of such a time-delay coincidence histogram. Noise and the filter-broadened gradual change in amplitude combine to give the breadth of the distribution. For comparison, Fig. 4(b) are the results of Ref. 4.

The results of Figs. 3 and 4(a) appear to preclude all physical explanations of the results of Ref. 4 [our Fig. 4(b)]. Indeed, the data of Ref. 4 were processed with a faulty computer program —an above-threshold event in one specific antenna which happened to fall in the last 0.1-sec bin of a 1000-bin data block was *always* counted as a prompt coincidence with a later above-threshold event in the other antenna.[11] This error was shown[11] to account for essentially all of the zero-delay excess events on a four-day tape of the data of Ref. 4.

We thank D. H. Douglass for many useful discussions.

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[4]J. Weber *et al.*, Phys. Rev. Lett. 31, 779 (1973).

[5]A. D. Whalen, *Detection of Signals in Noise* (Academic, New York, 1971), p. 204, Eq. (7–22). Our detection algorithm acts to "whiten" the correlated thermal noise of the lightly damped antenna, thus Eq. (7–22) is relevant. It reduces to Eq. (1a) above if the following substitutions are made: $E=q^2$, $P(E)\,dE=p_1(g)\,dq$, $E_g=(AT/2)^2$, and $2\sigma_T^2=kT_e$. Figure 7–5, p. 205 of the text may be used with the signal-to-noise ratio defined as $(S/N)=10\log(E_g/kT_e)$.

[6]R. S. Burington, *Handbook of Mathematical Tables and Formulas* (McGraw-Hill, New York, 1965), 4th ed.

[7]The amplitude change induced in the antenna depends on its orientation with respect to the (unknown) gravity wave propagation vector and polarization. We thus refer throughout to the final effect of a wave; i.e., the resulting pulse energy, for direct comparisons with other experiments.

[8]D. H. Douglass, private communication.

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Azimuthal Correlations in *pp* Interactions at 205 GeV/*c*

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Using data obtained from an exposure of the 30-in. Fermi National Accelerator Laboratory bubble chamber to a 205-GeV/*c* proton beam, results are presented on azimuthal distributions. Evidence is presented for dynamical azimuthal correlations whose behavior depends on the particular charge combination. The dependence of the azimuthal correlations on the rapidity difference and on the transverse momentum is also given.

The study of two-particle correlations has received considerable attention recently, both theoretically and experimentally. At the Fermi National Accelerator Laboratory (FNAL) and the CERN intersecting storage ring energies, most of the emphasis has been placed on the analysis of two-particle rapidity distributions, although some studies have been made of azimuthal correlations.[1,2] In this Letter we present new results on azimuthal correlations in *pp* interactions, at the inclusive and, for the first time at FNAL energies, at the semi-inclusive level. The depen-

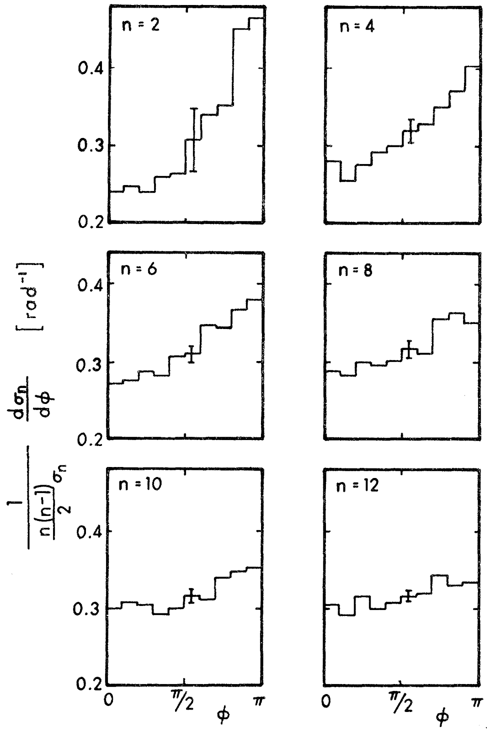

FIG. 1. Semi-inclusive azimuthal distributions for all charged-particle combinations.

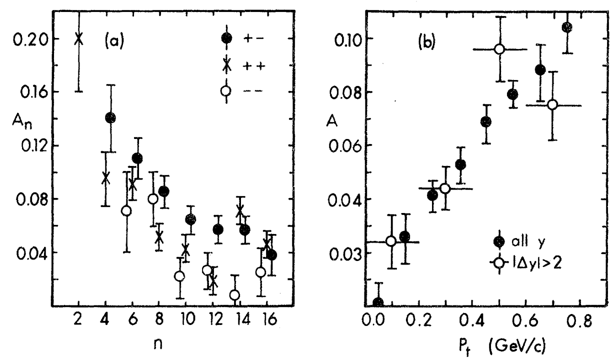

FIG. 2. (a) Asymmetry parameter A_n as defined in Eq. (1) as a function of charged-particle multiplicity n for different charge combinations. (b) Asymmetry A, integrated over all n, as a function of p_T for all y and for $|y_i - y_j| > 2$.

dence of the azimuthal distributions on the particle's charge, transverse momentum, and rapidity is also studied.

The results are based on an unbiased inclusive data sample of ~3500 completely measured events obtained from an exposure of the 30-in. FNAL hydrogen bubble chamber to a 205-GeV/c proton beam. Further details of this experiment are given elsewhere.[3] For the analysis of two-particle distributions we make the following choice of kinematic variables: the magnitude of the transverse momentum of particle i, P_{iT}; its center-of-mass rapidity, y_i; and the azimuthal angle between the transverse momenta of particles i and j,[4]

$$\varphi_{ij} = \cos^{-1} \frac{\vec{P}_{iT} \cdot \vec{P}_{jT}}{P_{iT} P_{jT}}.$$

In this paper we analyze the azimuthal distributions

$$\frac{d\sigma_n}{d\varphi} = \sum_{i=1}^{n} \sum_{j<i} \frac{d\sigma_n}{d\varphi_{ij}}$$

and

$$d\sigma/d\varphi = \textstyle\sum_n d\sigma_n/d\varphi,$$

where n is the charged-particle multiplicity.

In Fig. 1 we show the semi-inclusive azimuthal distributions for all charged-particle combinations. It is observed that (a) the distributions are asymmetric and peak near 180° and (b) the asymmetry becomes less pronounced as the charged multiplicity increases. As will be discussed later, these trends may be understood in part as a consequence of transverse-momentum conservation. To parametrize the distributions, we define, for fixed n, the asymmetry parameter A_n:

$$A_n = \left(\int_{\pi/2}^{\pi} \frac{d\sigma_n}{d\varphi} d\varphi - \int_0^{\pi/2} \frac{d\sigma_n}{d\varphi} d\varphi\right)\left(\int_0^{\pi} \frac{d\sigma_n}{d\varphi} d\varphi\right)^{-1}. \quad (1)$$

Table I and Fig. 2(a) present A_n as a function of n for the different charge combinations. In general, for each n, the asymmetry for the (+ −) combination is larger than that for either the (− −) or (+ +) combination. This holds as well for the inclusive values listed in Table I.

There exist simple model calculations which predict a definite relation between the asymmetry A and the total number (n_T) of particles produced in the final state. For example, a statistical model[5] which incorporates only transverse-momentum conservation and a Gaussian distribution in transverse momentum predicts $A = 1/(n_T - 1)$. Applying this model to our measured asymmetry parameters for all charged particles,[6] A_n^{cc}, we give in Table I estimates of n_T for a given number of charged particles, n. We also list in Table I the values of n_T calculated using the experimentally observed average number of π^0, K^0, and Λ

TABLE I. Asymmetry parameter A_n [Eq. (1)] for different charge combinations and leading particles. The total number of particles, n_T, estimated from A_n^{cc} and experimentally observed.

n	A_n^{cc}	A_n^{+-}	A_n^{++}	A_n^{++} leading	A_n^{--}	n_T from A_n^{cc}	n_T observed
2	0.20±0.04	———	0.20±0.04	0.20±.04	———	6±1	4±1
4	0.11±0.01	0.14±0.03	0.09±.02	0.04±.03	———	10±1	6±1
6	0.10±0.01	0.11±0.02	0.09±0.01	0.09±.03	0.07±0.03	11±1	11±1
8	0.072±0.006	0.08±0.01	0.05±0.01	0.05±.04	0.08±0.02	15±1	13±1
10	0.051±0.005	0.06±0.01	0.04±0.01	0.05±.06	0.02±0.01	21±2	17±1
12	0.040±0.005	0.06±0.01	0.02±0.01	}	0.03±0.01	26±3	18±1
14	0.050±0.006	0.06±0.01	0.07±0.01	} 0.00±.03	0.01±0.01	21±2	21±2
16	0.035±0.008	0.04±0.02	0.05±0.01	}	.02±0.02	29±6	24±2
Inclusive	0.056±0.003	0.067±0.006	0.048±0.005		0.027±0.009		

per inelastic event.[7] We have assumed that on the average there is one neutron or antineutron per inelastic event.[8] We observe that the model's predictions for n_T are consistently larger than the values experimentally determined, which may be a manifestation of dynamical effects not included in the model.

In order to study the transverse-momentum dependence of the azimuthal correlations, we consider the distribution

$$\frac{1}{\sigma_{\text{incl}}}\frac{d^2\sigma}{d\varphi\, dp_T}$$

in which we have integrated over the transverse momentum of one of the two particles. Figure 2(b) shows the asymmetry A, integrated over all n, as a function of p_T. The asymmetry is seen to increase monotonically with p_T, an effect due at least in part to energy-momentum conservation. In Fig. 2(b) we also show the asymmetry as a function of p_T for particle pairs with a large rapidity difference, $\Delta y = |y_i - y_j| > 2$. We observe that the above trend continues even over large rapidity differences.

Correlation effects between the longitudinal and transverse momenta can be analyzed by studying the distribution

$$\frac{1}{\sigma_{\text{incl}}}\frac{d^3\sigma}{dy_i\, dy_j\, d\varphi_{ij}}. \tag{2}$$

We note that in the independent-particle emission the asymmetry in φ for the distribution (2) depends only very weakly on the rapidity difference Δy.[9] In Figs. 3(a)–3(d) we show the asymmetry parameter as a function of Δy for the charged-charged, + −, + +, and − − cases, respectively. We observe that for the (+ −) combination (which, as noted above, shows larger correlations in both the inclusive and semi-inclusive cases), the asymmetry is large for small rapidity differences and decreases as the rapidity difference increases. That is, for this combination, particles which are close together in rapidity tend to balance their transverse momenta. By contrast, for the (+ +)[10] and (− −) combinations, we find that the asymmetry is small for small Δy. The asymmetry then increases to a maximum as the rapidity difference increases ($\Delta y \simeq 1-2$) and then drops to zero again.

The suppression of the azimuthal asymmetry at

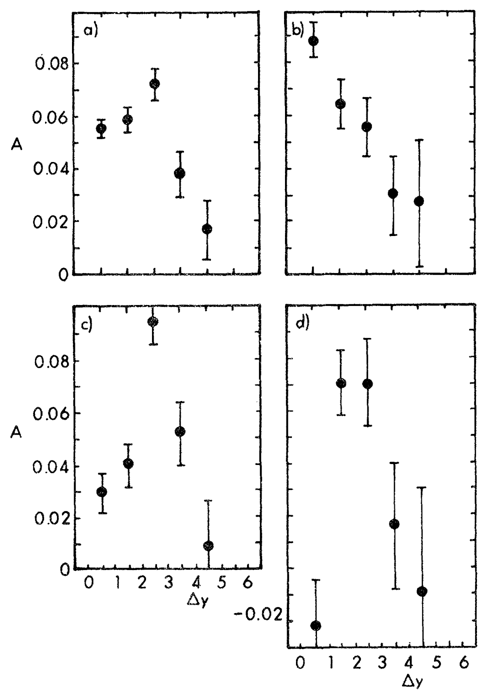

FIG. 3. Asymmetry parameter as a function of rapidity difference Δy for the (a) charged-charged, (b) $+-$, (c) $++$, and (d) $--$ combinations.

small Δy of like pions ($++$ and $--$) may be partly caused by correlations due to Bose-Einstein statistics.[11] However, it is tempting to go a step further and compare our observations with a short-range cluster model of particle production which has been successful in explaining two-particle rapidity distributions. In one such picture,[12] the clusters are assumed to be neutral and contain three pions. Hence, within such a simple cluster, there are no ($++$) or ($--$) combinations. This implies that the ($++$) and ($--$) correlations are due to cluster-cluster correlations alone, whereas the ($+-$) combination has, in addition to the cluster-cluster contribution, correlations between the particles within a single cluster. Qualitatively, this is consistent with the observation that the ($++$) and ($--$) correlations are small at small Δy, rise with increasing Δy [as correlations between (neighboring) clusters become important], and then fall toward zero (beyond the range of correlations between clusters). In a multiperipheral model with only single-pion emission the above effects can only be produced by imposing strong isospin constraints.

As has been shown above, as two particles get farther apart in rapidity, their transverse momenta become less correlated. It is natural to examine the extreme case where one studies the azimuthal correlation between "leading" particles (either π^+ or proton). In order to look for such correlations, we have defined the leading particles in an event as the positive particles with the smallest and largest angle (with respect to the beam direction) in the laboratory. Table I gives the asymmetry parameter as a function of topology. We obtain an asymmetry of 0.04 ± 0.02 averaged over events with four or more prongs. Thus, we observe that the asymmetry for the leading particles is nonzero but consistently smaller than the overall asymmetry. (Of course, for the two prongs, the two distributions are the same.) For $n > 8$, the average leading-particle asymmetry is consistent with zero.

We conclude that there are azimuthal correlations present which cannot be explained solely by energy-momentum conservation, since the asymmetry has a substantial charge dependence. The asymmetry is also a strong function of the rapidity difference between the two particles.

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[1]C. M. Bromberg *et al.*, Phys. Rev. D 9, 1864 (1974).

[2]H. Dibon *et al.*, Phys. Lett 44B, 313 (1973).

[3]Y. Cho *et al.*, Phys. Rev. Lett. 31, 413 (1973); R. Singer *et al.*, Phys. Lett. 49B, 481 (1974).

[4]Errors in φ_{ij} are typically $\pm 5°$. To calculate y_i we have assumed all negative tracks to be π^-, positive tracks with momentum greater than 120 GeV/c to be protons (resulting in less than a 5% misidentification of pions as protons), and the remainder of the positive tracks (except for identified slow-proton tracks with momentum less than 1.4 GeV/c) to be pions.

[5]M. C. Foster *et al.*, Phys. Rev. D 6, 3135 (1972); M. Pratap and J. C. Shaw, Phys. Rev. D 8, 3938 (1973).

[6]One can measure the asymmetry parameter using only the charged particles if the emission of particles is truly statistical.

[7]K. Jaeger *et al.*, "Characteristics of V^0 and γ Production in pp Interactions at 205 GeV/c," Argonne National Laboratory Report (to be published); K. Jaeger, private communication. Note that we have made the approximation

$$\left\langle \frac{1}{n_T - 1} \right\rangle_n \simeq \frac{1}{\langle n_T \rangle_n - 1}.$$

[8]This is consistent with estimates in the FNAL ener-

gy range given by J. Whitmore, FNAL Report No. NAL-Pub-73/70-EXP (unpublished); E. Malamud, Bull. Amer. Phys. Soc. 19, 467 (1974).

[9]D. Z. Fredman, C. E. Jones, F. E. Low, and J. E. Young, Phys. Rev. Lett. 26, 1197 (1971); D. Sivers, ANL Report No. ANL/HEP 7246, 1972 (unpublished).

[10]The mass identification for positive tracks leaves 12% protons misidentified as pions in the region $|y| \leq 3$ with a resulting shift of about one unit in Δy and affects Fig. 3(c) only slightly.

[11]G. Goldhaber *et al.*, Phys. Rev. 120, 300 (1960).

[12]C. Quigg and G. H. Thomas, Phys. Rev. D 7, 2752 (1973); A. W. Chao and C. Quigg, Phys. Rev. D 9, 2016 (1974).