# New Technique for the Measurement of Lifetimes of Heavy-Ion Inner-Shell Vacancies

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(Received 4 February 1974)

We report a new technique for the measurement of short lifetimes ($10^{-14}$ sec) of inner-shell vacancies in heavy-ion projectiles moving through thin, solid targets. The technique is applicable whenever the yield of projectile x rays does not increase in proportion to target thickness. Lifetime values of $1.5\times10^{-14}$ and $3.3\times10^{-14}$ sec have been found for $K\alpha$ and $K\beta$ transitions, respectively, in highly stripped sulfur ions.

When heavy ions penetrate through matter, interaction between the ions and target atoms lead to excitation of both collision partners. Intensive efforts have been devoted to the investigation of inner-shell vacancies which are created in heavy-ion–atom encounters and decay by emission of x rays and Auger electrons. These phenomena have been reviewed by several authors.[1] It has become evident that heavy-ion collisions generally lead to multiply ionized and multiply excited states of great complexity and, because of the large number of possible states, identification of measured Auger- and x-ray lines becomes difficult. Deduction of primary vacancy production cross sections from experimental x-ray production cross sections is particularly complicated since it requires calculation of fluorescent yields and, thus, lifetimes, of the complex states involved. It is the purpose of this Letter to demonstrate a simple method for the experimental determination of such lifetimes which are short ($10^{-14}$ sec) and do not allow the methods of beam-foil spectroscopy to be used. Our technique is applicable to foil-induced projectile vacancies in wide ranges of ion species, ion velocity, and transition energy, and requires merely measurement of relative x-ray yields and absolute differences of target thicknesses. Lifetimes which result from this technique are not only useful for comparison with theoretical calculations, but may also be helpful to identify observed x-ray transitions.

Sulfur ions with energies of up to 110 MeV from the tandem Van de Graaff facility at the University of Munich have been directed onto foil targets (Be, C, Al, Cu, Ge) with effective thicknesses $x_0$ from $(0.3 \text{ to } 7)\times10^{18}$ atoms/cm$^2$ (5 to 140 $\mu$g/cm$^2$ for C). Thicknesses were obtained from weights. X rays above ~1 keV were recorded with a Si(Li) detector placed perpendicular to the beam direction. Rutherford scattering served as monitor to determine absolute x-ray production.

Figure 1 shows $K$ x-ray yields of 95-MeV sulfur ions in carbon. The resolution of the Si(Li) detector was sufficient (~130 eV near 2.5 keV) to resolve four major S $K$ satellite lines which are labeled $K\alpha^1$, $K\alpha^2$, $K\beta^1$, and $K\beta^2$, respectively. Figure 2 presents these satellites for 110-MeV sulfur ions in 100 $\mu$g/cm$^2$ targets of Be, C, Al, and Cu. The x-ray energy scale was calibrated with an accuracy of better than ~2 eV by means of eight characteristic x-ray lines which were obtained from electron bombardment of Mg, Al, Si, S, Cl, K, and Ti.

The number of projectile x rays observed per incident ion does not necessarily increase in proportion to target thickness. In some cases, x-ray production becomes suppressed for target thicknesses above ~30 $\mu$g/cm$^2$; the extent of suppression depends on the nature of the target and is smallest in very light materials such as C or Be. In the present experiment, this effect can not be attributed to energy loss of projectiles or absorption of x rays in the target, but is readily understood from the rate equation for projectile x-ray production. We will explain this in the following.

For the sake of simplicity, let us assume that the ion beam can be considered to consist of two fractions of ions, those carrying a vacancy of interest which can decay by Auger and/or x-ray transitions ($Y$), and those which do not carry such a vacancy ($1-Y$). Then, $\sigma_v$ is the cross section to produce the vacancies in collisions with target atoms. Destruction of the vacancies proceeds via Auger and radiative decays involving lifetimes $\tau_A$ and $\tau_R$, respectively. However, as long as the ions move inside the target, a competing process must be taken into account, namely capture of target electrons directly into the inner-shell vacancy of the moving ion. These capture events may be nonradiative or radiative and can be summarized by a cross section $\sigma_c$. When we use $\tau^{-1}=\tau_R^{-1}+\tau_A^{-1}$, $\sigma=(nv\tau)^{-1}$, and $\sigma_T$

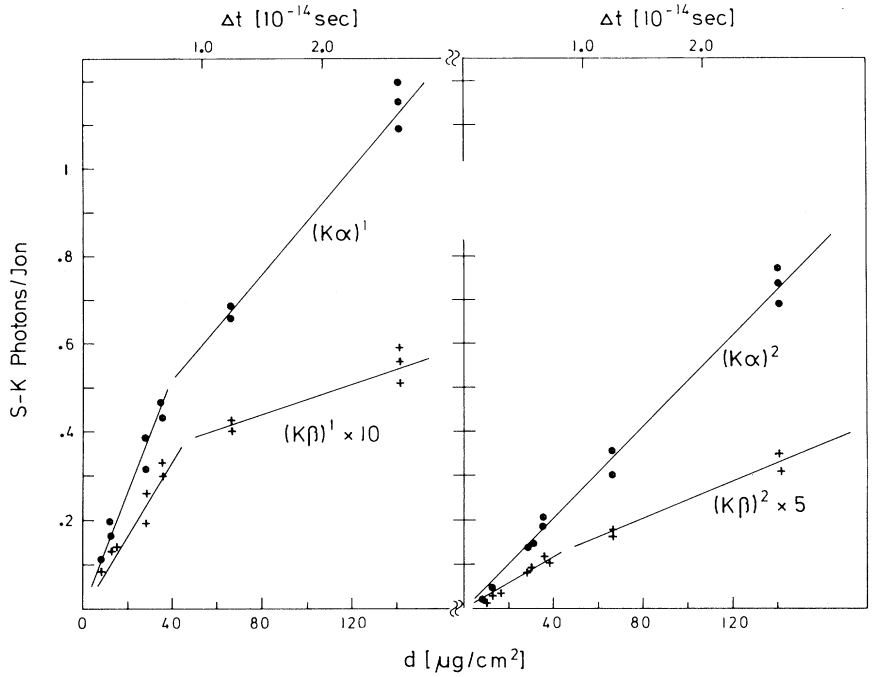


FIG. 1. Absolute yield of four resolved $K$ x-ray lines of 95-MeV sulfur in carbon, as a function of target thickness in $\mu g/cm^2$. The upper scale gives the time $\Delta t$ which the ions need to traverse the foil.

$= \sigma_v + \sigma_c + \sigma$, the correct rate equation for $Y(x)$ and its solution become

$$dY/dx = (1 - Y)\sigma_v - (\sigma + \sigma_c)Y, \quad (1)$$

$$Y(x) = (\sigma_v/\sigma_T)[1 - \exp(-\sigma_T x)], \quad (2)$$

where $n$ stands for the number of target atoms per cubic centimeter and $v$ is the ion velocity. Thus, the fraction $Y_0 = Y(x_0)$ emerges from the foil with vacancies and gives rise to emission of x rays *outside* the foil, in number $N_a = \omega Y_0$, where $\omega$ denotes the fluorescent yield, whereas the contribution from *inside* the foil amounts to $N_i = (\sigma_v x_0 - Y_0)/(nv\tau_R\sigma_T)$. With this, the total number of x rays produced per incident ion is

$$N(x_0) = \omega'\sigma_v x_0 + (\omega - \omega')Y_0 \xrightarrow[\sigma_T x_0 >> 1;\ \omega >> \omega']{} \omega'\sigma_v(x_0 + nv\tau), \quad (3)$$

where

$$\omega' = (nv\tau_R\sigma_T)^{-1} = \omega\sigma/\sigma_T. \quad (4)$$

It is illustrative to discuss Eq. (3) for different extreme cases. When the target thickness is chosen

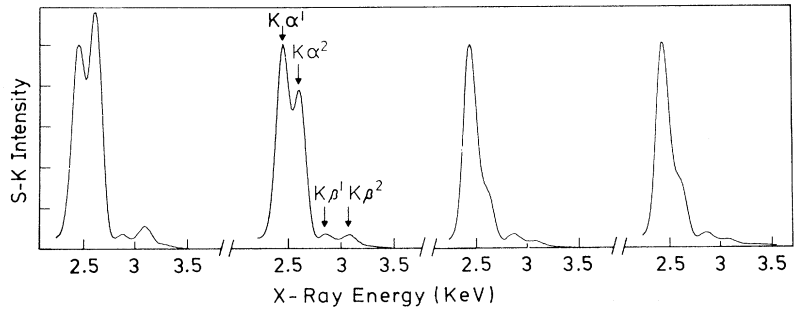


FIG. 2. $K$ x-ray spectrum of 110-MeV sulfur in various targets of thickness 100 $\mu g/cm^2$. $K\alpha^1$, $K\alpha^2$, $K\beta^1$, and $K\beta^2$ denote satellites for which x-ray yield curves have been determined (see Fig. 1).

small enough so that single collisions prevail ($\sigma_T x_0 \ll 1$), one obtains $N = \omega \sigma_v x_0$. This familiar relation will, however, remain effective for any higher target thickness as long as $\sigma_v + \sigma_c \ll \sigma$, i.e., $\omega \simeq \omega'$. Under these circumstances, measurement of projectile x-ray production cross sections is possible with the use of a single foil of any thickness, provided that effects of energy loss by the ions and absorption of x rays in the target are accounted for. By contrast, when $\sigma_v + \sigma_c$ is not small compared to $\sigma$, one obtains $\omega > \omega'$ and the x-ray yield exhibits nonproportional behavior as soon as $\sigma_T x_0$ reaches or exceeds unity (see Fig. 1).

Further interpretation of Eq. (3) is particularly simple for the case $\sigma_T x_0 \gg 1$ (equilibrium excitation) and $\omega \gg \omega'$. Then, the total yield is composed of two parts, (i) a constant contribution due to x rays emitted behind the foil, and (ii) a yield due to decays inside the foil which is proportional to target thickness, $N \simeq \omega' \sigma_v x_0$. Here, the replacement of $\omega$ by $\omega'$ indicates that the effective fluorescent yield depends not only on intrinsic atomic properties of the ion ($\omega$), but also on the probabilities for capture and loss of electrons in collisions, i.e., on the *dynamics* of collisions. It appears practical, therefore, to interpret $\omega'$ as a generalized or *dynamic fluorescent yield.*

The necessary and sufficient condition for observing nonproportionality of $N(x_0)$ can be expressed by

$$\sigma_v + \sigma_c \gtrsim 1/(nv\tau), \quad (\sigma_T x_0 \gg 1). \tag{5}$$

When yields are measured for target thicknesses below and above equilibrium, $N(x_0)$ can be fitted by Eq. (3) with three parameters $\omega' \sigma_v$, $\sigma_T$, and $\tau$. In cases of strongly pronounced nonproportionality ($\omega \gg \omega'$), measurement of $N(x_0)$ above equilibrium allows one to determine the lifetime from $\tau = d_1/v$, where $d_1 = x_1/n$ denotes the particular foil thickness for which the increasing part of the x-ray yield equals the constant one ($N_i = N_a$).

Use of the two-fraction system and its solution Eq. (3) is correct for cases in which there is only a single channel to form decaying ions. It can be shown, however, that the general yield curve for a specific satellite line will be similar to Eq. (3). In particular, for cases with pronounced nonproportionality one finds $N = Y_\infty (x_0 + nv\tau)/(nv\tau_R)$, where $Y_\infty$ is the equilibrium fraction of ions with a vacancy, regardless of the mechanisms which lead to this equilibrium excitation; thus, $\tau$ is still determined by $d_1/v$ just as in the simplified case discussed above.

Our data yield lifetimes of (1.4, 1.7, 1.5, and 1.8) $\times 10^{-14}$ sec for $K\alpha_1$ line of sulfur in C, Al, Cu, and Ge, respectively, with a best value of $(1.5 \pm 0.15) \times 10^{-14}$ sec, and $3.3 \times 10^{-14}$ sec for the $K\beta^1$ satellite in C. By comparison, the lifetime of a $K$ vacancy in singly ionized sulfur is $1.5 \times 10^{-15}$ sec.[2] It must be expected that our experimental lifetimes reflect decay of various satellites which could not be resolved with the Si(Li) detector. Use of a spectrometer with higher resolving power is desirable in order to obtain lifetimes for specific multiplet transitions. We comment briefly on the identification of the four satellites resolved in the present experiment. Comparison between experimental centroid energies and theoretical multiplet transition energies calculated with a relativistic Hartree-Fock program[3] reveals that the observed lines are essentially heliumlike ($K\alpha^1, K\beta^1$) and hydrogenlike ($K\alpha^2, K\beta^2$) transitions. We also found that the position of the two $K\alpha$ lines was independent (within ~5 eV) of target thickness and species (a dozen other targets have also been investigated) though the intensity ratios of the lines varied considerably (see Figs. 1 and 2). Even when the beam energy was decreased from 110 to 55 MeV, no shift of the $K\alpha$ lines greater than ~5 eV could be detected.

It is interesting to demonstrate that the major part of the $K\alpha^1$ satellite can not be associated with a metastable state. In foils with thicknesses above equilibrium (~30 $\mu$g/cm$^2$ C) the increasing part $N_i$ of the yield curve is due to fast transitions in ions inside the foil. The yield $N_a$ from behind the foil could, in principle, contain contributions from metastable transitions. In order to check this, we replaced a 60-$\mu$g/cm$^2$ C foil by two 30-$\mu$g/cm$^2$ C foils mounted on a frame such that the two foils did not touch and were separated by a small fraction of a millimeter. The x-ray yield from the double foil exceeded the one from the single foil by exactly the amount $N_a$. It must be concluded that this observed increase of the yield is due to x rays emitted within the short space between the two foils and, thus, signifies short-lived transitions.

Refinements of the presented technique for lifetime measurements consist of more precise determination of target thicknesses, use of an x-ray spectrometer with higher resolution to allow x-ray yield measurements for a unique transition, and consideration of secondary effects in

Eq. (3). The latter includes consecutive decay in ions with multiple vacancies in the inner shell and metastable decays which in some cases may contribute to $N_a$ but not to $N_i$.

Condition Eq. (5) holds for $K$ x rays of sulfur ions with energies above ~30 MeV, mainly because of relatively large electron-capture cross sections $\sigma_c$. It must be expected that Eq. (5) is validated when the ion velocity reaches an appreciable fraction of the orbital velocity associated with the projectile vacancy. We estimate, for example, that specific sulfur $L$ x-ray satellites should exhibit a nonproportional yield curve at energies above a few hundred keV. Finally, we note that the phenomenon of nonproportionality between projectile x-ray yield and target thickness affects measurement of projectile x-ray production cross sections and allows determination of the number of ions which emerge from solids in certain states of excitation.

The authors are greatly indebted to Professor Č. Zupančič for his support and stimulating discussions throughout this work.

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# NMR Shift and NMR Linewidth in Superfluid $^3$He†

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(Received 27 June 1974)

A set of equations is given, which allows us to rederive both the results of Leggett (hydrodynamic regime) and of Maki and Ebisawa (collisionless regime) for NMR in superfluid $^3$He. The NMR linewidths for both transverse and longitudinal resonance are derived by introducing a relaxation-time approximation. The results are in good agreement with experimental data.

NMR has been recently shown to be a very powerful probe of the superfluid phases of liquid $^3$He. The most striking feature is the shift from the Larmor frequency in transverse-resonance experiments.[1] Leggett has given a theory[2,3] accounting for this shift very well. Moreover, his prediction for a longitudinal resonance has been confirmed recently by experiment,[4,5] strongly indicating that the $A$ phase is the axial state, in agreement with the theory by Anderson and Brinkman.[6] Recent NMR experiments[4] also support the hypothesis that the $B$ phase is the isotropic state first considered by Balian and Werthamer.[7]

In this Letter, we rederive Leggett's results in the limit of small fluctuations and obtain linewidths of the NMR lines. This is done by using a completely microscopic theory which enables us to treat both the collisionless and the hydrodynamic regime. In this way, the results of Maki and Ebisawa[8,9] (collisionless regime) and the results of Leggett[2,3] (hydrodynamic regime) are deduced from the same set of equations. Moreover, we take into account collisions between quasiparticles by introducing a relaxation time in our equations. We obtain in the axial state both transverse and longitudinal linewidths which are in good agreement with the existing experimental data.[4,5] The surprisingly large resulting linewidths, induced by quasiparticle collisions, have the same physical origin as the NMR shift, namely the symmetry-breaking effect of the dipole interaction. We also give the result for the longitudinal linewidth in the isotropic state, but there are no published data on the $B$ phase.

The method that we use is based on the kinetic equation for the distribution function, which is derived by using the Hartree-Fock approximation from a Hamiltonian that includes pairing, Fermi liquid, and Zeeman terms in addition to kinetic energy. This equation is expanded in lowest order in $\omega/q$ and $qv_F/\Delta$ and diagonalized. The small fluctuations of the direction of the order parameter are accounted for by a space-time-dependent rotation in spin space, $U=\exp[-i\vec{\sigma}\cdot\vec{\theta}(\vec{r},t)]$. This method has been described extensively in another paper[10] and used to study spin waves in