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Dissipative Trapped-Electron Instability in Cylindrical Geometry*

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We show that the essential ingredients of the trapped-electron instability can be present in a cylindrical geometry. A linear instability theory, a simple physical explanation, and the results of an experiment are presented to support this.

The dissipative trapped-electron instability like all other trapped-particle instabilities has always been discussed in toroidal systems.[1-3] In toroidal systems magnetic field properties such as inhomogeneity, curvature, and helicity are all interrelated and the resulting zero-order particle trajectories of drifting bananas are very complex. These geometrical complexities also make experimental identification difficult. This Letter considers the simplest magnetic field sufficient for the instability and clarifies the underlying physics. We also report the results of an experiment which demonstrates that a dissipative trapped-electron instability can be excited in a cylindrical geometry.

It is well known that collisionless trapped-particle instabilities are driven by magnetic-curvature drift.[3] But dissipative trapped-particle instabilities are similar to collisional drift waves in the sense that the excitation of both classes depends on collisions. In the case of the trapped-electron instability one needs an appropriate electron collision frequency which should be higher than the wave frequency.[1] Therefore one can conceive of the dissipative trapped-electron instability in an essentially straight magnetic field, but to produce a trapped population of particles one needs at least two localized magnetic mirrors (Fig. 1). A density gradient, say in the radial direction, is always necessary for a drift-like mode and finally, an electron-temperature gradient coincident with the density gradient is essential.[1] We will show that the cylindrical system shown in Fig. 1 is capable of sustaining the dissipative trapped-electron instability.

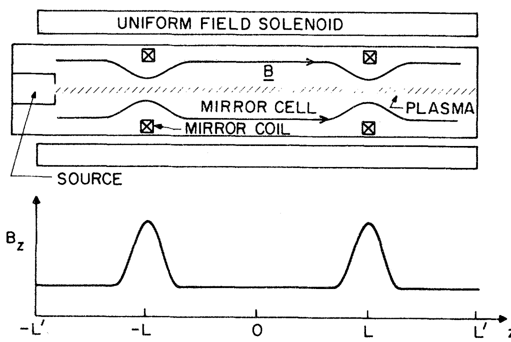

FIG. 1. Schematic of the mirror cell in cylindrical geometry (above) and the axial magnetic field intensity (below) for mirror ratio $R=3$. The plasma source which produces a radial electron-temperature gradient is an $\vec{E}\times\vec{B}$ source (Ref. 5).

For electrons trapped between the mirrors, we write the drift kinetic equation in cylindrical coordinates r, θ, z with the Krook model for the collision term as

$$\frac{\partial f_t}{\partial t}+v_z\frac{\partial f_t}{\partial z}+\frac{e}{m_e}\frac{\partial\varphi}{\partial z}\frac{\partial F_t}{\partial v_z}-\frac{c}{B_z r}\frac{\partial\varphi}{\partial\theta}\frac{\partial F_t}{\partial r}=\nu_e(v)\left(\frac{e\varphi}{kT_e}F_t-f_t\right), \qquad (1)$$

where f and φ are the perturbed electron distribution function and potential, respectively; v_z, $\nu_e(v)$, and F are the axial velocity, velocity-dependent electron collision frequency, and equilibrium distribution function, respectively; and the subscript t refers to the trapped population. Integrating Eq. (1) along the straight zero-order trajectory between sharp mirrors, averaging over the fast bounce period $2L/v_z$, and integrating in velocity space yields the perturbed trapped-electron density

$$n_t=\frac{e\varphi\delta N}{kT}-\frac{e}{2kTL}\int_{-L}^{L}\varphi(z')\,dz'\int\frac{(\omega-\omega_e{}^*)F_t d^3v}{\omega+i\nu_e v_{\rm th}{}^3/\epsilon v^3},$$

where F_t is taken as Maxwellian, $\nu_e(v)\approx\nu_e v_{\rm th}{}^3/\epsilon v^3$, $\delta=N_t/N$, $\epsilon=1-R^{-1}$, R is the mirror ratio,

$$\omega_e{}^*=\frac{-cm_e kT}{reB_z}\left[\frac{1}{N}\frac{dN}{dr}+\left(\frac{m_e v_e{}^2}{2kT_e}-\frac{3}{2}\right)\frac{1}{T_e}\frac{dT_e}{dr}\right],$$

and we assume standing-wave solutions in the z direction:

$$f_t=f_t(z)\exp(-i\omega t+im\theta),\quad \varphi=\varphi(z)\exp(-i\omega t+im\theta).$$

We note that, unlike the toroidal case, the trapped fraction $\delta\neq\sqrt{\epsilon}$ here. The perturbed transit electron density n_u is assumed to be a perturbed Maxwellian since the transits are in equilibrium with the source. The ions behave essentially as in a drift wave, so that their perturbed density is given by

$$n_i=\frac{\omega^*}{\omega}\frac{Ne\varphi(z)}{kT}-\frac{Ne}{m_i\omega^2}\frac{\partial^2\varphi(z)}{\partial z^2},$$

where $\omega^*=(cmkT/eB_z r)N^{-1}dN/dr$. Using the quasineutrality condition $n_t+n_u=n_i$ yields

$$A(\omega)\varphi(z)-B(\omega)\partial^2\varphi(z)/\partial z^2=C(\omega)\int_{-L}^{L}\varphi(z')\,dz'[u(z-L)-u(z+L)],\tag{2}$$

where

$$A(\omega)=\frac{Ne}{kT}\left(\frac{\omega^*}{\omega}-1\right),\quad B(\omega)=\frac{Ne}{m_i\omega^2},\quad C(\omega)=\frac{-e}{2LkT}\int\frac{(\omega-\omega_e{}^*)F\,d^3v}{\omega+i\nu_e v_{\rm th}{}^3/v^3}\approx-\frac{iNe}{kT}\frac{\epsilon\delta\omega_T{}^*}{\nu_e L}.\tag{3}$$

The unit step function u was included so that Eq. (2) correctly represents the entire system length $-L'<z<L'$. Outside the mirror region, the right-hand side is zero and the equation reduces to that of a stable drift wave.

Expecting only even modes we expand $\varphi(z)$ in a Fourier cosine series over the system length $2L'$ as $\varphi(z)=\sum_n a_n\cos(n\pi z/L')$. Substituting this series in Eq. (2) we obtain an infinite set of homogeneous equations for the Fourier coefficients a_n with ω as the eigenvalue parameter. It can be shown that the Fourier series is rapidly converging. Therefore truncating the set of equations to three and setting its determinant to zero yields a good approximation both for the eigenvalue ω and the spatial structure of $\varphi(z)$. With the use of the parameter relations $L'=2L$, $\nu_e/2=\omega^*=\omega_T{}^*=(cmkT/eB_z r)T_e{}^{-1}dT_e/dr$, $\delta=0.2$, $\epsilon=0.6$, $\omega_{be}/\omega^*=6$, the results are

$$\omega=(0.992+i0.12)\omega^*;$$
$$\varphi(z)\sim1+1.27\cos\frac{\pi z}{L'}-0.28\cos\frac{3\pi z}{L'},\tag{4}$$

where $\omega_T{}^*$ is the drift frequency based on electron-temperature gradient and $\omega_{be}=\frac{1}{2}L^{-1}(KT_e/m_e)^{1/2}$ is the electron bounce frequency. The growth rate is seen to be more than 10% of the real frequency and can be shown to vary roughly as L/L'. The spatial structure seen in Eq. (4) indicates a high degree of localization in the trapped-particle region (Fig. 2). This feature is not dissimi-

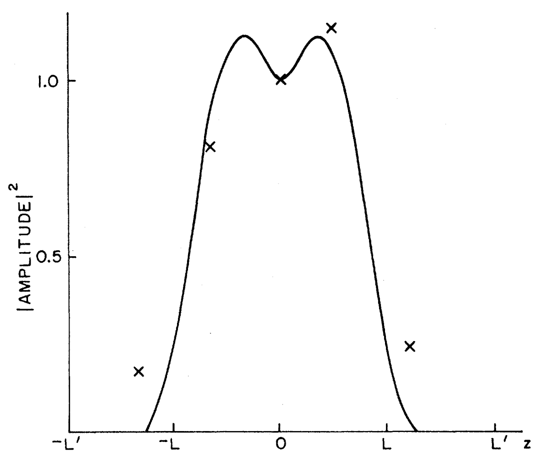

FIG. 2. Dependence of the normalized, relative, square of the fluctuation amplitude upon axial position. The theoretical result is from Eq. (4), and the experimental data are obtained from the output of an autocorrelator driven by a Langmuir probe located in the region of maximum density gradient. Plasma radius ≈ 1 cm, $T_e\approx6$ eV, $N^{-1}dN/dr\approx T_e{}^{-1}dT_e/dr\approx1$ cm^{-1}, $B=1200$ G, $N\sim2\times10^{11}$ cm^{-3}.

lar to that of a collisionless trapped-particle instability.[5] But the reasons are quite different, as the localization of the collisionless instability is due to its flutelike nature and occurs in the bad-curvature region of the magnetic field. If we take the limit $L \sim L'$ so that the entire system is populated uniformly by trapped particles, only the first term of the Fourier series is necessary because φ will be constant and the above result will reduce to $\omega/\omega^* = 1 + i\delta\epsilon\omega_T^*/\nu_e$ which is essentially the same as that of Kadomtsev and Pogutse.[1]

Deschamps *et al.*[4] report a driftlike instability in an ODE machine[6,7] which produces a cylindrical plasma with radial density and electron-temperature gradients in a corrugated magnetic field. However, at their low operating density, $\nu_e < \omega^*$, one should not expect a dissipative trapped-electron instability. Guided by the above analysis we have performed a similar experiment using a Q machine converted to an ODE-type device. The parameters T_e, T_i, B_z, $N^{-1}\,dN/dr$, $T_e^{-1}dT_e/dr$, plasma radius, and hydrogen-gas fill are approximately the same as reported in their paper. However, in our experiment (a) there is one mirror cell ($2L = 50$ cm, $2L' = 100$ cm) and the magnetic field is constant over 80% of its length, (b) the mirror ratio R is controllable and is made large in order to enhance electron trapping, and (c) the electron density is much higher ($N_e \sim 2 \times 10^{11}$ cm^{-3}) so that the electron mean free path is of the order of the mirror length and $\nu_e > \omega^*$. This environment can support a dissipative trapped-electron instability and we observe a monochromatic wave at frequency $\omega \approx \omega^*$ and azimuthal mode number $m = 1$. The instability, which is localized in the mirror cell (Fig. 2), is observed when the mirror coils are energized and its amplitude is at least an order of magnitude larger than the background level of drift-wave fluctuations in the absence of the mirror field. The instability amplitude became maximum at $n/N \sim 10\%$ when $\nu_e > \omega^* \sim 3 \times 10^5$ sec$^{-1} \ll \omega_{be}$, $R \sim 3$, and the electron mean free path ~ 200 cm. These conditions not only favor electron trapping in the mirror cell but are appropriate for a positive growth rate of the dissipative trapped-electron instability as shown by the above theory.

The fraction δ of electrons trapped can be estimated experimentally by observing the plasma-density change when the mirror farthest from the source is switched on or off, the other mirror left on. Since the ions have a very short mean free path (<10 cm) their containment is not affected by the magnetic mirrors: We conclude this from the observation that the plasma density is not affected by the presence of only the mirror farthest from the source. Thus, the plasma-density change is due to electron trapping in the mirror cell, with ions being drawn in to maintain neutrality. (Since $T_e/T_i \sim 6$, ambipolar effects are not important to electron trapping.[8]) The fraction of electrons trapped $\delta \approx \frac{1}{2}$. Clearly, as R increases the growth rate should increase since both the trapped fraction δ as well as ϵ increase. In Fig. 3(a) we see that the wave amplitude increases as R increases. Furthermore in Fig. 3(b) we see that the experimental wave amplitude tends to follow qualitatively the growth rate as a function of ν_e/ω; the theoretical curve is obtained with the help of the detailed calculation of the velocity-space integral in $C(\omega)$, Eq. (3). The mechanism for nonlinear limitation of wave growth has not been identified, but it is possible for the steady saturation amplitude of an instability of the soft-onset type to vary as the linear growth rate.[9]

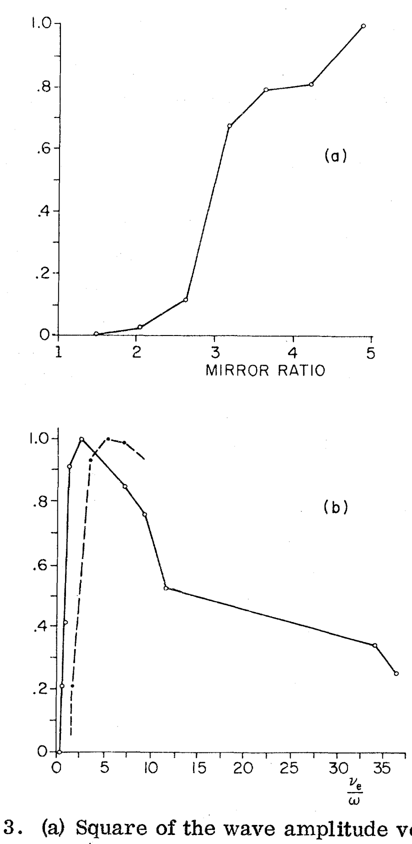

FIG. 3. (a) Square of the wave amplitude versus mirror ratio R, $\nu/\omega = 3$. (b) Square of the amplitude (solid line) and the theoretical growth rate (broken line) versus collision frequency.

In order to eliminate ∇T_e, we converted the source to a hollow-cathode arc containing a hot filament. For this source $\vec{E} \parallel \vec{B}$ and no temperature gradient was produced in the plasma. In the same range of plasma parameters as with the $\vec{E} \times \vec{B}$ discharge, the magnetic mirrors had no effect on the fluctuation level.

Since toroidal drifts do not play a causal role in this instability a description of the physical mechanism applies equally well to cylindrical and toroidal systems. If $\omega_{be} \gg \omega$ the trapped electrons experience a relatively steady electric field during a bounce period. Therefore one can consider all the trapped particles to drift radially at the speed E_θ/B_z, where E_θ is the perturbed electric field. The radial flux of trapped electrons of velocity v at position r is proportional to $F_t E_\theta/B_z$. The perturbed trapped-electron density buildup, Δn_t, at r due to this flux will be proportional to the flux, the length of trapping time $\nu_e^{-1}(v)$, and $\partial F_t/\partial r$. The velocity integral of this product yields the integral $C(\omega)$, Eq. (3). For simplicity we consider a perturbation with $k_\parallel = k_z = 0$, so that the density-gradient contribution to the electron buildup is zero as appropriate for a stable drift wave. Let the sign of E_θ be such that the trapped-electron flux is radially outward. Since $\nu(v) \sim v^{-3}$ the high-velocity particles are trapped longest and contribute most to the flux. Since the temperature decreases radially outward, the number of high-velocity particles also decreases radially outward. Thus there is a greater radial outward flux of trapped electrons to a point than outward flux from the point, thereby leading to an electron buildup. The direction of E_θ which produces this accumulation corresponds to an original ion perturbation which depletes ions at that point. Thus the trapped-electron buildup enhances the original perturbation, resulting in an instability. If either electron-neutral collisions dominate or electrostatic trapping occurs,[10] low-velocity electrons contribute most to the net flux, more electrons leave than arrive at the point, and the perturbation decays.

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