Nuclear Magnetic Resonance of Superfluid He^3 near T_c in High Magnetic Fields

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We present the first measurements of NMR behavior of superfluid He^3 in high magnetic fields at melting pressures in the A_1 region in which only one spin species has undergone pairing. We observe a linear shift in the square of the transverse resonant frequency in A_1 with a pressure dependence of $(9.52\pm0.19)\times10^7$ $(\text{Hz})^2/\text{mbar}$, which is 0.188 ± 0.004 times the low-field result. This and other features that we report are in detailed agreement with the theory presented.

In the presence of a static magnetic field H_0, the transition of He^3 to the superfluid A phase at melting pressures splits into two transitions, A_1 and A_2, which separate linearly with field at a rate of 6.4 μK/kOe.[1,2] This results from the altered density of states of the spin populations at the Fermi surface, and creates a region A_1 in which only one spin species is paired. Such behavior was first considered by Ambegaokar and Mermin,[3] who predicted three transitions for condensates with $m_s = \pm 1, 0$, and by Varma and Werthamer.[4] Brinkman and Anderson[5] (BA) first showed that if the A phase were the Anderson-Brinkman-Morel (ABM) state,[6,7] the "axial" $L=1$ state, a splitting in agreement with experiment was expected, since the ABM state may be described in terms of equal-spin pairing only.

We have studied the transverse-frequency shift $\Delta\nu_0 \equiv \nu_0 - \gamma H_0$ near the A_1 region at 4.93 and 7.40 kOe, the longitudinal resonant frequency ν_L near the A_2 transition at 15.1 kOe, as well as $P(A_1)$ and $P(A_2)$, the melting pressures of the transitions, in several fields up to 15.1 kOe. The behavior we observe is in detailed agreement with the calculations we present based on the ABM state when we include the effects of spin fluctuations as worked out by Brinkman, Serene, and Anderson (BSA).[8]

By considering the general expression for the free energy of the ABM state near T_c as given by Brinkman and Anderson, using the generalized gap for the ABM state,

$$\Delta(\vec{k}) = (k_x + ik_y)\begin{pmatrix}\Delta_1 & 0\\ 0 & \Delta_2\end{pmatrix},$$

one can show that the gaps for the two spin species must have the form

$$(\Delta_1)^2 = \begin{cases}\alpha(t'+1), & -1 \leq t' \leq 0,\\ \alpha + \beta t', & 0 \leq t',\end{cases}$$

$$(\Delta_2)^2 = \begin{cases}0, & t' \leq 0,\\ \beta t', & 0 \leq t',\end{cases} \qquad (1)$$

where t' is a new temperature scale linear in T such that $t'(A_1) = -1$ and $t'(A_2) = 0$. In this new temperature scale, $t'(T_c) = -\alpha/2\beta$ as H_0 is decreased to zero. [We define T_c as $T(A)$ in zero field.] Using the field dependence of the A_1-A_2 splitting given above yields

$$t' = (1.56\times10^5\ \text{kOe/K})(T_c - T)/H_0 - \alpha/2\beta.$$

The constants α and β are determined from the free energy in terms of the Brinkman-Anderson parameter $\delta = (\Delta F^s)_{\text{BW}}/(\Delta F^0)_{\text{ABM-BW}}$ by using the results of BSA. We find $\beta/\alpha = (1 - 7\delta/80)/(1 - 21\delta/40)$. In the same sense, the fractional specific-heat jump at T_c, $\Delta C/C_{\text{F.L.}}$, is $5(1.43)/6(1-21\delta/40)$.

To calculate $\Delta(\nu_0)^2 \equiv \nu_0^2 - (\gamma H_0)^2$ in the A_1-A_2 region we repeat the calculations of Leggett[9] for motion of the spin system of the superfluid. We derive the d matrix $d_{\alpha i}$[5] and calculate the dipole energy $\Gamma(d_{ii}d_{jj}^* + d_{i\alpha}d_{\alpha i}^*)$ as a function of orientation of the spin coordinate axis. By rotating $\vec{d}$ toward $-\hat{H}_0$ we find $\Delta(\nu_0)^2 = (2\Gamma/\chi)[\frac{1}{2}(\Delta_1+\Delta_2)]^2$. Similarly, by rotating $\vec{d}$ about $\hat{H}_0$ we discover that $(\nu_L)^2 = 2\Gamma\Delta_1\Delta_2/\chi$. Only when $\Delta_1 = \Delta_2$ does $\Delta(\nu_0)^2 = (\nu_L)^2$.[10] Using the form of $(\Delta_i)^2$ from (1) we find the temperature dependences of $\Delta(\nu_0)^2$ in $A(t' \gg 0)$ and A_1 to be in the ratio $4\beta/\alpha$.

All these results, including also the dependence of T_{AB}/T_c upon δ obtained in interpolation between the behavior at the $T_{\text{AB}}/T_c = +1$ and 0 limits, are shown in Fig. 1.

By using Eq. (1) one finds

$$\Delta(\nu_0)^2 = \begin{cases}(\Gamma/2\chi)\alpha(t'+1), & t' \leq 0,\\ (\Gamma/2\chi)\{2\beta t' + \alpha + 2[(\beta t')^2 + \alpha\beta t']^{1/2}\}, & t' \geq 0.\end{cases}$$

Note that the second expression exhibits a square-root singularity at A_2, and has an asymptotic slope for $t' \gg 1$ of $(\Gamma/2\chi)4\beta$ which it reaches to within about 2% for $t' \geq \sim 1$.

To measure what amounted to exceedingly mi-

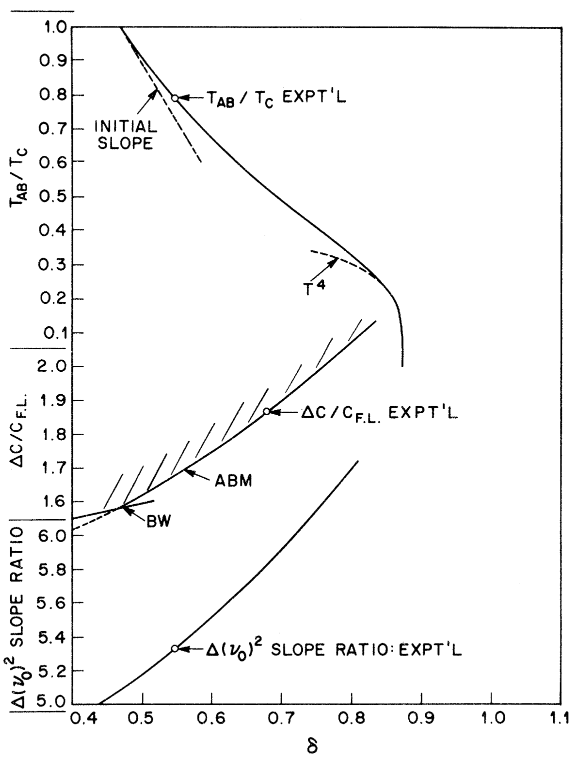

FIG. 1. Theoretical dependence of the $\Delta(\nu_0)^2$-slope ratio, the specific-heat jump (Ref. 11), and T_{AB}/T_c upon the spin-fluctuation parameter δ. The curve given is a freehand interpolation between the low- and high-temperature limits given by the dashed curves. The high-temperature portion is a linear approximation in $T-T_c$ as stated by BSA, while the T^4 curvature at $T\rightarrow 0$ follows as a result of the nodes of the gap in the ABM phase, essentially as calculated by Anderson and Morel.

nute frequency shifts in an extremely narrow temperature interval we utilized a highly corrected superconducting solenoid to provide our static field H_0 whose field homogeneity over about 10% of our sample (above 5 kOe) was about 4×10^{-8}. By using a high-stability frequency synthesizer to generate the rf field, and by adequately compensating for magnetic-field drifts, we could determine our resonant frequency shifts to within ± 0.1 Hz. The compression cell we used to make these measurements[12] was magnetically inert, and will be described in detail elsewhere.

We used the He^3 melting pressure as our thermometer, utilizing a Straty-Adams–type capacitive transducer which exhibited short-term reproducibility of ± 0.02 mbar and long-term drift of typically ± 0.05 mbar (at T_c, $dP_M/dT \simeq -35.65$ mbar/mK)[13] to measure pressure. We find in magnetic fields as high as 15.1 kOe that $P(A_2) - P(A_1)$ is linear in H_0 to within about 7%. Since for fields above about 1 kOe we expect $T(A_1) - T(A_2)$ to vary linearly with H_0, and that the depression of the solid He^3 entropy will be at least quadratic in H_0, we interpret this behavior of $P(A_2) - P(A_1)$ as evidence that the slope of the He^3 melting curve in fields below 7.5 kOe is nearly independent of H_0 (to within ~3%) at T_c. (A more complex set of measurements of pressure intervals to be presented elsewhere substantiates this conclusion.)

The cell temperature was first regulated near $T(A)$ for about 2 h during a given compression, after which time the cell warm-up rate at constant volume was about 0.05 μK/sec. Just prior to cooling below $T(A_1)$, the rate of field decay was measured to ± 0.1 Hz over a half-hour period. Then, for the next hour, data were obtained by sweeping the frequency while regulating the cell pressure to ± 0.02 mbar. This process was repeated several times at 16 and 24 MHz resonant frequencies.

Values of $\Delta(\nu_0)^2$ corrected for the field drift were plotted as a function of pressure. Such individual plots were combined by offsetting the pressure scales so that the data just below $T(A_2)$ would be coincident. In this way minor pressure errors were eliminated. Typical offsets at 24 MHz were only 0.02 mbar, but at 16 MHz, because of a faulty electrical switch, offsets as large as 0.1 mbar had to be made. The final data sets are shown in Figs. 2 and 3.

The solid curves in Figs. 2 and 3 are best fits to the theory in which the scale of t', $P(A_2) - P(A_1)$, and the slope of $\Delta(\nu_0)^2$ below $T(A_2)$ were allowed to vary. The slopes in the A_1 phase at 16 and 24 MHz were found by linear regression on the data points to be 9.59×10^7 and 9.44×10^7 Hz^2/mbar, respectively. From the fits, the values of $4\beta/\alpha$ at 16 and 24 MHz were found to be 5.35 and 5.28. From these we find that the asymptotic slopes ($t' \gg 1$) of $\Delta(\nu_0)^2$ are 5.13×10^8 and 4.98×10^8 Hz^2/mbar for 16 and 24 MHz. These are to be compared with a slope of $(5.14\pm0.02)\times10^8$ Hz^2/mbar measured at 2.8 MHz (low field) near T_c. Also from the fits we obtain values of $[P(A_2) - P(A_1)]/H_0$ of 0.229 and 0.239 mbar/kOe at 16 and 24 MHz, where theory shows that this quantity should be field independent. We compare these values with the average value of 0.227 mbar/kOe obtained from the pressurization curves (P versus time) through A_1 and A_2 at magnetic fields of 4.93, 7.40, and 7.51 kOe). From these comparisons we find the 16-MHz data to be

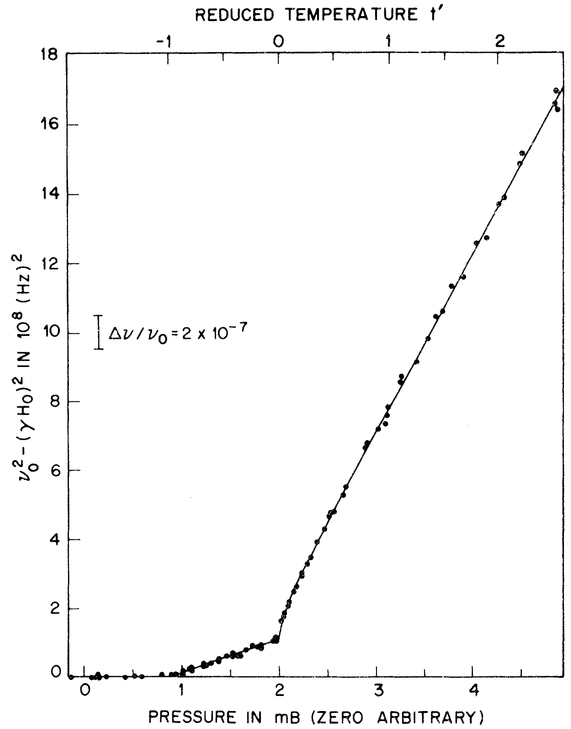

FIG. 2. Transverse-frequency-shift data, $\Delta(\nu_0)^2$ (from four runs), versus pressure at $\nu_0=16$ MHz. The solid curve is the theoretical fit to $\Delta(\nu_0)^2$ versus t'. See text.

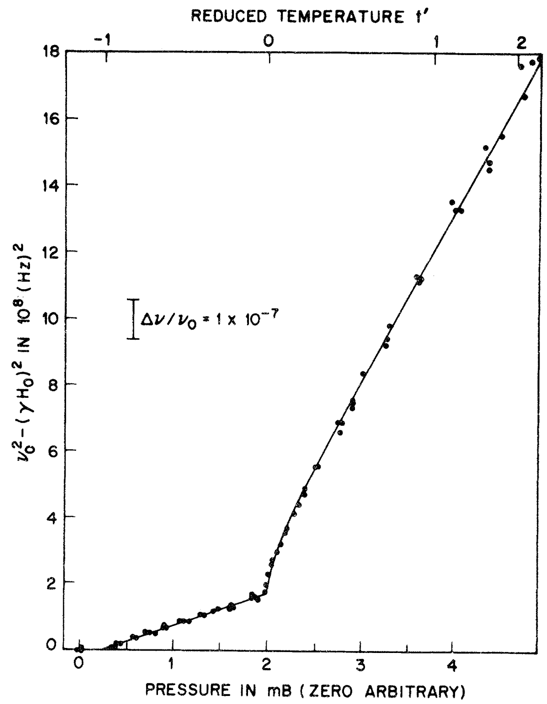

FIG. 3. Transverse-frequency-shift data, $\Delta(\nu_0)^2$ (from four runs), versus pressure at 24 MHz. The solid curve is the theoretical fit to $\Delta(\nu_0)^2$ versus t'. See text.

very reliable, and obtain a weighted-average value of $4\beta/\alpha=5.33\pm0.10$.

From our value of β/α one expects $P(T_c)$ to lie at a pressure $0.625P(A_2)+0.375P(A_1)$. When we plot this pressure, as obtained in a number of magnetic fields from the pressurization curves, as a function of $(H_0)^2$, we obtain a straight line of slope $(5.3\pm0.1)\times10^{-2}$ mbar/(kOe)^2 which passes directly through $P(T_c)$ in zero field. This is consistent with T_c defined in this fashion being a fixed temperature.

Our value of δ is in close agreement with the value obtained from T_{AB}/T_c in Fig. 1. The disagreement with the specific-heat determination may be due to further strong coupling effects not accounted for by theory.

Finally, we have also observed the behavior of ν_L at 16 and 22 kHz in a field of 15.2 kOe and find that behavior in total agreement with the theory. The pressure dependence of ν_L at pressures very close to $P(A_2)$ is very steep and we conclude that no longitudinal resonance will occur in the A_1 phase.

In conclusion, we find that the NMR behavior we observe is in detailed agreement with a rather complex theory which is based entirely upon the Leggett equations, and the BSA results. In the framework of BSA, the behavior we observe in this tiny temperature interval allows us to independently evaluate δ, and we show that this estimate of δ is entirely consistent with other estimates based on T_{AB} measurements. This agreement supports both the BSA theory and the major conclusions of that theory, that is, that the A phase is to be identified as the ABM state and the B phase is to be identified as the Balian-Werthamer (BW) state.[14] For those who do not wish to acknowledge the successes of BSA as sufficient proof that the new superfluid phases are of the $L=1$ manifold, this work provides precise experimental measurements of behavior in the A_1 region which appears at present to be the only region in which free-energy calculations can be easily carried out for higher odd-L angular momentum states.

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[1]D. D. Osheroff, Ph.D. thesis, Cornell University, 1972 (unpublished).

[2]W. J. Gully, D. D. Osheroff, D. T. Lawson, R. C. Richardson, and D. M. Lee, Phys. Rev. A 8, 1633 (1973).

[3]V. Ambegaokar and N. D. Mermin, Phys. Rev. Lett. 30, 81 (1973).

[4]C. M. Varma and N. R. Werthamer, Bull. Amer. Phys. Soc. 18, 23 (1973). These authors considered only the low-field limit and hence predicted only two transitions.

[5]W. F. Brinkman and P. W. Anderson, Phys. Rev. A 8, 2732 (1973).

[6]P. W. Anderson and W. F. Brinkman, Phys. Rev. Lett. 30, 1108 (1973).

[7]P. W. Anderson and P. Morel, Phys. Rev. 123, 1911 (1961).

[8]W. F. Brinkman, J. W. Serene, and P. W. Anderson, to be published. A calculation along similar lines by Y. Kuroda (to be published) gives results which agree with BSA for the purpose of this paper.

[9]A. J. Leggett, Phys. Rev. Lett. 31, 352 (1973), and Ann. Phys. (New York) 85, 11 (1974).

[10]Since the submission of this work for publication we have received a preprint of an article by S. Takagi (to be published) which also derives the frequency shift in A_1 for a more general class of states and arrives at the same result.

[11]J. C. Wheatley, Physica (Utrecht) 69, 218 (1973).

[12]D. D. Osheroff and W. F. Brinkman, Phys. Rev. Lett. 32, 584 (1974).

[13]W. P. Halperin, private communication.

[14]R. Balian and N. R. Werthamer, Phys. Rev. 131, 1553 (1963).

Ion Heating by an Intense Relativistic Electron Beam

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A high-current electron beam is used to produce a plasma from neutral hydrogen. Doppler–line-broadening and Thomson-scattering measurements show that the ion energy is 100 ±15 eV compared with only 12 ±2 eV for the electron energy. It is shown that this ion energy can be supplied by an inverse pinch effect. It is suggested that much higher ion energies will result for electron densities n_e lower than 10^{15} cm^{-3} measured here.

Several experimenters[1-7] have reported that when a high-current relativistic electron beam is injected into a plasma, a fraction of the beam energy is transferred to the plasma through the excitation of streaming instabilities. The energy is transferred primarily to the plasma electrons. In this paper we report measurements which show that direct energy transfer to the ions can occur if the self-magnetic field of the beam is not completely neutralized by the counter-streaming plasma current—for, in such a situation, there exists a force $\vec{j}\times\vec{B}$, associated with the plasma current and the magnetic field due to the net current, which accelerates the plasma radially outward. This is the main force acting on the plasma since the coupling between the beam and plasma electrons is weak. The energy for the radial motion is of course provided by the beam.

The experimental arrangement is shown in Fig. 1. A 36-kA, 350-kV, 100-nsec electron beam is compressed by a cone to a current density of 20 kA/cm^2 and injected through a Mylar foil into a test chamber, which is filled with neutral hydrogen. There is no external magnetic field. The current and energy of the beam are reproducible

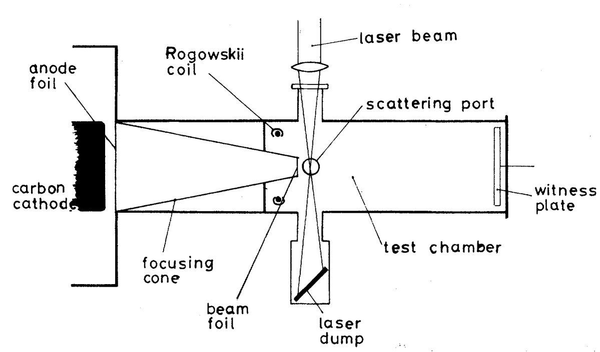

FIG. 1. Diagram of the apparatus. The line-broadening measurements are made by using the laser scattering ports.