, but the fixed position of a reference system removes the arbitrariness.

Thus the relevant consideration[3] in assessing the significance of the charge superselection rule is an empirical one. One has to ask whether there exists in nature any system which is already a coherent superposition of different charge states, and can therefore act as a reference system for the phase. In their derivation of charge superselection, Wick, Wightman, and Wigner state,[3] "It may be well, however, to reemphasize the critical assumption on which our analysis is based: that we have no states naturally given which are superpositions (rather than mixtures) of states with different charges." Recently Kibble[2] argued that a local region of a superfluid or a superconductor, or each side of a Jo-