Heavy-Ion Transfer Reactions at Low Energies*

Z. E. Switkowski, R. M. Wieland, and A. Winther†
California Institute of Technology, Pasadena, California 91109
(Received 23 May 1974)

Simple asymptotic expressions are obtained for the optimum Q value and for the total heavy-ion transfer cross section with optimum Q value at energies well below the barrier. For such reactions transfer processes are predicted to be more important than compound reactions in agreement with measurements performed on the $^{10}B+{}^{18}O$ system. Calculations on transfer reactions between ^{12}C and ^{16}O, of current interest in nucleosynthesis, indicate direct reactions to be of no significance.

While a semiclassical theory[1] can be used to describe direct reactions between heavy ions at bombarding energies close to the Coulomb barrier, this is not the case for much lower energies, where the cross section is mainly determined by penetration through the barrier.

This is readily seen from the distorted-wave Born approximation where the cross section for the reaction $a+A\rightarrow b+B$ is proportional to the square of the matrix element

$$M=\int_0^\infty F_{l_\beta}(k_\beta r)\exp(-\kappa r)F_{l_\alpha}(k_\alpha r)\,dr, \tag{1}$$

between the regular Coulomb wave functions, F_l, in the entrance and exit channels α and β, respectively. Since only the tail of the form factor is important, in the energy region considered, we have assumed that it is proportional to an exponential function where κ is related to the charge and the binding energy of the transferred particle. For very low bombarding energies it is seen from a WKB approximation for $F_{l_\alpha}(k_\alpha r)$ that the product of the last two factors in Eq. (1) has a rather sharp maximum at

$$r_{\max}=\frac{1}{q}\frac{\eta_\alpha}{k_\alpha}\{1+[1+q(l_\alpha+\tfrac{1}{2})^2/\eta_\alpha{}^2]^{1/2}\}, \tag{2}$$

where η_α is the Coulomb parameter

$$\eta_\alpha=(Z_aZ_Ae^2)/\hbar v_\alpha, \tag{3}$$

and q is defined by

$$q=1+\kappa^2/k_\alpha{}^2. \tag{4}$$

When for low energies the wave number k_α becomes small, $r_{\max}$ approaches a constant. Since the maximum in the first factor in (1) appears close to the classical turning point $r_{0\beta}$ in the exit channel defined by

$$r_{0\beta}=\frac{\eta_\beta}{k_\beta}\{1+[1+(l_\beta+\tfrac{1}{2})^2/\eta_\beta{}^2]^{1/2}\}, \tag{5}$$

the optimum Q value $Q_{\rm opt}$ for the reaction $\alpha\rightarrow\beta$ is obtained when $r_{0\beta}$ coincides with $r_{\max}$. Furthermore, since only low angular momenta contribute, i.e., $l_\alpha\simeq l_\beta<\eta_\alpha k_\alpha/\kappa$, this means

$$\frac{1}{q}\frac{\eta_\alpha}{k_\alpha}=\frac{\eta_\beta}{k_\beta}. \tag{6}$$

Noting that

$$\frac{\eta_\beta}{k_\beta}=\frac{Z_bZ_Be^2}{2(E_\alpha+Q_{\rm opt})}, \tag{7}$$

we obtain

$$Q_{\rm opt}=\left(\frac{Z_bZ_B}{Z_aZ_A}-1\right)E_\alpha+\frac{Z_bZ_B}{Z_aZ_A}\frac{\hbar^2\kappa^2}{2m_\alpha}, \tag{8}$$

where E_α is the center-of-mass energy and m_α the reduced mass in the entrance channel. For low values of E_α this result deviates significantly from the semiclassical expression in which the last term in Eq. (8) is missing.[2]

Evaluating the integral (1) in the WKB approximation, the total cross section for a reaction of optimum Q is found to be proportional to

$$\sigma_{\alpha\rightarrow\beta}\sim[E_\alpha]^{-1}\exp(-4\eta_\alpha\arctan\kappa/k_\alpha). \tag{9}$$

For very low energies where $\kappa/k_\alpha>1$, we thus find that the S factor for the reaction is of the form

$$S_{\alpha\rightarrow\beta}\equiv\sigma_{\alpha\rightarrow\beta}E_\alpha\exp(2\pi\eta_\alpha) \tag{10}$$

$$\simeq\exp(4Z_aZ_Ae^2m_\alpha/\hbar^2\kappa), \tag{11}$$

using the first-order expansion for the arctangent function. Since for heavy-ion reactions the exponent in (11) is a large number, the cross section for an optimum-Q-value transfer reaction will normally exceed the cross section for compound reactions at very low energies. This conclusion is not correct for Q values much different from $Q_{\rm opt}$, especially if the Q value is less than $Q_{\rm opt}$.

In order to verify some of these conclusions we have studied the neutron transfer reaction $^{10}B(^{18}O,$

^{17}O*)^{11}B leading to the first excited state (0.871 MeV) of ^{17}O. For this reaction the Q value (2.538 MeV) is rather close to the optimum value.

Thick enriched ^{10}B targets were bombarded with ^{18}O(4+) beams of 0.1–3.0 μA from the Office of Naval Research–California Institute of Technology tandem accelerator. The beam was produced from ^{18}O-enriched (81.84%) water vapor in a duo-plasmatron ion source, and its identity verified by elastic-scattering coincidence measurements prior to bombardment. The total transfer cross section to the 0.871-MeV state was obtained by measuring the yield of γ rays from its decay with a large-volume Ge(Li) detector placed close to the target at 0°. No significant yields of γ rays from de-excitation of higher energy states in ^{17}O were observed. This is consistent with the nonoptimum Q value for such neutron transfers. Hence the yield of 871-keV γ rays is an accurate absolute measure of the transfer cross section of interest. Furthermore, transfers leading to the ground state of ^{17}O, to which our detection system is insensitive, would increase the transfer cross section and only strengthen the conclusions of this paper discussed latter. Data were obtained for center-of-mass energies in the range 3.0–7.7 MeV. Shown in Fig. 1 are the reduced data plotted in the form $S = \sigma E \exp(2\pi\eta_\alpha)$ in order to remove the Coulomb-barrier effects dominant at these low energies. Also shown is the S factor obtained from Eq. (9) and from a Coulomb-wave Born-approximation calculation using a value of $\kappa = 0.7\ \mathrm{fm}^{-1}$.

Several γ-ray yields which corresponded to reactions proceeding via the compound nucleus ^{28}Al were also observed. These included transitions in the following nuclei: ^{26}Mg ($E_\gamma = 1.809$ MeV), ^{26}Al (0.417), ^{25}Mg (0.390, 0.585, 0.975), and ^{24}Na (0.472). In the case of the reaction ^{10}B(^{18}O, 2α) forming ^{20}F, the 1.634-MeV β-delayed γ ray was measured. The energy dependence of the total compound-reaction cross section was determined from these yields by applying corrections based on Hauser-Feshbach calculations. This method has been successfully applied to a number of other low-energy heavy-ion reactions and details will be published in a separate paper.[3] These data were then normalized at 7.5 MeV to an optical-model calculation using the strong-absorption potential of Michaud,[4] $V = 50$ MeV, $W = 10$ MeV, $r_0 = 1.34$ fm, and $a = 0.4$ fm. The reliability of the optical model near the Coulomb barrier is well established (see, for example, Kuehner and Almqvist[5] and Cujec and Barnes.[6]

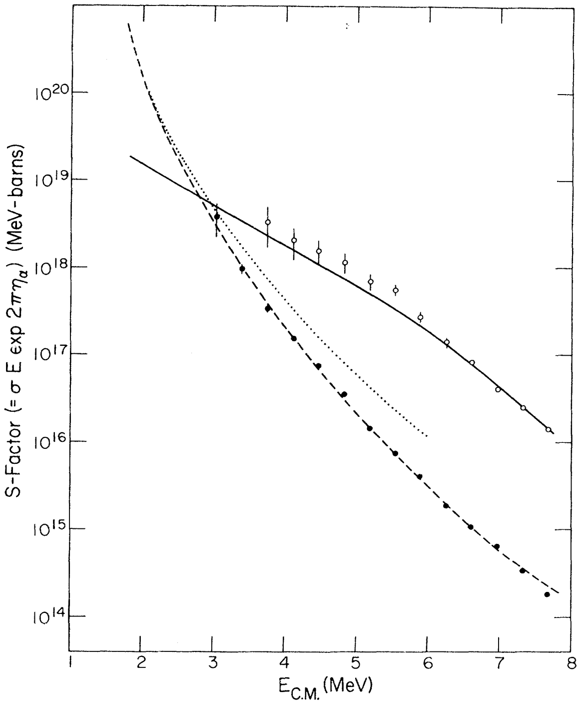

FIG. 1. S factors for the transfer reaction $^{10}\mathrm{B} + {}^{18}\mathrm{O} \rightarrow {}^{11}\mathrm{B} + {}^{17}\mathrm{O}^*$ (closed circles) and the total compound nuclear reaction $^{10}\mathrm{B} + {}^{18}\mathrm{O} \rightarrow {}^{28}\mathrm{Al}^*$ (open circles). The asymptotic form, determined from Eq. (9), of the transfer S factor (dotted curve) is normalized at low energies to an undistorted Coulomb-wave Born-approximation calculation (dashed curve). The latter is normalized to the experimental n-transfer data at 4.1 MeV. The absolute value of the experimental S factor for the total compound nuclear reaction was obtained by normalizing to the optical-model prediction (solid curve) at 7.5 MeV.

These results appear in Fig. 1. It is seen that both the transfer and compound reaction data are well described by the calculations described above, enabling us to infer that the direct contribution to the $^{18}\mathrm{O} + {}^{10}\mathrm{B}$ reaction will exceed the compound-nucleus contribution for $E_{\mathrm{c.m.}} \lesssim 3.0$ MeV.

A similar investigation of the reaction ^{11}B(^{18}O, ^{17}O*)^{12}B ($Q = -5.549$ MeV) yielded cross sections down by several orders of magnitude from ^{10}B, consistent with the nonoptimum Q for this reaction. Furthermore, our study of the reaction ^{14}N(^{18}O, ^{17}O*)^{15}N, a system for which the Q value is close to optimum, resulted in good agreement with the prediction of Eq. (9).

We have extended these calculations to the heavy-ion reactions currently thought to be of the greatest importance in nucleosynthesis and it is our conclusion that transfer reactions among

^{12}C and ^{16}O nuclei, which all have negative Q values, are not important in the low-energy nucleosynthesis region. This conclusion is supported by Kozlovsky[7] for the α-transfer reaction ^{12}C(^{12}C, ^{8}Be)^{16}O. In the case of ^{12}C + ^{16}O it has been suggested that the α-transfer reaction producing ^{20}Ne accounts for $\lesssim 3\%$ of the total reaction cross section at 10 MeV[8] and that at even lower energies this contribution may be greater. Our calculations show that the transfer process may increase in proportion to the total reaction cross section as the energy is lowered but below 6 MeV the nonoptimum nature of the Q value drastically reduces the transfer cross section. At 7.2 MeV (the effective energy for explosive nucleosynthesis[9] at temperatures around 3.6 $\times 10^9$ °K) our calculations and more exact distorted-wave calculations of Nilsson and Barnes[10] yield an upper limit of 10% to the fraction of the cross section proceeding by transfer. This contribution is less than the uncertainties in the measured absolute reaction cross section for ^{12}C + ^{16}O (Ref. 6 and Patterson *et al.*[11]).

The authors would like to thank Professor William A. Fowler and Professor C. A. Barnes for valuable discussions, and Barbara Zimmerman for her computational support.

---

*Work supported in part by the National Science Foundation, Grant No GP-28027.

†Permanent address: Niels Bohr Institute, Copenhagen, Denmark.

[1] R. A. Broglia and A. Winther, Phys. Rep. 4C, 153 (1972).

[2] P. A. J. Buttle and L. J. B. Goldfarb, Nucl. Phys. A176, 299 (1971).

[3] R. G. Stokstad, Z. E. Switkowski, and R. M. Wieland, to be published.

[4] G. Michaud, Phys. Rev. C 8, 525 (1973).

[5] J. A. Kuehner and E. Almqvist, Phys. Rev. 134, B1229 (1964).

[6] B. Cujec and C. A. Barnes, to be published.

[7] B.-Z. Kozlovsky, Astrophys. Space Sci. 8, 114 (1970).

[8] T. W. Conlon, Nature 247, 268 (1974).

[9] S. E. Woosley, W. D. Arnett, and D. D. Clayton, Phys. Rev. Lett. 27, 213 (1971).

[10] B. Nilsson and C. A. Barnes, private communication.

[11] J. R. Patterson, B. N. Nagorcka, G. D. Symons, and W. M. Zuk, Nucl. Phys. A165, 545 (1971).

Entrance-Channel Effects in the ^{32}S System: Comparison of ^{12}C + ^{20}Ne and ^{16}O + ^{16}O Elastic Scattering*

R. Vandenbosch, M. P. Webb, and M. S. Zisman
University of Washington, Seattle, Washington 98195

(Received 15 July 1974)

^{12}C + ^{20}Ne elastic-scattering angular distributions have been measured at several energies, and an excitation function has been obtained at 70° center of mass from $E_{\mathrm{c.m.}} = 17$ to 28 MeV. The ^{12}C + ^{20}Ne data are greatly damped compared with ^{16}O + ^{16}O data in this energy range. This demonstrates that the absorption is not determined primarily by compound-nuclear properties. The differences are interpreted in terms of the more favorable angular-momentum matching conditions for the ^{12}C + ^{20}Ne direct-reaction channels.

The observation[1,2] of prominent gross structure in ^{16}O + ^{16}O elastic scattering has led to considerable interest in the nature of the mechanisms and interaction potentials in heavy-ion reactions. The periodicity of the structure is readily understood in terms of shape resonances. The most dramatic aspect of the structure, however, is the large peak-to-valley ratio together with the large absolute magnitude of the cross sections at angles near 90°. A conventional optical-model analysis[1] easily reproduced the periodicity but not the magnitude of the oscillations. Chatwin and co-workers[3,4] showed that the magnitude of the oscillations could also be reproduced if the imaginary potential was made explicitly l dependent. They introduced a smooth cutoff in the strength of the absorptive potential as l gets larger than a certain critical value. This cutoff becomes important if the heavy nuclei in the entrance channel bring in a greater amount of angular momentum than any of the exit channels can carry away. The parameters of the l-dependent potential were