a second Maxwellian distribution with temperature $T_2 \gg T_i$ and density $n_2 \ll n_i$. The above emission formulas are valid for a double Maxwellian velocity distribution provided that, for the nth harmonic,[9] we replace T_e in I_B by $(n_1T_1^n+n_2T_2^n)/(n_1T_1^{n-1}+n_2T_2^{n-1})$ and $n_eT_e^{n-1}$ in $\int\alpha_e ds$ by $n_1T_1^{n-1}+n_2T_2^{n-1}$. The measured ratios I_3/I_2 and I_4/I_2 together with $n_1=n_e$ and $T_1=T_{e0}$ determine independent values $T_2 \sim 36(1+2\Delta T_e/T_e)$ keV and $n_2 \sim 8\times10^{12}(1-7\Delta T_e/T_e)$ m^{-3}, where $\Delta T_e/T_e$ is the uncertainty in the measured electron temperature (±10%). Although sensitive to the peak value of T_e (and to the assumed value of $1-r$), these results are not significantly modified by different profiles of $T_e(s)$.

One important characteristic of the spectrum observed for high-runaway conditions (Fig. 2, curve c) is the dominant broad peak at $\omega \sim 3.3\omega_{ce}$. Overlapping of I_3 and I_4 in the region $3.2<\omega/\omega_{ce}<3.6$ could occur for flat profiles $n_2(s)$ and $T_2(s)$. The above values of n_2 and T_2 are consistent with the ratio of this broad peak to I_2.

Discussion.—It is possible, in view of the uncertain absolute calibration, to assume that the second harmonic in Fig. 3 is fitted by the predicted curve for unpolarized emission [i.e., $2I_B(\omega)$] for the measured $T_e \sim 300$ eV. This change of calibration implies that the fundamental emission should be above the limit of detectability. However, the extraordinary-mode fundamental would be reflected in the decreasing B_φ at the upper-hybrid region,[10,11] leaving the ordinary mode below detectability, consistent with observation.

On the other hand, if I_2 really is well above $2I_B$ (Fig. 3) then we have to explain the suprathermal emission even for $n=2$.

The main implications of this work are that (i) diagnostic applications may be confused by runaway phenomena, and (ii) the power balance of a fusion reactor may be adversely affected by the enhanced cyclotron emission arising from the depolarization and the runaway effects.

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Rayleigh-Taylor Instability and Laser-Pellet Fusion

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The Rayleigh-Taylor instability in laser-driven spherical implosions can be stabilized by convective flow and by the "fire-polishing" effect, but the size of the stabilization effect depends on details of the thermal conductivity near the ablation surface.

In the basic concept for laser-pellet fusion, a dense cold shell of compressed deuterium-tritium is accelerated radially inward, while being compressed in a nearly adiabatic fashion.[1] When the shell reaches the center, it heats, ignites, and produces thermonuclear burning. The dense shell accelerates inward because of a sharp temperature front that continually ablates the outer

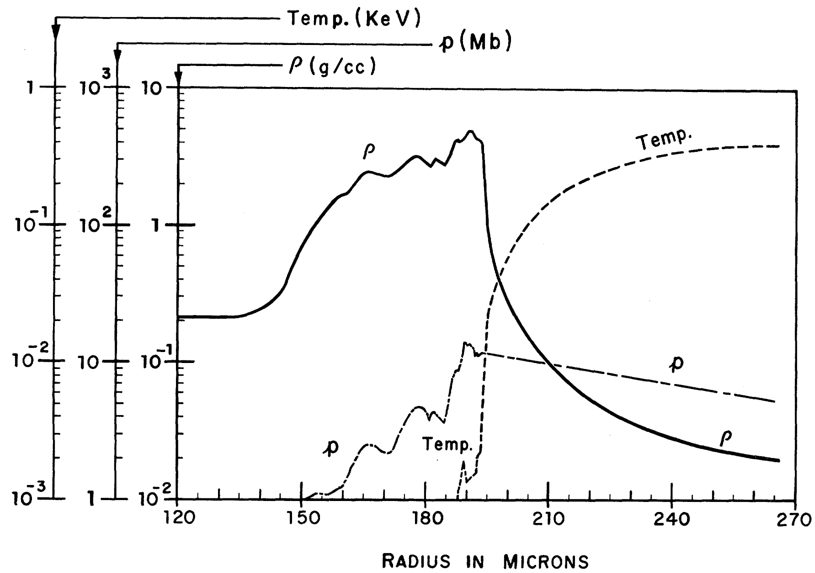

FIG. 1. Density, pressure, and temperature versus pellet radius, at one time, for a typical pellet implosion calculation.

edge of the cold compressed shell. Figure 1 shows sample radial profiles at one time of the mass density, electron temperature, and pressure, taken from a recent calculation by Nuckolls and Thiessen.[2]

In the accelerating frame moving with the ablation front, we have a dense fluid adjacent to a light fluid, with an effective outwardly directed gravitational force. This is the standard situation for the occurrence of the Rayleigh-Taylor instability, but with numerous modifications that are unique to the laser-pellet fusion concept. There have been several recent, and apparently conflicting, computer studies of Rayleigh-Taylor instability as it relates to laser-pellet fusion.[3] The only analytic work so far has been an unpublished derivation of the stabilizing effect of "fire polishing."[4] As the Rayleigh-Taylor instability grows, the peaks will be closer than the valleys to the laser deposition surface. The temperature gradients will thus be larger and the ablation faster at the peaks. This is the stabilizing "fire-polishing" effect.

Consider a model problem to isolate the effects of convective flow and of fire polishing. Also consider a slab geometry with two incompressible fluids connected by an ablation surface at $x = \xi(y,t)$. In this model problem, the equilibrium will be time independent, in the frame moving with the ablation front. In the laser-pellet fusion designs, the equilibrium actually changes on the nanosecond time scale. In order that our model be applicable to the pellet designs, attention is restricted to instability wavelengths such that the equilibrium does not change on the time scale of the growth rate. Without stabilization effects, $\gamma = (kg)^{1/2}$. At the sample time shown in Fig. 1, $g = 1.5\times10^{15}$ cm/sec^2. Thus I restrict my attention to wavelengths much less than 100 μm. I find, in fact, that under some circumstances the large-k modes are stabilized. More rigorously, I can only claim that I will show that large k modes grow no more rapidly than on the nanosecond time scale, because this is the time scale for changes in the equilibrium.

The perturbations in the high- and low-density regions can be treated as incompressible if the velocity of the plasma relative to the ablation surface is subsonic. In Fig. 1, the plasma becomes supersonic at about 230 μm. This supersonic flow will reduce the growth rate of perturbations with wavelength greater than or of the order of 35 μm.

The scale height at the ablation surface is less than 1 μm in Fig. 1. The finite gradient will limit the growth rate of perturbations with wavelength less than 1 μm.

Viscosity is important for perturbations such that $\gamma/(\nu k^2)<1$ or $\gamma d^2/\nu<1$. Here γ is the growth rate, ν is the specific viscosity at the ablation surface, k is the perturbation wave number, and d is the thickness of either the dense or the light fluid. At the ablation surface the ion temperature is less than 100 eV, so that viscosity is unimportant.

For all x and y use the one-fluid equations

$$\partial\rho/\partial t + \nabla\cdot(\rho\vec{v}) = 0, \tag{1}$$

$$\partial/\partial t(\rho\vec{v}) + \nabla\cdot(\rho\vec{v}\vec{v}) = -\nabla p + \rho\vec{g}. \tag{2}$$

For $x \neq \xi$, the fluids will be treated as incompressible:

$$\nabla\cdot\vec{v} = 0, \quad x \neq \xi, \tag{3}$$

For the jump conditions across the ablation front, use the model heat equation

$$\partial/\partial t(\rho e + \tfrac{1}{2}\rho v^2) + \nabla\cdot[\vec{v}(\rho e + \tfrac{1}{2}\rho v^2 + p)] = \rho\vec{v}\cdot\vec{g} + I(\xi)\delta(x-\xi). \tag{4}$$

I have modeled the electron thermal diffusion by a $\delta(x-\xi)$ deposition, with an $I(\xi)$ dependence to model the fire-polishing effect.

In the zeroth-order, time-independent equilibrium, the fluids have uniform density and velocity: ρ_0' and v_0' for $x<0$, ρ_0'' and v_0'' for $x>0$; $\vec{g} = +g\hat{e}_x$, with $\rho_0'' \ll \rho_0'$. Because the Rayleigh-Taylor instability involves surface motion, we write the hydrodynamic terms, with first-order perturbation, as

$$\psi(x,y,t) = \psi_0(x) - \xi(y,t)\Delta(\psi_0)\delta(x) + \tilde{\psi}(x,y,t), \tag{5}$$

where $\Delta(\psi_0)$ is the jump in the zeroth-order quantity across $x=0$, and $\tilde{\psi}(x)$ is bounded for all x and goes to zero as $x \to \pm\infty$. Here ψ represents any of the quantities appearing in Eqs. (1)–(4): ρ, v_x, v_y, p, ρv_x^2, $\rho v_x v_y$, etc. The fluid equations have been written in the form of a complete derivative in x to avoid ill-defined quantities such as $\delta(x)S(x)$, where $S(x)$ is the step function.

I look for solutions of the perturbed quantities of the form $\exp(\gamma t + iky)$, and obtain a set of jump conditions:

$$-\gamma\xi\Delta(\rho_0) + \Delta(\rho_0\tilde{v}_x + \tilde{\rho}v_0) = 0, \tag{6}$$

$$\Delta(\tilde{p} + \tilde{\rho}v_0^2 + 2\rho_0 v_0\tilde{v}_x) = -\xi g\Delta(\rho_0), \tag{7}$$

$$\Delta(\tilde{v}_y + ik\xi v_0) = 0, \tag{8}$$

$$\gamma\xi\rho_0 v_0\Delta(v_0) + \Delta(\tfrac{5}{2}\tilde{p}v_0 + \tfrac{5}{2}p_0\tilde{v}_x + \tfrac{1}{2}\tilde{\rho}v_0^3 + \tfrac{3}{2}\rho_0 v_0^2\tilde{v}_x) = I(\xi) - I(0) \equiv \xi I(0)/L. \tag{9}$$

For $x<0$, the perturbed quantities satisfy the relations

$$\tilde{v}_y' = i\tilde{v}_x', \tag{10}$$

$$\tilde{p}' = -\rho_0'(\gamma/k + v_0')\tilde{v}_x', \tag{11}$$

$$\tilde{\rho}' = 0. \tag{12}$$

For $x>0$, there is one relation between the perturbed quantities, in the limit $x \to 0$:

$$\tilde{p}'' + i\rho_0'' v_0''\tilde{v}_y'' - \frac{\gamma\rho_0''}{k}\tilde{v}_x'' + \frac{g v_0''}{\gamma + k v_0''}\tilde{\rho}'' = 0. \tag{13}$$

Note that $\tilde{\rho}''$ is not necessarily zero, because it can satisfy the relation $\gamma\tilde{\rho}'' + v_0''\partial\tilde{\rho}''/\partial x = 0$ with the form $\exp(-\gamma x/v_0'')$.

We now have eight equations, (6)–(13), for nine quantities: $\tilde{\rho}'$, $\tilde{\rho}''$, $\tilde{v}_x'$, $\tilde{v}_x''$, $\tilde{v}_y'$, $\tilde{v}_y''$, $\tilde{p}'$, $\tilde{p}''$, and ξ. If we expand the ablation region to a nonzero thickness, $(\xi - a) < x\ (\xi + a)$, then at $x = \xi - a$ the density, velocity, and pressure should be continuous. The density is ρ_0'. If we specify the temperature at $x = \xi - a$, we will have our ninth equation. The temperature just inside the ablation region is determined by the previous adiabatic compression, the thermal heat transport, and by suprathermal energy transport. As the ninth equation, we will assume that

$$p_1'(\xi) = \tilde{p}'(\xi) + \rho' g\xi = \beta p_0'(0)\xi/L, \tag{14}$$

where β is some unknown constant, of order 1, or perhaps much less than 1, depending on the unspecified details in the energy transport.

We now obtain a cubic dispersion relation,

$$u\sigma^3 + [u^2 - (1-P)]\sigma^2 + [u - 2u(1-P) + \epsilon u^3(1+\beta)]\sigma + [(1-u^2)(1-P) + \epsilon^2\beta u^4] = 0, \tag{15}$$

with $\sigma = \gamma/(kg)^{1/2}$, $\epsilon = \rho_0'/\rho_0''$, $u = kv_0''/(kg)^{1/2}$, and $P = \beta k p_0'(0)/(\rho_0' g)$. In deriving this equation, we have assumed that v_0'' is small compared to the acoustic velocity, and $\epsilon \ll 1$, but with no ordering assumptions on u or σ. If the distance between the ablation surface and the laser energy deposition surface is D, and if $kD \gg 1$, then one can show that $kL \cong 1$. This approximation has been used also.

If we set $u=0$ (a singular point in the cubic equation), the solution is $x = \pm 1$, the standard Rayleigh-Taylor result when $\epsilon \ll 1$.

In the standard Rayleigh-Taylor theory, $\gamma \to \infty$ as $k \to \infty$. From Eq. (15) we find that γ now has a maximum and then goes to zero for finite k. The maximum k for instability is the larger of $g/v_0''^2$ and $g\rho_0'/\beta p_0'(0)$.

If we ignore the fire-polishing effect by letting $\beta \to 0$ and $kL \to \infty$, instead of $kL = 1$, we obtain the stabilization due solely to convection:

$$\gamma = (kg)^{1/2} - kv_0'. \tag{16}$$

If we add in thermal conduction, $kL = 1$, but keep

$\beta = 0$, we obtain a weaker stabilization effect:

$$\gamma \cong -\tfrac{1}{2} k v_0' + (kg + \tfrac{1}{4} k^2 v_0'^2)^{1/2}. \tag{17}$$

Thus under some circumstances, fire polishing can be destabilizing. Even though the peaks ablate faster than the valleys, the perturbed flow in the dense fluid is such as to move the peaks outward.

The stabilization phenomena depend on the parameter β, which in turn depends on details in the thermal conductivity in the region of the ablating surface. This result is similar to the general theory of weak deflagrations,[5] where one concludes that the fluid velocity relative to the ablation surface depends upon heat conductivity, and is not totally determined by the Rankine-Hugoniot relations. In weak deflagrations, such as in the laser-pellet implosion, one can derive the approximate relation

$$\tfrac{5}{2} p_0'(0) v_0'' \cong I_0.$$

Combining this with the definition of β, we obtain

$$\frac{1}{\beta} \cong 1 + \left.\frac{p_0' \, dv_0''/dt}{v_0'' \, dp_0'/dt}\right|_{x=0}. \tag{18}$$

There are a number of inherent assumptions that give lower k limits to our theory's validity. Because the dense fluid should have a finite thickness d, we have $k > 1/d$. Incompressibility in the dense fluid requires that an acoustic wave be able to travel one wavelength in one e-folding time: $k\bar{v}_{ac} > \gamma$, where $\bar{v}_{ac}$ is the mean acoustic velocity $1/k$ in from the ablation surface. If $\gamma^2 \cong kg$, we find that $k > g/\bar{v}_{ac}^2$.

Finally, for small P, there is another unstable root of Eq. (15), which is driven by the convective behavior, and is not of the Rayleigh-Taylor type. It satisfies the dispersion relation

$$\gamma = g/v_0'' - kv_0''. \tag{19}$$

The physical mechanism behind this mode is unknown.

This research was intended as a model calculation to examine some stabilization mechanisms for Rayleigh-Taylor instability, to give insight into some of the important physics, to give guidelines to interpreting computational results, and to indicate the basic parameters of interest. In the case shown in Fig. 1, the acceleration is 1.6×10^{15} cm/sec^2. Using Eq. (18), find that $\beta \cong 0.7$. The maximum wave number is then 700 cm^{-1}. But because of the many assumptions in this theory, this value should be treated with some skepticism. Also because of the flexibility in laser-pellet fusion designs, one cannot draw any overall conclusions at the present time about the importance of Rayleigh-Taylor instability in laser-fusion, except to note that there are stabilization mechanisms.

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