to the lower-energy n-p data. They indicate a smooth rise of about 1.5 mb in the n-p total cross section between 50 and 280 GeV/c. In general, the n-p and p-p total cross sections seem to be approximately equal over the range 30 to 300 GeV/c. Our highest-energy n-p points agree farily well with the p-p total cross sections at 200 and 300 GeV/c measured previously by our group.[10] The combined n-p and p-p data above 20 GeV/c can be well fitted with the expression $\sigma = 38.4 + 0.85|\ln(s/95)|^{1.47}$ mb.

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[1]For a detailed description of the beam see, M. J. Longo *et al.*, University of Michigan Report No. UM HE 74-18 (unpublished).

[2]The target length was measured at room temperature and at liquid-nitrogen temperature. Since the flask was made of a known aluminum alloy, the length at hydrogen temperature could be easily calculated from thermal-expansion data. The change in length from nitrogen to hydrogen temperature was only 0.2%.

[3]H. M. Roder *et al.*, *Survey of the Properties of the Hydrogen Isotopes below Their Critical Temperatures*, U. S. National Bureau of Standards Technical Note No. 641 (U. S. GPO, Washington, D. C. 1973).

[4]Because of the finite opening angle of the cone containing the charged secondaries, the effective size of the transmission counters was somewhat larger than their geometrical size. The effective size was determined by scanning the beam stepwise across the iron converter. This gave the probability of detecting a neutron as a function of distance from the beam axis.

[5]L. W. Jones *et al.*, University of Michigan Report No. UM HE 73-24 (to be published).

[6]This is a partial coincidence of two pulses from the calorimeter which shifts one or both pulses to a higher energy bin.

[7]W. F. Baker *et al.*, "Measurement of π^+, $K^\pm$, p, and $\bar{p}$ Production by 200 and 300 GeV/c Protons" (to be published).

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Pion-Nucleon Form Factor in the Chew-Low Theory*

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We suggest a form of the off-shell pion-nucleon amplitude which is motivated by simple field-theoretic considerations. This amplitude contains a momentum-dependent form factor $v(k)$ which we determine by solving an inverse scattering problem, using the experimental phase shifts and inelasticities as input.

Phenomenological pion-nucleon interactions have been of the form of zero-range interactions,[1] of separable energy-independent potentials,[2] and of separable energy-dependent potentials.[3] Each approach yields a different off-shell two-body T matrix, even though the on-shell data may be reproduced. The sensitivity of the pion-nucleus cross sections to the off-shell behavior has been explored by various authors.[4]

Here, we attempt a somewhat more fundamental view. We seek as much guidance as possible from an underlying field theory, even though such guidance may be incomplete. We use a nonrelativistic pion-nucleon interaction Hamiltonian $H_{\rm int}$ which couples the pion field linearly to the nucleon density, viz.,

$$H_{\rm int}=(if/\mu)\sum_{p,k,\alpha}\sum_{\nu,\nu'}v(k)(\vec{\sigma}\cdot\vec{k})_{\sigma\sigma'}(\tau_\alpha)_{\tau\tau'}a_{p+k,\nu'}{}^{\dagger}a_{p\nu}Q_{k\alpha}, \tag{1}$$

where $Q_{k\alpha}=[2\omega_\pi(k)]^{-1/2}(B_{k\alpha}+B_{-k\alpha}{}^{\dagger})$; $B_{k\alpha}{}^{\dagger}$ and $a_{k\nu}{}^{\dagger}$ are the creation operators for a pion or nucleon of momentum k and spin-isospin state α or $\nu=\{\sigma,\tau\}$, respectively. The *ad hoc* form factor $v(k)$ provides a high-energy cutoff. Our objective is to obtain $v(k)$ directly from the πN-scattering data. Thus we view $H_{\rm int}$ as a semiphenomenological model. The flexibility allowed by $v(k)$ suffices to enable the model to establish contact with πN data. Such a treatment is not meant to replace a more fundamental theory.[5]

In the static limit, this Hamiltonian leads to a Low equation for the pion-nucleon scattering amplitude,[6]

$$\langle p|T(\omega_\pi)|q\rangle=-\textstyle\sum_n[\langle p|T^{\dagger}|n\rangle\langle n|T|q\rangle/(E_n-\omega_\pi-i\epsilon)+\langle q|T^{\dagger}|n\rangle\langle n|T|p\rangle/(E_n+\omega_\pi)], \tag{2}$$

where $|n\rangle$ and E_n represent a complete set of states and their corresponding energies. These states $|n\rangle$ correspond to a physical nucleon plus n physical pions. If we now truncate the sum over n to include only the single-nucleon and the nucleon-plus-*one*-pion intermediate states (thereby ignoring pion production in pion-nucleon collisons), and further ignore $n>0$ contributions to the crossing term, we can write the solution to the Low equation as[6,7]

$$\langle q|T_\alpha(\omega_\pi)|q'\rangle=\left(-\frac{2\pi\lambda_\alpha qq'v(q)v(q')}{\omega_\pi{}^2}\right)\left(1-\frac{\lambda_\alpha\omega_\pi}{\pi}\int_0^\infty\frac{p^4dp}{\omega_\pi{}^3(p)}\frac{v^2(p)}{\omega_\pi(p)-\omega_\pi-i\epsilon}\right)^{-1}, \tag{3}$$

where $\alpha=\{2J,2T\}$ indexes the spin-isospin channel and $\lambda_\alpha=(f^2/4\pi)(2/3\mu^2)[-4,-1,-1,2]$ for $\alpha=[11,13,31,33]$. The *off-shell* dependence of T_α indicated by Eq. (3) cannot be reproduced by an *energy-independent* potential, although an *energy-dependent* potential can be found which will do so.[8]

We would like to determine $v(q)$ from the pion-nucleon elastic-scattering data. However, Eq. (3) is not adequate for that purpose, since it yields real phase shifts beyond the pion-production threshold. Thus, one must go beyond the one-meson approximation and take account of inelastic channels.

We propose to replace the energy-independent coupling constant λ_α, by $\lambda_\alpha\gamma_\alpha(\omega_\pi)$, where $\gamma_\alpha(\omega_\pi)$ is a complex function.[9] This leads to a modification of Eq. (3) in which the denominator becomes

$$D_\alpha(\omega(q))=\frac{1}{\gamma_\alpha(\omega_\pi(q))}-\frac{\lambda_\alpha\omega_\pi(q)}{\pi}\int_0^\infty\frac{p^4dp}{\omega_\pi{}^3(p)}\frac{v^2(p)}{\omega_\pi(p)-\omega_\pi(q)-i\epsilon}. \tag{4}$$

Such a replacement results, in the static limit, from the inclusion of a large subset of the neglected multipion terms on the right-hand side of Eq. (2). Thus our approximation is similar to the Chew-Low approximation with a much less radical truncation of Eq. (2). However, this argument is not compelling, because the static limit becomes a poorer approximation as more pions are produced.

Inclusion of the factor $\gamma_\alpha(\omega_\pi)$ permits us to satisfy unitarity, so that the T matrix will have the appropriate discontinuity across the inelastic cut. As a consequence of unitarity, $\gamma_\alpha(\omega_\pi)$ and $[\gamma_\alpha(\omega_\pi)]^{-1}$ satisfy dispersion relations. We assume that $\gamma(\omega_\pi)\to1$ as $\omega_\pi\to\infty$. The behavior of $\gamma(\omega_\pi)$ is discussed in Ref. 3 for potential scattering. Similar considerations obtain here. Thus we write

$$\frac{1}{\gamma_\alpha(\omega_\pi)}=1-\frac{1}{\pi}\int\frac{\mathrm{Im}[1/\gamma_\alpha(\omega_\pi{}')]}{\omega_\pi-\omega_\pi{}'+i\epsilon}d\omega_\pi{}'. \tag{5}$$

The above theory is derived for infinitely massive nucleons. We wish, however, to make contact with the scattering from finite-mass nucleons. Customarily, this is done by identifying q in Eqs. (3) and (4) as the momentum of the pion in the c.m. frame. With this identification, the T matrix satisfies the unitarity relation

$$T_\alpha(q)-T_\alpha{}^{\dagger}(q)=-\frac{i}{\pi}[q\omega_\pi(q)]T_\alpha(q)T_\alpha{}^{\dagger}(q)\left(1-\frac{\mathrm{Im}[1/\gamma(\omega_\pi(q))]}{\lambda_\alpha q^3v^2(q)/\omega_\pi(q)}\right). \tag{6}$$

The physical T matrix for the scattering from a finite-mass nucleon, however, satisfies a unitarity relation of the form of Eq. (6), with $[q\omega_\pi(q)]$ replaced by $[q\omega_\pi(q)]\omega_N(q)/\omega(q)$, where $\omega(q)\equiv\omega_\pi(q)+\omega_N(q)$

$= (q^2+m_\pi^2)^{1/2} + (q^2+m_N^2)^{1/2}$. To make contact with the data, we thus modify Eqs. (3) and (4) to satisfy the finite-mass unitarity relation. This can be done by replacing $\omega_\pi(q)$ in Eqs. (3) and (4) by $\tilde\omega_\pi(q) = \omega(q) - m_N$.

With this modification, we may invert Eqs. (3) and (4) to determine $v(q)$ from the elastic-scattering T matrix. Using Eqs. (3) and (4) we may write

$$\mathrm{Im}\frac{1}{\gamma_\alpha(\tilde\omega_\pi(q))} = \frac{\lambda_\alpha}{\pi}\frac{q^2v^2(q)}{\tilde\omega_\pi^2(q)}\left[-\mathrm{Im}\left(\frac{2\pi^2}{T_\alpha(\tilde\omega_\pi(q))}\right) + \frac{\pi q\omega_\pi(q)\omega_N(q)}{\omega(q)}\right]. \tag{7}$$

If we introduce the inelasticity parameter $\hat\eta$ and the real phase shift $\hat\delta$ via[3]

$$T_\alpha(q) = -[2\pi\omega(q)/q\omega_\pi(q)\omega_N(q)]\hat\eta_\alpha(q)\exp[i\hat\delta_\alpha(q)]\sin\hat\delta_\alpha(\epsilon), \tag{8}$$

we find

$$D_\alpha(\omega) = 1 - \frac{\lambda_\alpha}{\pi}\int_0^\infty \frac{p^4\,dp}{\tilde\omega_\pi^2(p)}\frac{v^2(p)[1-\hat\eta_\alpha^{-1} - (\omega - m_N)/\tilde\omega_\pi(p)]}{\omega - \omega(p) + i\epsilon}. \tag{9}$$

Since $D_\alpha(\omega)$ does not approach unity as $\omega\to\infty$, we consider the ratio $D_\alpha(\omega)/D_\alpha(\infty)$. The analytic structure of $D_\alpha(\omega)$ implies that the logarithm of this ratio satisfies a dispersion relation, i.e.,

$$\mathrm{Re}\ln\frac{D_\alpha(\omega)}{D_\alpha(\infty)} = \frac{1}{\pi}\,\mathrm{P}\int d\omega'\,\frac{\mathrm{Im}\ln[D_\alpha(\omega')/D_\alpha(\infty)]}{\omega'-\omega}. \tag{10}$$

Using the relation $\mathrm{Im}\ln[D_\alpha(\omega(p))/D_\alpha(\infty)] = -\hat\delta_\alpha(p)$, we find that Eq. (10) has the solution

$$D_\alpha(\omega)/D_\alpha(\infty) = \exp\{\pi^{-1}\textstyle\int\hat\delta_\alpha(p)\,d\omega(p)/[\omega-\omega(p)+i\epsilon]\}. \tag{11}$$

If we equate the imaginary parts of Eq. (9) and (11) we find

$$\frac{v^2(q)}{D_\alpha(\infty)} = -\frac{\hat\eta_\alpha\omega_\pi(q)\omega(q)}{\lambda_\alpha q^3\omega_N(q)}\sin\hat\delta_\alpha(q)\exp\left(\frac{1}{\pi}\mathrm{P}\int\frac{\hat\delta_\alpha(\omega')d\omega'}{\omega(q)-\omega'}\right). \tag{12}$$

The limit of Eq. (9) then yields

$$D_\alpha(\infty) = 1 + \frac{\lambda_\alpha}{\pi}D_\alpha(\infty)\int_0^\infty\frac{p^4\,dp}{\tilde\omega_\pi^3(p)}\frac{v^2(p)}{D_\alpha(\infty)}. \tag{13}$$

Equations (12) and (13) represent the solution of the problem.

We have numerically evaluated the form factor $v(q)$ for the $\{3,3\}$ channel according to Eqs. (12) and (13). We have used the CERN theoretical fit[10] for the phase shifts for c.m. momentum less than 692 MeV/c, and have attached a tail of the form $A/(p-p_0)^2$ to bring the phase shifts $\hat\delta(p)$ smoothly to zero. The phase shift $\hat\delta(p)$ and inelasticity parameter $\hat\eta(p)$ are depicted in Fig. 1.

In Fig. 2, we have plotted $[\gamma(\tilde\omega_\pi=0)]^{1/2}v(q)$. We

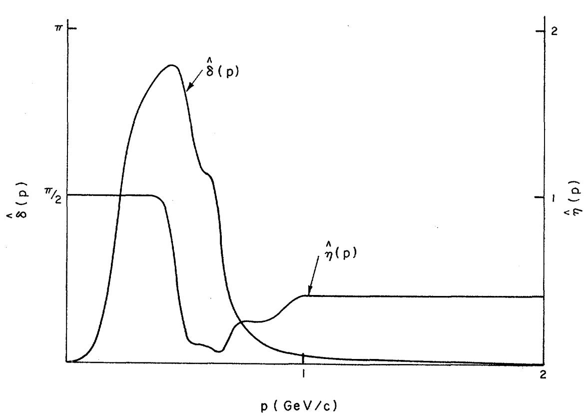

FIG. 1. Experimental parameters $\hat\delta(p)$ and $\hat\eta(p)$ as defined in Eq. (8).

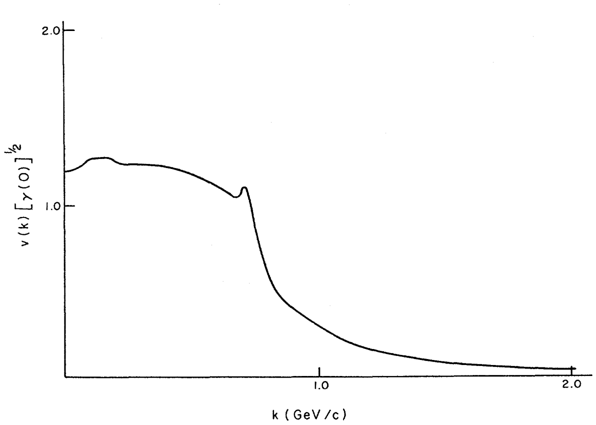

FIG. 2. Form factor $[\gamma(0)]^{1/2}v(k)$ as calculated from Eqs. (12) and (13). The values of $\hat\delta$ and $\hat\eta$ used in the calculation are depicted in Fig. (1).

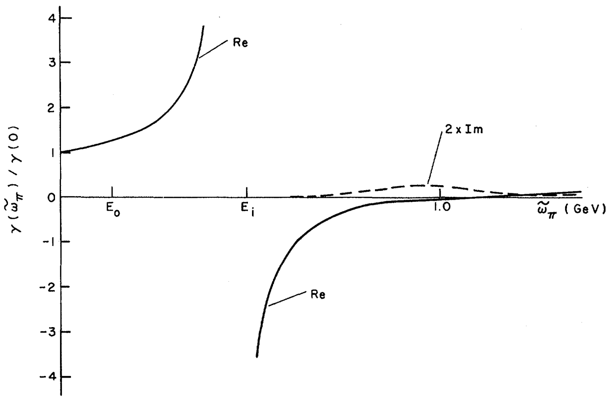

FIG. 3. Energy dependent coupling constant $\gamma(\tilde{\omega}_\pi)/\gamma(0)$. The elastic threshold is marked by E_0 and the inelastic threshold by E_i. The solid line is the real part of $\gamma(\tilde{\omega}_\pi)/\gamma(0)$ and the dashed line is the imaginary part.

plot $[\gamma(\tilde{\omega}_\pi=0)]^{1/2}v(q)$, as this yields the residue of the T matrix [cf. Eqs. (3) and (4)] as $\tilde{\omega}_\pi$ approaches zero. We have taken $f^2/4\pi=0.08$, and find that inclusion of the inelastic cut in the T matrix yields an effective coupling constant of $(f^2/4\pi)\times\gamma(\tilde{\omega}_\pi=0)v^2(0)=0.115$.

That this value differs from 0.08 is not surprising if we look at $\gamma(\tilde{\omega}_\pi)$ as shown in Fig. 3. At low energies $\gamma(\tilde{\omega}_\pi)$ varies rapidly with energy. The extrapolation to $\tilde{\omega}_\pi=0$ is, therefore, sensitive to details of the theory (such as how one includes the finite mass of the nucleon or how one treats the high-energy tail of the phase shifts). Thus, we do not place undue emphasis on the details of these calculations at this time, but rather present a phenomenology which is consistent with the Chew-Low theory and can also fit the elastic-scattering data.

From Fig. 3, we observe that $\gamma(\tilde{\omega}_\pi)$ has a singularity between the resonance and the inelastic threshold. This singularity in $\gamma(\tilde{\omega}_\pi)$ does not imply any special properties of the elastic-scattering T matrix. Again, the existence and position of this singularity is sensitive to details of our model, and perhaps one should not ascribe any special significance to it.

At the same time it is important to not that $v(q)$ (at momenta below ~800 MeV/c) is insensitive to these details, so that we have greater confidence in the form factor. But it is just this form factor which we seek to determine in order to tie down the interaction Hamiltonian of Eq. (1). Moreover, it is this interaction Hamiltonian which we require to build a theory of pion-nucleus scattering.

Retardation effects[11] and the energy dependence of the pion-nucleon interaction might be expected to play a more important role in pion-nucleus interactions than in nucleon-nucleus interactions. For the nucleon-nucleon case, field theory suggests that an energy-independent interaction is plausible. For the pion-nucleon case, however, field theory does not yield energy-independent potentials. Therefore, we have attempted an approach to the pion-nucleon interaction which is capable of fitting exactly the pion-nucleon data and of maintaining some contact with an underlying field theory. We suggest that studies of pion-nucleus scattering be based on an interaction Hamiltonian of the type we propose, rather than upon two-body potentials.

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