prior to publication.

*Work supported by the U. S. Atomic Energy Commission.

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Solitons and Resonant Absorption

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(Received 24 June 1974)

Simulations and theory of driven plasmas show the existence of long-lived, extremely intense spikes of high-frequency electric field, accompanied by a self-consistent depression in density. These entities are the limit to which solitons collapse in a driven plasma when the wave-particle interaction is included. Alterations in the density profile due to such entities is especially important for the absorption of radiation in inhomogeneous plasmas.

The role of solitons in strong electron-plasma-wave turbulence is receiving widespread attention. Zacharov[1] has shown that electron plasma waves can be described by a Schrödinger equation, which admits such localized solutions. Kingsep, Rudakov, and Sudan[2] have pointed out that the saturated spectrum of plasma waves observed both in beam-plasma simulations[3] and in simulations of plasmas driven by intense fields[4] indicate the presence of solitons. Morales, Lee, and White[5] have recently modified the Schrödinger equation to include a monochromatic, homogeneous pump $E_0 \cos(\omega_0 t)$ with $\omega_0 = \omega_{pe}$, the electron plasma frequency. They find then that the soliton width collapses and its intensity increases faster than exponentially. The final soliton intensity is limited in their formalism only by pump depletion. We present here the results of particle-simulation studies in which we investigate the final state of solitons in a driven plasma. Strong dissipation due to the onset of the wave-particle interaction prevents the complete soliton collapse predicted by the Schrödinger equation model. With the inclusion of the wave-particle interac-

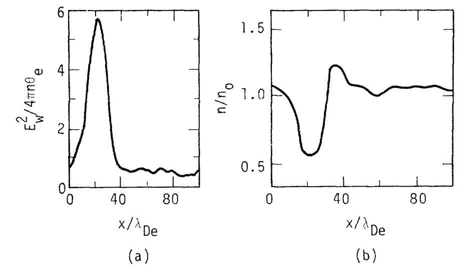

FIG. 1. The spatial dependence of (a) the electric field intensity, and (b) the ion density observed in a one-dimensional electrostatic computer simulation. The parameters are $eE/m\omega_{pe} = 0.8V_{te}$, $\omega_0 = 0.9\omega_{pe}$, $m/M = 0.01$, and $\omega_0 t = 600$.

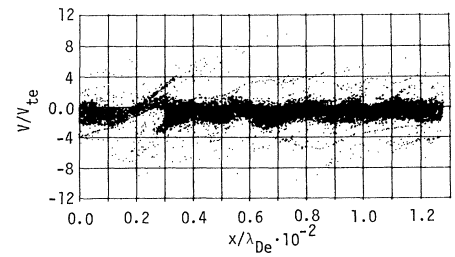

FIG. 2. Electron phase space (v versus x) from the computer simulation. The parameters are the same as those for the previous figure.

tion, solitons become very intense and narrow spikes of high-frequency electric field accompanied by a self-consistent density depression.

We first show an example of a soliton as observed in a one-dimensional simulation of an initially uniform plasma driven by a spatially homogeneous pump field. The pump frequency ω_0 is equal to $0.9\omega_{pe}$. This choice is not essential but enables us to isolate the soliton formation from other mode-coupling processes which occur simultaneously when $\omega_0 > \omega_{pe}$. The initial electric field and density fluctuations are random. Figure 1(a) shows a plot of the square of the peak self-consistent plasma-wave field versus position at $\omega_{pe}t = 600$, and Fig. 1(b) depicts the ion density versus position at that time. Note the narrow width ($l \sim 15\lambda_{De}$) and the very large peak intensity maximum ($E_p^2/4\pi n\theta_e \sim 5$). Depending on the initial random noise and the size of the simulated plasma, one or more of these entities are observed to form. As long as they are well separated, they can coexist, although they appear to interact through the radiation of sound waves. This interaction is particularly apparent for two solitons separated by a distance comparable to their width. Then the larger soliton can annihilate the other by "filling in" the latter's density depression.

The pump field intensity $E_0^2/4\pi n\theta_e$ equals 0.64 which is about 1.5 times the threshold intensity for the oscillating two-stream instability[6] when $\omega_0 = 0.9\omega_{pe}$. This threshold is the accessibility criterion for the formation of solitons in a quiescent plasma. However, the simulations show that this threshold can be significantly lowered if an elevated level of turbulence or noise is present. This can be a particularly important consideration for real plasma, since a number of instabilities often operate simultaneously.

The plasma is heated very effectively after the onset of the wave-particle interaction. The plasma temperature increases linearly with time in this simulation to 25 times its initial value in a time $\Delta t = 100/\omega_{pe}$. The dissipation rate, defined in terms of the electrostatic wave energy and the rate of temperature increase by

$$\frac{d}{dt}\int dx\, nT = \nu^* \int dx \frac{E^2(x)}{8\pi},$$

is thus seen to be substantial, about $0.4\omega_{pe}$. Figure 2 shows an electron-phase-space plot (v versus x) from the simulation, again at $\omega_{pe}t = 600$. Electrons are accelerated through the narrow, intense structure, forming streamers in phase space which are similar to those seen in simulations of resonant absorption.[7] Indeed, this heating can be viewed as resonant absorption on nonlinearly produced density channels. The velocity increment of an electron which samples the maximum field as it transits the soliton is $\sim\sqrt{2}V_W$, where $V_W \equiv eE_p/m\omega$. If we add to this increment the initial particle velocity, we see that velocities in excess of five thermal velocities are attainable in a single pass. The efficient heating observed is not surprising when one considers that some particles, albeit only the fastest, suffer velocity changes of $O(V_W)$ in a transit time which is $O(\pi/\omega_0)$. Even in the presence of strong heating, solitons are observed to persist for times in excess of $500\omega_{pe}^{-1}$, after which time the plasma temperature has increased by almost 2 orders of magnitude.

In these periodic simulations, soliton lifetime is principally determined by large zero-order changes in the plasma temperature, which, for example, can drive the oscillating two-stream instability below threshold. We are currently examining the formation of solitons in simulations which model a heating region of finite length. Escaping heated particles are replaced by colder ones, allowing a steady state to be reached. These simulations will enable us to isolate the effects of ion inertia on the lifetime.

The final soliton width appears to be universal and is determined by very basic considerations. The adiabatic invariant $mv^2/2+e\varphi$, where φ is the ponderomotive potential $[e\varphi=\frac{1}{4}e^2E_0^{\,2}(x)/m\omega^2]$, can be used to describe the motion of a particle of velocity v in an almost monochromatic electric field if the inequality $v/L\omega \ll 1$ is satisfied. Here L is the scale of spatial variation of the field and ω is its frequency. A sufficiently slow particle transits a localized region of oscillating field with only an exponentially small energy change (if the spatial variation of the field is analytic). It is just the breakdown of the constancy of this invariant for a large number of particles which signals the onset of strong heating. These considerations allow us to estimate the width l of a soliton by setting $v/l\omega=1$ or $l/\sqrt{3}\,v_{Te}=2\pi/\omega_0$. Here the $\sqrt{3}\,v_{Te}$ arises from the choice of a rectangular velocity distribution of the form $f_e(v)=n/2\sqrt{3}\,v_{Te}$ for $|v|\leq\sqrt{3}\,v_{Te}$, $=0$ otherwise. This estimate, $l\simeq 12\lambda_{De}$, is in reasonable agreement with the observed full width at half-maximum intensity of about $15\lambda_{De}$. In other simulations with Maxwellian distributions, essentially the same width is observed.

We have quantified this estimate by numerically calculating the energy change of particles which are incident upon a localized field structure of the form $E(x,t)=E_0\,\mathrm{sech}(kx)\cos(\omega t)$. The functional form $\mathrm{sech}(kx)$ provides a reasonable fit to the observed spatial dependence of E. The relevant amplitude parameter $k\Delta x\equiv keE_0/m\omega_0^{\,2}$ was chosen to be 0.3, the value observed in the simulations.

Many particles of each initial velocity class $v>0$ were distributed uniformly in initial position over an interval $x_0-\Delta x\leq x\leq x_0$ with $\Delta x\,\omega/v=2n\pi$ and $x_0k\ll -1$. Their motion was followed numerically until their interaction with the structure was complete. The phase-averaged energy change of each class was computed. Particles of velocity $v<v_1\equiv eE_0/\sqrt{2}\,m\omega_0$ were reflected with no energy change. Those with $v_1<v<v_2\simeq 0.3\omega_0/k$ passed through with no noticeable energy change. Particles with larger initial velocities gained energy. The maximum energy gain was for particles of velocity class $v_3\simeq 0.95\omega_0/k$. Their maximum velocity change (as a function of initial phase) was $\sim\sqrt{2}\,V_W$. The energy gain decreased monotonically thereafter with increasing initial velocity.

The amplitude of the energy change curve varied linearly with $E_0^{\,2}$ as $E_0^{\,2}$ decreased over 5 orders of magnitude, suggesting that the value $k\Delta x=\frac{1}{3}$ is small enough for the nonadiabatic particles to validate a linearized calculation of the wave-particle damping. We have analytically calculated the linear dissipation rate for the assumed form $E(x,t)=E_0\,\mathrm{sech}(kx)\cos(\omega t)$ as a function of $k\lambda_{De}$ for both rectangular and Maxwellian distributions. Briefly stated, both numerical and analytic analyses predict a smaller than observed value for ν^* unless $k\lambda_{De}$ is taken about 50% larger than observed. We expect that this discrepancy can be removed by including the self-consistent distribution function in such a calculation rather than the initial distribution function $[T_e(\omega_{pe}t=600)\simeq 2T_e(0)]$.

The maximum intensity is more difficult to estimate. In the sample simulation, the plasma temperature changes so rapidly that no simple estimate of the intensity is very convincing. The maximum intensity becomes large enough to initially reflect nearly all the particles from the soliton, i.e., $e_0E_p/m\omega\sim\sqrt{6}\,v_{te}$.

Both because the soliton grows so rapidly until the onset of heating and because the temperature increases so rapidly thereafter, the ion-density depression associated with the intensity does not follow from a balance between the plasma and ponderomotive pressure, i.e., $\ln(n/n_0)\neq e\varphi/\theta_e$. Another indication of the importance of ion inertia is obtained upon examination of ion phase space where ion flow velocities comparable to the acoustic velocity are observed. In the finite-interaction-length simulations currently under investigation, the long soliton lifetime and the attainment of steady plasma temperature allow a closer pressure balance to be maintained at long times after soliton formation. After the formation of the density channel, the ion flow velocity vanishes and pressure balance provides a good estimate of the relative density depression.

Intense, localized spikes of high-frequency field play a role in many practical applications. They appear to be a general feature of strong electron-plasma-wave turbulence. For example, Estabrook, Valeo, and Kruer have carried out many two-dimensional, electromagnetic, relativistic simulations[8] of resonant absorption of radiation in plasmas. A common feature observed in the simulations[8,9] is a pronounced steepening

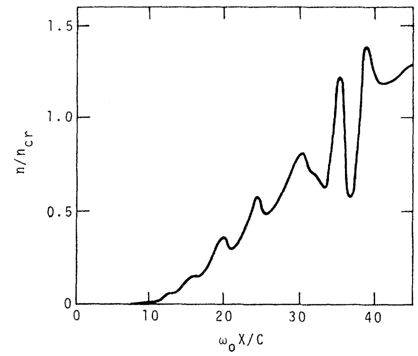

FIG. 3. An ion density profile from a two-dimensional electromagnetic simulation of resonant absorption of intense radiation. The profile initially had a linear rise to $1.5n_{cr}$ in $2\lambda_0$, where $n_{cr}=m\omega_0^2/4\pi e^2$ and λ_0 is the free-space wavelength. The energy density of the radiation is $\frac{1}{2}$ the plasma thermal energy density, and the angle of incidence is 24°. Plasma expansion later reduces the size of the secondary density maxima, leaving a density step from subcritical to supercritical.

of the density profile near the critical density at which resonant excitation occurs, as shown in Fig. 3. This density step is driven by a strong gradient in the resonantly excited electron-plasma wave intensity, as well as by a gradient in the electron temperature due to the plasma heating. The result is the production of an intense narrow spike of high-frequency electric field with a self-consistent sharp variation in plasma density, both of which have a scale length of (10–20)λ_{De}.

This nonlinear steepening of the density profile is found to have a number of important consequences. It reduces the region of plasma accessible to instabilities transverse to the density gradient and makes resonant absorption an effective mechanism for larger angles of incidence. The density modifications can also enhance the amount of absorption. For example, we have observed resonant absorption to occur multiply on a series of nonlinearly produced density channels near the critical density. Sizable modification of the density profile due to resonant absorption has been recently observed in microwave experiments,[10] even with rather modest incident power levels.

Finally we note that the source of the pump electric field can be an injected electron beam rather than incident radiation. We have run simulations which verify this assertion. For example, a beam of density $n_B/n_0=2\times10^{-2}$, velocity $v_B/v_{te}=25$, and temperature $\theta_B=\theta_e$ first excites the electron two-stream instability. This leads to the formation of a large amplitude, long wavelength ($kv_B\simeq\omega_{pe}$) plasma wave. This plasma wave then excites the oscillating two-stream instability,[3,11] which leads to soliton formation. The solitons then efficiently transfer their energy to the background electrons as described above. Again, the soliton width is approximately $15\lambda_{De}$. Solitons may have been observed in beam-plasma experiments,[12] where apparently stationary and intense structures of width ~ $10\lambda_{De}$ appear upon beam injection.

We gratefully acknowledge many useful discussions with K. G. Estabrook, J. J. Thomson, A. B. Langdon, and B. F. Lasinski. All work was performed under the auspices of the U. S. Atomic Energy Commission.

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