ment produced by a fixed drift. Numerical examples demonstrate that the comparatively weak drift current expected in a tokamak has almost no effect unless $\vec{k}$ and $\vec{B}$ are nearly perpendicular. Thus with $v_D = v_i$ and $\theta = 0$, $S(y)/S(y=0) - 1 = 10^{-4}$ near the maximum ($x = 0.18$) of the spectrum of the deuterium plasma containing 10^{-3} Mo^{42} ions per deuteron shown in Fig. 1. Enhancement as large as 1% is achieved only when θ reaches 89.7°. Similarly, the drift perturbation of the ST tokamak spectrum illustrated in Fig. 2 was computed at $x = 0.2$ to be less than 2% if θ was less than 60°. So, provided $\vec{k}$ and $\vec{B}$ are far enough from perpendicular, the influence of electron drift on the ion feature expected for a tokamak plasma appears to be negligible compared with impurity effects.

Since the Debye length in a tokamak plasma is 75 to 100 μm, the laser needed to reach the required α values at scattering angles large enough to retain the spatial resolution inherent in scattered-light experiments might be either HCN, operating at 337 μm for which a pulsed power of 1 kW has been reported,[7] or CO_2-pumped methyl fluoride at 496 μm at a pulsed power of 30 kW.[8] In either case, heterodyne detection[9] would be obligatory.

We conclude that contamination of a hydrogen plasma by heavy, high-Z ions can be measured by collective scattering of laser radiation down to about 10 ppm. In contrast to conventional spectroscopy, fully stripped ions are detectable. Moreover, a correctly designed scattering experiment could measure the effective charge $\bar{Z}$ of the plasma directly. Within the approximations of the linear theory of fluctuations, electron drift of the strength expected in a tokamak has negligible effect on these conclusions, provided that the angle between the magnetic field and the scattering vector is sufficiently far from perpendicular. Spatial resolution, without recourse to Abel inversion, is a feature inherent in the scattering technique.

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Ion Heating at Twice the Ion-Cyclotron Frequency in Reactor-Oriented Machines

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A systematic study is made of the efficiency of first-harmonic ion-cyclotron heating in hot, dense, large plasmas. Promising heating times are derived when the imposed axial wavelength satisfies the rather restrictive high-temperature, high-density limitation $k_z \geq \omega_{pi}^2 V_i \omega_{ci}^{-1} c^{-2}$.

Heating at the first harmonic of the ion-cyclotron frequency ($\omega = 2\omega_{ci}$) is considered to be one of the more attractive supplementary heating schemes for reaching ignition in large tori. As is well known,[1-3] this heating method depends upon the excitation of the fast hydromagnetic wave, whose electric field at $\omega = 2\omega_{ci}$ has an appreciable left-hand polarized component, i.e., in the sense of the ion rotation. Efficient excitation of the fast wave is, however, only possible above a threshold density, corresponding to the lowest radial magnetoacoustic resonance (see, e.g., Messiaen and Vandenplas[4]). This limit proves to be quite severe for most small-scale machines and little experimental evidence for heating is available until now. Dollinger *et al.*[5] reported wave damping and plasma heating in the beach of a linear device. Hosea and Hooke studied the fast-wave propagation in toroidal geometry[6] and reported ion heating.[7]

In the present Letter, we summarize a systematic study of the optimization of harmonic ion-cyclotron heating of large, dense, hot plasmas. The wave-particle interaction leading to such heating can be formulated by means of the collisionless dielectric tensor[8] expanded in powers of $\lambda_\alpha = k_\perp^2 KT_\alpha m_\alpha^{-1}\omega_{c\alpha}^{-2}$, where $k_\perp$ is the component of the wave vector perpendicular to the magnetic field, α denotes the species, T_α is the temperature (assumed to be isotropic), m_α the mass, and $\omega_{c\alpha}$ the cyclotron frequency. As usual,[2,3] the expansion parameter is then assumed to be such that terms of order higher than 1 need not be retained. A bounded-geometry description was first sketched by Vasileev *et at.*,[1] and a more thorough one given by Cato[9] with application to a particular experimental cold-plasma setup. The present treatment differs from the latter in its scope and in various amendments which will be systematically pointed out, but particularly through the boundary conditions which here apply to a magnetically confined plasma. First, we consider a homogeneous cylindrical plasma in a constant magnetic field, toroidal effects being discussed afterwards.

The infinite plasma column of radius a is surrounded by vacuum and excited from outside by a rf field periodic along the z axis, which can be expanded in a sum of transverse electric (TE) and transverse magnetic (TM) cylindrical waves, defined by their axial magnetic (H_{za}) and electric (E_{za}) field components[4]:

$$\begin{bmatrix} H_{za} \\ E_{za} \end{bmatrix} = \sum_{n=-\infty}^{\infty} \begin{bmatrix} F_n \\ G_n \end{bmatrix} J_n[(k_0^2 - k_{zn}^2)^{1/2} r] \times \exp[i(n\theta + k_{zn}z - \omega t)], \tag{1}$$

where r, θ, and z are the cylindrical coordinates, k_{zn} is the axial wave number corresponding to the nth azimuthal component, $k_0 = \omega c^{-1}$ is the vacuum wave number, and J_n is the Bessel function of order n. The applied field induces a rf field inside the plasma, given below, and a scattered field

$$\begin{bmatrix} H_{z\,\mathrm{sc}} \\ E_{z\,\mathrm{sc}} \end{bmatrix} = \sum_{n=-\infty}^{\infty} \begin{bmatrix} D_n \\ L_n \end{bmatrix} H_n^{(1)}[(k_0^2 - k_{zn}^2)^{1/2} r] \times \exp[i(n\theta + k_{zn}z - \omega t)], \tag{2}$$

where $H_n^{(1)}$ is a Hankel function. After imposing the boundary conditions, one can calculate the absorbed power P by applying Poynting's theorem over a surface S surrounding the plasma. The dielectric tensor elements K_{ij}, as given in Ref. 8, Eq. (9.9), are introduced in

$$c^2\vec{k}\times(\vec{k}\times\vec{E}) + \omega^2 \overleftrightarrow{K}\cdot\vec{E} = 0, \tag{3}$$

giving a dispersion relation of third order in $k_\perp^2$. By the inverse transformation from $k_\perp^2$ to $-\Delta_\perp^2$, a factorable sixth-order differential equation describing the perturbations inside the plasma is then found, the solution of which has the form

$$H_z = \sum_{j=1}^{3} A_{jn} J_n(k_{\perp j} r) \exp[i(n\theta + k_{zn}z - \omega t)], \tag{4}$$

where the $k_{\perp j}^2$ are the three roots of the dispersion relation. All other field components can then be expressed in terms of the same constants A_{jn}. For a given n, there must be five boundary conditions to determine the five unknowns: the A_{jn}, D_n, and L_n. Via the relationship between kinetic λ ordering and moment expansion of the Boltzmann equation (see also Ref. 10), we obtain the following constraints at the plasma (p)-vacuum (v) interface[11]:

$$\begin{aligned} &B_z^p - B_z^v B_0^v/B_0^p = -\mu_0 p_{rr}/B_0^p, \\ &E_z^p = E_z^v, \quad B_\theta^p = B_\theta^v, \quad p_{r\theta} = 0, \\ &B_r^p B_0^v/B_0^p - B_r^v = -\mu_0 p_{rz}/B_0^p. \end{aligned} \tag{5}$$

Aside from the familiar continuity of E_z and B_θ, these equations express the fact that like the equilibrium plasma pressure, the perturbed plasma-pressure components p_{rr}, $p_{r\theta}$, and p_{rz} must be balanced by a jump in magnetic field.

The numerical evaluation was carried out for typical thermonuclear parameters. The applied field is purely TE $(G_n = 0)$ and is characterized by a B_{za}^v of $10^{-4} B_0^v$ at the plasma boundary. We note the following:

(i) A typical dependence of the absorbed power per unit axial length P' on density (N) is shown in Fig. 1 for fixed k_z and $\omega = 2\omega_{ci}$. The absorption, which is low below the infinite-plasma cutoff $\omega_{pi}^2 k_z^{-2} c^{-2} = \frac{3}{4}$, is very much enhanced by the first magnetoacoustic resonances which are superposed on a background curve that increases roughly proportionally to density. At higher densities, it levels off or even drops. The overall behavior can be understood qualitatively on the basis of the infinite-plasma formula

$$p_i' = \pi^{3/2}a^2\epsilon_0\omega_{pi}^2V_ik_\perp^2 \exp\left[-\left(\frac{\omega-2\omega_{ci}}{k_zV_i}\right)^2\right]|iE_r - E_\theta|^2(8k_z\omega_{ci}^2)^{-1}, \qquad (6)$$

where ω_{pi} is the ion plasma frequency, ϵ_0 the vacuum permittivity, and $V_i=(2KT_im_i^{-1})^{1/2}$ the ion thermal velocity. It can be shown that

$$\alpha_n = \left|\frac{iE_r - E_\theta}{iE_r + E_\theta}\right| = \left|\frac{(iR+1)J_{n+1}(k_\perp r)}{(iR-1)J_{n-1}(k_\perp r)}\right|,$$

where $k_\perp$ is that of the fast wave. When $\omega_{pi}^2 > k_z^2c^2$, R can be approximated by $R=-i(\frac{4}{3}-S)/(-\frac{2}{3}+S)$, with

$$S=\tfrac{1}{4}(\omega V_i\omega_{pi}^2/c^2k_z\omega_{ci}^2)Z,$$

where Z is the plasma dispersion function of argument $(\omega-2\omega_{ci})k_z^{-1}V_i^{-1}$. For $n=0$ and cold plasmas, the left-hand component equals $\frac{1}{3}$ of the right-hand one. However, α_0 drops significantly when $\omega_{pi}^2V_i \gg k_z\omega_{ci}c^2$. For instance, for the case $T_i=10^8$ of Fig. 1, α_0 decreases from 0.39 at $N=10^{12}$ cm^{-3} to 0.05 at $N=10^{14}$ cm^{-3}. Physically, this very important hot dense plasma effect results from wave confluence (see Fig. 2) of the fast wave with an ion Bernstein wave. The third wave present is of lesser importance. Over the range where the fast wave is ion Bernstein like, its heating properties are very much reduced.

In Fig. 1, not only the relative importance of the left-hand component decreases with density, but also the magnitude of the average field inside the plasma. At fixed B_{za}^v, this can be due to two effects: a change of the surface impedance $(-E_\theta H_z^{-1})$ or a change of the field distribution inside the plasma which is characterized by an e-folding length proportional to Re$k_\perp$/Im$k_\perp$. As these are mainly bounded-plasma effects, general considerations are difficult to formulate.

(ii) A typical dependence of P' on k_z for fixed N (10^{14} cm^{-3}) and $\omega=2\omega_{ci}$ is shown in Fig. 3, where the k_z absorption band is strikingly clear. However, k_z values higher than 10 m^{-1} are entirely unsatisfactory for the large plasmas under consideration due to the exponentiallike $I_n(k_zr)$ falloff of the exciting field ($k_z>k_0$). As a result, the k_z bandwidth is even more reduced.

(iii) The absorption around $\omega=2\omega_{ci}$ is often very asymmetric. On the one hand, there is the influence of the ion Bernstein wave which can propagate for $\omega<2\omega_{ci}$ (see Fig. 2). On the other hand, the magnetoacoustic resonances which are excluded from the immediate vicinity of $2\omega_{ci}$ when $\omega_{pi}^2V_i>k_z\omega_{ci}c^2$ can be superposed anywhere on the $(\omega-2\omega_{ci})^2$ Gaussian of Eq. (6) when the inequality is reversed.

We now briefly turn to the implications for toroidal field geometry. Because of the magnetic inhomogeneity, absorption only occurs in a reso-

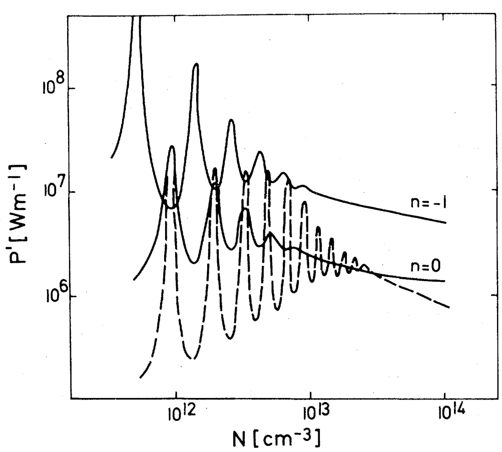

FIG. 1. P' versus density for a cylindrical plasma with parameters: deuterium, $a=1.3$ m, $B_0^v=5$ T, $k_z=1$ m^{-1}, $T_e=10^8$ °K. The solid lines apply to an ion temperature $T_i=10^8$ °K and azimuthal mode numbers $n=0$ and -1. For the dashed curve, $T_i=10^6$ °K and $n=0$. The influence of n is seen to be quite significant.

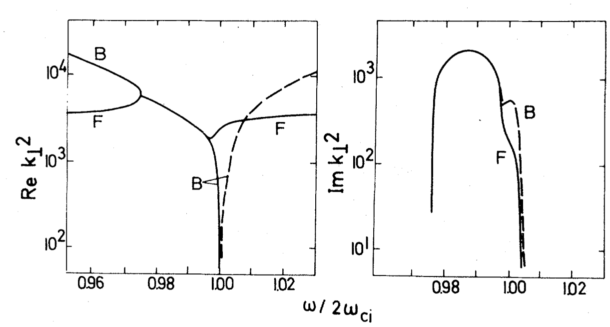

FIG. 2. Real and imaginary parts of the square of the perpendicular wave numbers of the first (F) and ion Bernstein (B) waves around $2\omega_{ci}$. The plasma parameters are those of Fig. 1, with $T_i=10^8$ °K and $N=10^{14}$ cm^{-3}. The solid lines represent positive values, the dashed lines negative ones. The waves are complex conjugate from the confluence up to close to $2\omega_{ci}$.

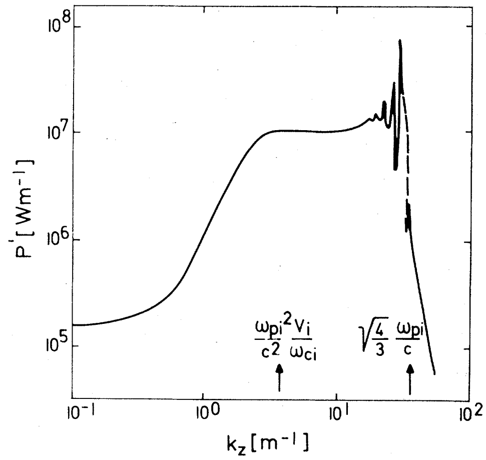

FIG. 3. P' versus axial wave number k_z for a cylindrical plasma whose parameters are those of Fig. 1, with $T_i = 10^8$ °K and $N = 10^{14}$ cm^{-3}. The arrows mark the position of the infinite-plasma cutoff $(\frac{4}{3})^{1/2}\omega_{pi}c^{-1}$ and the hot-plasma limit $\omega_{pi}{}^2 V_i \omega_{ci}{}^{-1} c^{-2}$.

nance zone of approximate width $\Delta = k_z V_i R \omega_{ci}{}^{-1}$ where R is the major radius. In Fig. 2, it can be seen that, when $\omega_{pi}{}^2 V_i \geq k_z \omega_{ci} c^2$, a fast wave approaching from the high-field side will encounter a zone of mode conversion before getting to this resonance zone. However, this is in no way restrictive, since the easiest way to launch waves experimentally is from the outside or low-field area of the torus. As regards the absorbed power, one can assume[3] that the energy is distributed over the whole plasma by means of the rotational transform, in which case the values found in the cylindrical approximation have to be multiplied by $\beta = (2/\pi)\Delta a^{-1}$. This leads to an important reduction when Δ, i.e., k_z is small. However, the values thus found are too pessimistic. Indeed, in our cylindrical approximation resonance exists everywhere and a wave entering the plasma will gradually damp as it proceeds. The equivalent field in Eq. (6) is then averaged over the whole plasma. In toroidal geometry, the wave remains undamped until it hits the resonance zone. While it will then damp with about the same e-folding length as before, the average field will now be higher.

On this basis, we attempt a final assessment of the heating efficiency for a deuterium plasma with $a = 1.3$ m, $N = 10^{14}$ cm^{-3}, $T_e = 10^8$ K, and $B_0{}^v = 5$ T and $R/a = 5$ under axisymmetric excitation. The normalization is now based on the assumption that for technological reasons the electric field E_θ at the plasma boundary must be limited to 3000 V/m. Table I shows typical optimal heating times τ for various T_i and k_z combinations. τ_1 is defined as $NKT_i \pi a^2/\beta P'$, while τ_2 is the average corrected value based on the computed e-folding lengths. As regards the $k_z = 10$-m^{-1} values, one should bear in mind that in order for a structure located at $r = 1.5$ m to create the assumed field, the E_θ value at that radius would have to be some 2×10^4 V/m. As τ scales as $E_\theta{}^{-2}$, very promising heating times could be achieved when higher fields are permitted.

TABLE I. Typical optimized heating times without (τ_1) and with (τ_2) field correction.

T_i (°K)	$k_z = 1$ m^{-1} τ_1 (sec)	$k_z = 1$ m^{-1} τ_2 (sec)	$k_z = 10$ m^{-1} τ_1 (sec)	$k_z = 10$ m^{-1} τ_2 (sec)
10^8	15.5	5.5	1.5	0.4
10^7	5.5	0.6	0.5	0.2
10^6	1.4	0.15	0.18	0.18

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