Coulomb Energy of He^3 and Charge Symmetry of Nuclear Forces

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The Coulomb energy of He^3 has been calculated using separable potentials with a repulsive term. The calculated value is in close agreement with the H^3-He^3 binding-energy difference. An expected 3–6% decrease, brought about mainly by the inclusion of a tensor term, can be explained by charge asymmetry due to η-π and ρ-ω-φ mixing, provided one takes $g_\rho/g_\omega > 0$. This also yields $a_{nn} - a_{pp} \gtrsim 0$, which is required by the present experimental situation, in contrast to $a_{nn} - a_{pp} \sim 1.4$ fm obtained by earlier authors with $g_\rho/g_\omega < 0$.

The purpose of this note is twofold. Firstly, we report a new calculation of the Coulomb energy of He^3 using rank-3 separable potentials which include a repulsive term in the singlet state, and secondly, we analyze the existing situation regarding charge asymmetry of nuclear forces in the light of our present result and a more definite value for the n-n scattering length.

The interest in the Coulomb energy of He^3 persists mainly because of its ramifications on the charge asymmetry of nuclear forces. A large number of variational calculations of the Coulomb energy (E_C) of He^3 have yielded results in the range $E_C = 0.6$–0.65 MeV.[1] In the recent past, calculations have also been performed via the solutions of Faddeev equations to yield $E_C \sim 0.62$–0.69 MeV.[1] However, the experimental difference in the binding energies of H^3 and He^3 is $\Delta E_B = 0.764$ MeV. The gap, $\Delta V = \Delta E_B - E_C$, between the two is usually attributed to the charge asymmetry of nuclear forces. For the calculations cited above, this gap is ~ 0.1–0.16 MeV, which is about 15–20% of ΔE_B. A gap of this order of magnitude requires a much larger charge asymmetry than is consistent with the evidence from many other sources.[2,3]

A few years ago, Gupta and Mitra[4] had calculated E_C within a separable-potential model, which has had a very successful career so far,[5] and had obtained $E_C = 0.84$ MeV. However, the repulsive core in the singlet state, which has a significant effect, had not been included in that study. We have now calculated E_C for potentials which do include such a repulsive term in the singlet state. In the triplet state, the potential is taken to be a purely attractive central potential:

$$-M\langle p\,|\,V_T|\,p'\rangle = \lambda_T g(p) g(p').$$

where $g(p)$ is taken to be of the usual Yamaguchi form,[6]

$$g(p) = (p^2 + \beta_c^2)^{-1}.$$

The singlet potential now consists of the sum of an attractive and a repulsive term:

$$-M\langle p\,|\,V_s|\,p'\rangle = \lambda_s [f(p) f(p') - f_1(p) f_1(p')],$$

where we take, as usual,

$$f(p) = (p^2 + \beta_s^2)^{-1},$$

$$f_1(p) = np^2 (p^2 + \beta_0^2)^{-2}.$$

The calculations have been performed with two different sets of parameters for the singlet part of the potential (referred to as N and G_1' in the notation of Ref. 5), each of which gives a fairly good representation of the N-N singlet phase shifts up to sufficiently high energies.[7] The results of the present calculation, along with those of Ref. 4 are presented in Table I. The calculations have been performed for point as well as extended protons.[8] For the case of extended protons, we obtain $E_C = 0.77$ MeV (for both the sets employed), a result which is embarrassingly close to the experimental value 0.764 MeV for ΔE_B. This striking agreement, however, is not to be taken too literally. For completeness, one must include a tensor term and (perhaps) a small repulsive term in the triplet part of the potential, both of which would tend to reduce the Coulomb energy. The more important repulsive term, which we have included in the present anal-

TABLE I. The Coulomb energy of He^3 with separable potentials for point as well as extended protons. N and G_1' refer to two different potential sets for the singlet state (for notations see Ref. 5). Experimental value of ΔE_B, 0.764 MeV.

Rank of the potential	Potential set	E_C (MeV) Point protons	Extended protons
Rank 2, no repulsion included	$C_Y^{\mathrm{eff}}+S_Y$	1.060	0.856
(Ref. 4)	C_N+S_N	0.952	0.830
Rank 3, repulsion in the singlet	$C_Y^{\mathrm{eff}}+(S+H)_N$	0.905	0.775
state included (present work)	$C_Y^{\mathrm{eff}}+(S+H)_{G_1'}$	0.900	0.771

ysis, has reduced E_C by ~10%, not unexpectedly. Once this repulsive term has been included, the tensor term is expected to further reduce E_C by 3–6%,[5,9] which should bring E_C, at best, down to 0.7–0.73 MeV.[10] This is much higher than E_C ~0.6 MeV obtained from variational calculations by Okamoto *et al.*

Though there is some evidence for charge-symmetry breaking in nuclear forces, and on theoretical grounds such symmetry breaking must exist,[11] we believe the last word has not been said on this problem *vis-à-vis* the H^3-He^3 Coulomb-energy-binding-energy difference. Since E_C turns out to be less than ΔE_B, whatever the magnitude of this difference, it follows that the n-n potential is somewhat stronger than the p-p potential, at least as far as its effect on E_C is concerned. Naively, one should then expect the n-n singlet scattering length a_{nn} to be (numerically) larger than a_{pp}. Indeed, the earlier analyses of charge-asymmetric potentials were based on the assumption that $|a_{nn}| \gtrsim |a_{pp}|$. Until a couple of years ago, this assumption was consistent with the then experimental situation because of the large uncertainties in the determination of a_{nn}. However, these uncertainties have been narrowed down considerably, and Henley and Wilkinson quote the following values for a_{nn} and a_{pp}[12]:

$$a_{nn} = -16.4 \pm 0.9 \text{ fm};$$

$$a_{pp} = -17.1 \pm 0.2 \text{ fm}.$$

Thus, in all probability, $|a_{nn}| < |a_{pp}|$, contrary to what one expected from the above argument.

There have been a number of calculations of the effect of charge-asymmetric potentials, which follow from the electromagnetic mixing of mesons, to explain the relevant two-body and three-body data.[13] Apart from many other uncertainties in these analyses, one crucial factor is the relative sign of the ρ and ω coupling constants, i.e., the sign of g_ρ/g_ω. Indeed, so far, the sign of g_ρ/g_ω was chosen to be negative so as to yield $|a_{nn}| > |a_{pp}|$ and then the contribution of such a charge-asymmetric potential to the binding energy was calculated perturbatively. It was then possible to obtain $\Delta V \sim 0.1$–0.15 MeV for this contribution.[11] Since, according to the latest experimental situation, $\Delta a = a_{nn} - a_{pp} = 0.7$ fm (taking the errors into account, it could lie between -0.4 and 1.8 fm), the sign of g_ρ/g_ω, in fact, *must be chosen to be positive,* so as to yield $|a_{nn}| \lesssim |a_{pp}|$ (rather than the other way round). Now the contributions to Δa and ΔV (the energy that must be *added* to E_C to make it equal to the experimental binding-energy difference ΔE_B) come from η-π as well as ρ-ω-φ mixing,[14,15] and both are quite model dependent (depend upon many ill-determined parameters which enter into the theory, as well as upon the three-nucleon wave function assumed for H^3). Okamoto and Pask[1] have determined ΔV for Stevens's η-π mixing model,[14] as well as Downs and Nogami's η-π and ρ-ω-φ mixing models,[15] but for the wrong sign for the quantity g_ρ/g_ω. If we take $g_\omega = 6$ instead of $g_\omega = -5$, ΔV will change sign, though its magnitude will remain almost the same. One can see that changing the sign of Downs and Nogami's ρ-ω-φ contribution in Table VI of Ref. 1 gives $\Delta V \simeq 0.025$–0.03 MeV. (The actual magnitude could vary considerably depending upon the parameters chosen.) This is much more gratifying from our point of view (than $\Delta V \sim 0.1$–0.15 MeV), since we predict ΔV to lie between 0.03 and 0.06 MeV. This is also in accord with the situation for higher mirror nuclei where one finds that theory and experiment agree within a few per cent (rather than 15–20%), and, in fact, may even be consistent with charge symmetry.[16,17] Also, the overall charge asymmetry now required will be ~$\frac{1}{3}$ of what Okamoto estimated,[18] and hence in much better agreement with the calculations of Blin-Stoyle and Yalgin,[19] as well as with the evidence coming from other sources.[2]

As for the scattering length, the contribution to Δa from ρ-ω-φ mixing will now be positive whereas that from η-π mixing will be negative, so that $\Delta a_{\rho\omega\varphi}+\Delta a_{\eta\pi} \gtrsim 0$ (since $|\Delta a|_{\rho\omega\varphi} \gtrsim |\Delta a|_{\eta\pi}$), which is just what the data require. Since the scattering length is rather sensitive to small differences in the potential, because of the singlet state being nearly a bound state, it will be hazardous to make a more definite commitment as to the actual magnitude of Δa until a much more precise charge-asymmetric potential is available. But, in general, one can say that charge-asymmetric potentials are able to explain both $a_{nn}-a_{pp}$ and $\Delta E_B - E_C$, if one takes E_C ~ 0.7–0.73 MeV. It will be extremely difficult, on the other hand, to reconcile $|a_{nn}| \lesssim |a_{pp}|$ and $E_C \lesssim 0.6$ MeV[3] obtained from variational calculations.

In conclusion, we find that our calculation with separable potentials which include a repulsive term in the singlet state yields $E_C \approx 0.77$ MeV, in agreement with the experimental value of ΔE_B. Inclusion of tensor and small repulsive parts in the triplet state should lower E_C to around 0.7–0.73 MeV. This small deviation of E_C from ΔE_B can be explained from charge-asymmetric potentials obtained from the mixing of η-π isosinglets and the ρ-ω-φ isotriplets, *provided* one takes $g_\rho/g_\omega > 0$. This will also simultaneously make $|a_{nn}| \lesssim |a_{pp}|$, which is *required* by the present experimental situation, resolving the long-standing ambiguity in the simultaneous explanation of both Δa and ΔV. On the other hand, it will be very hard to understand ΔV ~ 0.1–0.15 MeV along with $|a_{nn}| \lesssim |a_{pp}|$. Though $g_\rho/g_\omega < 0$ yields ΔV ~ 0.1–0.15 MeV, it simultaneously gives $\Delta a \sim -1.4$ fm, which is firmly ruled out by the present data, making ΔV ~ 0.1–0.15 MeV also highly unlikely. Also, the overall percentage of charge asymmetry required to explain the present value of Δa along with our value for ΔV is much less than was envisaged by Okamoto, but agrees with what is required from many other considerations.

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[2] E. M. Henley, in *Isospin in Nuclear Physics,* edited by D. Wilkinson (North-Holland, Amsterdam, 1969).

[3] Since the wave functions used for the calculation of E_C usually underbind triton by 1–2 MeV, those which yield the right value for E_B will further decrease E_C significantly.

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[10] There are, of course, a large number of corrections which must be applied to E_C. These have been analyzed in detail in Ref. 1. But most of them are negligible, and of others, some contribute with a positive and some with a negative sign. The net result is not very significant.

[11] For excellent reviews on charge symmetry, refer to articles by D. H. Wilkinson and E. M. Henley, in *Few Particle Problems in the Nuclear Interaction,* edited by I. Slaus (North-Holland, Amsterdam, 1972).

[12] Wilkinson and Henley, in a note added to the articles quoted in Ref. 11.

[13] An alternative suggestion by R. T. Folk [Phys. Lett. 28B, 159 (1968)] to explain ΔV in terms of "dynamical polarization" of protons was found by Okamoto and Pask [Ref. 1] to give $a_{nn} \sim -20$ fm, and so is untenable.

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