phases of the data taking is gratefully acknowledged.

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[1]M. Sakai and K. Kubo, Nucl. Phys. A185, 217 (1972).

[2]T. Ishimatsu, S. Hayashibe, N. Kawamura, T. Awaya, H. Ohmura, Y. Nakajima, and S. Mitara, Nucl. Phys. A185, 273 (1972).

[3]T. Ishimatsu, S. Hayashibe, T. Awaya, and H. Ohmura, J. Phys. Soc. Jpn. 35, 1579 (1973).

[4]F. E. Bertrand and R. W. Peelle, Phys. Rev. C 8, 1045 (1973), and private communication.

[5]M. Sakai, M. Sekiguchi, F. Soga, Y. Hirao, K. Yagi, and Y. Aoki, Phys. Lett. 51B, 51 (1974).

[6]W. H. A. Hesselink, B. R. Kooistra, L. W. Put, R. H. Siemssen, and S. Y. van der Werf, Nucl. Phys. A226, 229 (1974).

[7]E. R. Flynn, D. D. Armstrong, J. G. Beery, and A. G. Blair, Phys. Rev. 182, 1113 (1969).

[8]A. M. Lane, R. G. Thomas, and E. P. Wigner, Phys. Rev. 98, 693 (1955).

[9]B. Bloch and H. Feshbach, Ann. Phys. (New York) 23, 47 (1963).

[10]C. Shakin, Ann. Phys. (New York) 22, 373 (1963).

Resolving Ambiguities in Heavy-Ion Potentials*

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It is shown that nuclear (as opposed to Coulomb) rainbow-scattering data would be useful in discriminating among heavy-ion optical potentials. Acquisition of such data is facilitated by going to higher bombarding energies and using low-A targets.

There has recently been a great deal of interest in obtaining optical potentials which will satisfactorily describe the elastic scattering of heavy ions and which can be used in distorted-wave Born-approximation calculations for transfer reactions.[1] Most of the experiments performed to date appear to be sensitive only to the extreme tail of the nuclear potential, and hence, if one assumes that the potential is of the Woods-Saxon form,

$$U(r)=\frac{-(V+iW)}{1+\exp[(r-R)/a]},$$

one can only determine a and the combined constant $(V+iW)e^{R/a}$. The situation is very similar to that which existed for the early α-scattering experiments.[2] It therefore appears reasonable that recent approaches which have proven helpful in unraveling the ambiguities in α-particle scattering may also prove useful for heavy ions and possibly prevent some retracing of familiar, if not altogether fruitful, ground as well.[3]

In a recent paper by the present authors[4] it was shown that discrete ambiguities in the α-particle optical potential could be resolved[5] by taking data beyond the maximum *negative* deflection angle or *nuclear* rainbow angle $\Theta_r^{(N)}$.[6] This result can be explained in classical terms. Since the maximum deflection angle increases with the strength of the potential, measurement of the former quantity determines the latter. In both the classical and quantum cases, the measurement is effected by measuring the cross section in the region beyond the maximum deflection angle $\Theta_r^{(N)}$. In the classical case, the cross section falls abruptly to zero beyond $\Theta_r^{(N)}$; in the quantum case, it exhibits the exponentiallike falloff characteristic of rainbow scattering.

One might well expect that nuclear-rainbow-scattering data would be useful in eliminating ambiguities other than the discrete kind. In particular, for $Ve^{R/a}$ ambiguities, the parameter a is well determined, and so the maximum deflecting force, which occurs at $r=R$, is simply proportional to V. It is our purpose to show, by using a series of model calculations, that the above expectation appears to be justified, and to examine the experimental conditions which would facilitate the observation of nuclear rainbow scattering.

It was shown in Ref. 4 that there is a minimum energy ϵ_{crit} required for nuclear rainbow scattering to occur, and that as the energy is increased above ϵ_{crit}, the nuclear rainbow angle $\Theta_r^{(N)}$ decreases to smaller (and experimentally more accessible) angles. To verify this for the case of heavy ions, we have performed model calculations for the case of ^{16}O incident on ^{28}Si.

The potentials used in the calculations are those which Cramer *et al.*[7] obtained by fitting elastic-scattering data over the range 33–81 MeV; these potentials form a $Ve^{R/a}$ ambiguity (referred to by Cramer *et al.* as a "class") for which V can range from 10 MeV to $\gg 1$ GeV with essentially no change in χ^2. We initially selected the potential in this class for which $V = 100$ MeV. The calculated ϵ_{crit} for this potential is 140 MeV (lab). However, as stated above, for the nuclear rainbow scattering to be readily observable it is necessary to go to higher energies. Hence we extended our calculations up to 200 MeV (lab) which is about the maximum energy currently available for ^{16}O ions.

The results of the calculations are shown in Fig. 1; the curves are quite similar to those for α particles on heavy nuclei. At $E = 40$ MeV, the differential cross section is that typical of *Coulomb* rainbow scattering. As the energy is increased to 80 MeV the Coulomb rainbow peak moves to smaller angles and one observes the onset of diffraction oscillations, a transition similar to that observed in low-energy α scattering from heavy targets. As the energy is increased still further, the diffraction oscillations begin to die out at larger angles and give way to the structureless falloff associated with rainbow scattering.

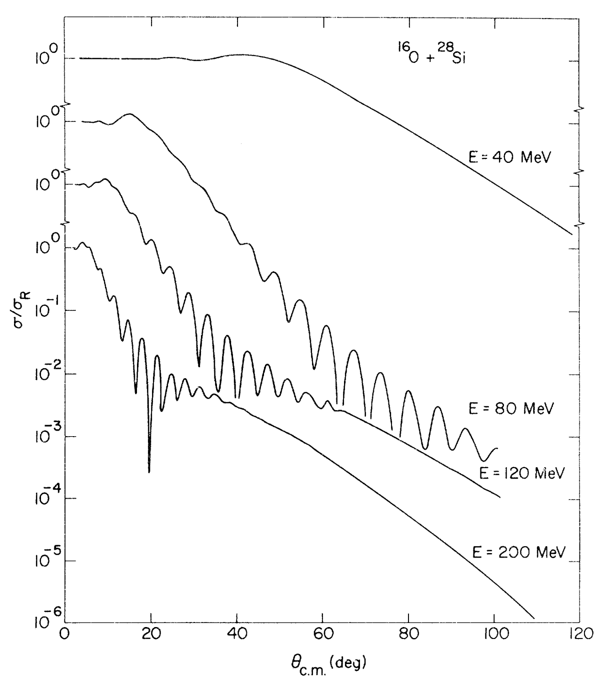

FIG. 1. Model calculations for elastic-scattering cross section predicted by the 100-MeV-deep heavy-ion potential of Cramer *et al.*

In fact at $E = 200$ MeV one is able to see *both* the Coulomb *and* the nuclear rainbows, the former being masked somewhat by the presence of diffraction oscillations.

The existence of a falloff region, which usually begins to appear at energies slightly *below* ϵ_{crit} (e.g., 120 MeV for $^{16}O + ^{28}Si$) is not in and of itself definitive evidence of rainbow scattering. As pointed out in Ref. 4, the *shape* (but not the magnitude) of the cross section beyond the rainbow angle is *independent of absorption.* Hence, short of doing a full-blown semiclassical phase-shift calculation, a simple test for the existence of rainbow scattering can be performed by calculating the angular distribution in the *absence* of absorption. If the falloff persists, as it does for such a calculation in the 200-MeV case, then rainbow scattering is in fact occurring. Moreover, the independence of the shape of the cross section on W means that the determination of the parameters of the real part of the well is relatively independent of absorption.

Having established that nuclear rainbow scattering will indeed occur at sufficiently high incident energies, we now consider its usefulness in distinguishing among various optical potentials. We consider the scattering predicted for two other potentials of the above class, namely, those for $V = 50$ and 20 MeV. To maximize the region in which nuclear rainbow scattering dominates, we choose the incident energy to be 200 MeV.

The results of the calculations for the three potentials are shown in Fig. 2. As expected, the 50-MeV well has a smaller nuclear rainbow angle than the 100-MeV well, although the rainbow-scattering region is still largely distinct from the diffraction region. For the 20-MeV potential, the real well is so weak that the nuclear rainbow angle has fallen well within the diffraction region and the resulting cross section becomes a steeply falling diffraction pattern.[8] Comparing the 50- and 100-MeV cross sections we make two further observations. First, the smaller $\Theta_r^{(N)}$ for the former potential results in a smaller cross section at larger angles. Secondly, the cross sections predicted by the two potentials are quite similar at angles less than $\Theta_r^{(N)}$. Thus the results bear out the earlier contention that *it is the data in the nuclear-rainbow-scattering region which enable one to distinguish among various optical potentials.*

We now consider the question of how to facilitate rainbow-scattering measurements. We have shown earlier, that such measurements are

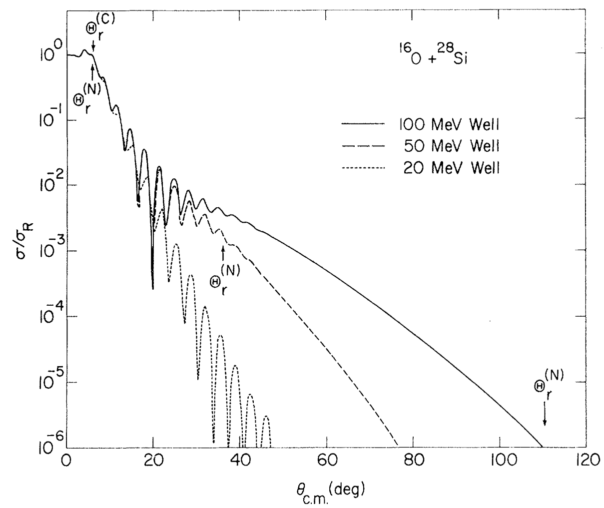

FIG. 2. Comparison of predicted cross sections at $E = 200$ MeV for three different potentials which give equivalent fits to scattering data in the region 30–81 MeV. The radii for the wells shown are 6.86, 6.31, and 5.87 fm for the 20-, 50-, and 100-MeV wells, respectively, and $a = 0.64$ fm in each case (Cramer *et al.* use a slightly modified version of $Ve^{R/a} = \text{const}$ in obtaining different potentials within the class); the real and imaginary geometries are the same and W is related to V by the relation $W = 0.44V + 2.7$. The respective *nuclear* rainbow angles for the three cross sections, calculated as described in Ref. 3, are indicated as $\Theta_r^{(N)}$. The *Coulomb* rainbow angles denoted $\Theta_r^{(C)}$, are essentially identical in all three cases; this reflects the fact that $\Theta_r^{(C)}$ depends only on the extreme tail of the nuclear potential.

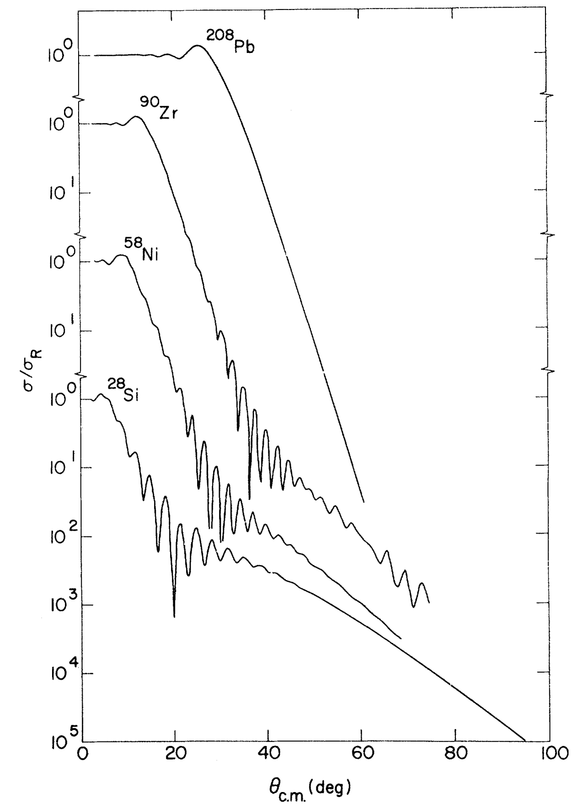

FIG. 3. Comparison of predicted cross sections for scattering of 200-MeV ^{16}O from four different nuclides. The potential used was the 100-MeV potential of Cramer *et al.*, with the radius scaled as $16^{1/3} + A_T^{1/3}$.

facilitated (more importantly, even made possible) by increasing the incident energy so that $\Theta_r^{(N)}$ is reduced. Moreover, the higher incident energy also increases the rainbow cross section, both absolutely and relative to the Rutherford cross section, thereby further improving the ease of measurement. For example, additional model calculations for the case of ^{16}O + ^{28}Si show that as the incident energy is increased from 200 to 300 MeV the cross section at $\Theta_r^{(N)}$ increases by a factor of about 4×10^3, and σ/σ_R increases some fortyfold.

A second, and equally important factor is the choice of target. Based on arguments made in Ref. 4 on the A dependence of ϵ_{crit} one can easily show that, for a given potential, i.e., one in which the A dependence is simply due to the increase of R with A, the nuclear rainbow angle increases with A, a result that recently has been verified experimentally for the case of intermediate-energy α scattering.[9] To demonstrate the effect in heavy-ion scattering we have done model calculations for 200-MeV ^{16}O incident on four different targets. The results are shown in Fig. 3. For ^{58}Ni, the rainbow-scattering region sets in at slightly larger angles, and, more importantly, has a smaller cross section. As A increases further, so does $\Theta_r^{(N)}$ until, in the case of ^{208}Pb, it exceeds 180° (and accordingly ϵ_{crit} exceeds the incident energy). In fact for ^{208}Pb, the scattering pattern is essentially that of pure *Coulomb* rainbow scattering, i.e., the scattering is only sensitive to the extreme tail of the potential; stated slightly differently, for ^{16}O incident on ^{208}Pb, $E_b = 200$ MeV still represents the low-energy limit.

The conclusions from the above calculations seem clear. Nuclear-rainbow-scattering data are useful in discriminating among nuclear potentials[10]; such data are most easily obtained by using high incident energies and low-A targets. Once one has "tied down" the potential at these higher energies one can extrapolate experimentally to lower energies, as has recently been done by Put and Paans[11] for the case of α particles. Moreover, recent calculations[12] appear to indicate that the energy dependence of heavy-ion optical potentials is expected to be small.

The arguments leading to these conclusions involve quite general characteristics of nuclear scattering, which we have described elsewhere as refractive behavior,[9,13] and are based on a simple classical analog. In particular the arguments are not predicated on the potential having a Woods-Saxon shape, a form whose widespread usage is at least as much the result of its convenience as its "correctness." Whatever the parametrization, rainbow-scattering data will give one a good estimate of the overall strength of the interaction. (In fact, the data may even result in an increased knowledge of the form of the potential, as has already been the result in the case of α scattering.[14]) The main point is that once relatively unambiguous results can be obtained for the lighter targets, studies of fine detail, e.g., using observations of resonances in excitation functions, can be more meaningfully performed and results extended to heavier nuclei. We feel that the present work indicates what, in the near term, is likely to be the most fruitful direction for experimental activity to take.

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[1]See, e.g., A. Gobbi, R. Wieland, L. Chua, D. Shapira, and D. A. Bromley, Phys. Rev. C 7, 30 (1973), and references therein.

[2]R. M. Eisberg and C. E. Porter, Rev. Mod. Phys. 33, 190 (1961), and references therein.

[3]G. Santayana, *The Life of Reason* (Charles Scribner, New York, 1953), p. 82, line 29.

[4]D. A. Goldberg and S. M. Smith, Phys. Rev. Lett. 29, 500 (1972).

[5]At low energies, ambiguities have been removed by reducing the number of optical parameters [see, e.g., B. D. Watson *et al*., Phys. Rev. C 4, 2240 (1971)]. In our view, this represents a *suppression* of the ambiguity, rather than a *resolution* of it; it is nonetheless a reasonable approach in analyzing elastic-scattering data taken at energies where the scattering is so insensitive to anything but the extreme tail of the potential that even as few as four parameters are excessive.

[6]Low-energy α and heavy-ion scattering is characterized by a maximum *positive* deflection angle at the point where the attractive nuclear force begins to overcome the repulsive Coulomb force. To avoid confusion, we will refer to this as the *Coulomb* rainbow angle, $\Theta_r^{(C)}$, and to the maximum negative *deflection* angle, caused solely by the attractive nuclear force, as the *nuclear* rainbow angle, $\Theta_r^{(N)}$. The magnitude of the former is generally much smaller than that of the latter.

[7]J. G. Cramer, R. M. DeVries, M. S. Zisman, K. G. Nair, K.-L. Liu, and Y.-D. Chan, in *Proceedings of the International Conference on Nuclear Physics, Munich, Germany, 1973,* edited by J. de Boer and H. J. Mang (North Holland, Amsterdam, The Netherlands, 1973), p. 342, and to be published. We are grateful to J. Cramer for communicating his recent results to us prior to publication.

[8]A similar effect is observed in the case of elastic proton scattering, where the strength of the real well is also relatively weak.

[9]D. A. Goldberg, S. M. Smith, and G. F. Burdzik, Phys. Rev. C (to be published).

[10]An illustration of this result "by converse" may be seen in the results of M. L. Halbert *et al*., Bull. Amer. Phys. Soc. 18, 1387 (1973), and to be published; Halbert *et al*. succeeded in eliminating an overly *shallow* potential by taking data beyond *its* rainbow angle, and noting that the experimental cross section did *not* fall as predicted.

[11]L. W. Put and A. M. Paans, Phys. Lett. 49B, 266 (1974).

[12]D. F. Jackson and R. C. Johnson, Phys. Lett. 49B, 249 (1974).

[13]S. M. Smith and D. A. Goldberg, Bull. Amer. Phys. Soc. 17, 591 (1972).

[14]D. A. Goldberg, Bull. Amer. Phys. Soc. 19, 595 (1974).