Fluctuation Conductivity in the Incommensurate Peierls System*

Bruce R. Patton
Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

and

L. J. Sham
Department of Physics, University of California San Diego, La Jolla, California 92037
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The period of the Peierls distortion in a one-dimensional system is assumed to be incommensurate with the lattice. The fluctuation conductivity just above the mean-field transition temperature is calculated in the presence of electron-impurity scatterings. In the clean limit for sufficiently strong electron-phonon interaction, the conductivity is enhanced by fluctuations whose contribution is impurity limited and goes like $(T-T_P)^{-1/2}$ in the leading approximation. In the dirty limit, the conductivity is reduced by fluctuations.

Recent experiments on quasi one-dimensional conductors have excited much interest in the properties of a one-dimensional electron-phonon system which exhibits a Peierls instability.[1] The conductivity behavior of such a system varies drastically depending on its commensurability. If the first reciprocal lattice vector G of the undistorted lattice is commensurate with the Fermi diameter q_0 $(=2k_F)$, then the system becomes an insulator below the Peierls transition temperature T_P. The half-filled case $(G=2q_0)$ is a special example; its fluctuation conductivity just above T_P is insulatorlike,[2] i.e., the conductivity is depressed. On the other hand, if G is incommensurate with q_0, then the system possesses infinite conductivity at zero temperature.[3] In this case, the fluctuation conductivity is found to be positive and proportional to $(T-T_P)^{-1/2}$ by Allender, Bray, and Bardeen,[4] by means of a phenomenological theory. We report here the results of a microscopic calculation for the incommensurate case.

Random impurities are included to provide finite conductivity in the normal phase, and have two types of effects. The first is to introduce the usual finite lifetime into the single-electron states. The second is to cause a static distortion of the lattice, an effect which is enhanced by the Kohn singularity in one dimension. The importance of this second effect depends on the relative magnitude of the impurity potential and the electron-phonon interaction.

Consider the extreme incommensurate case of an ideal one-dimensional chain of uniformly spaced atoms where the Fermi energy measured from the bottom (or top) of the conduction band is only a small fraction of the bandwidth. The electron-phonon interaction in standard form is

$$H_{ep}=N^{-1/2}\sum_{kq}g(q)\,c_{k+q}^{\dagger}c_k(b_q+b_{-q}^{\dagger}), \qquad (1)$$

where c and b are the electron and phonon operators and N is the number of atoms in the chain. Electron-impurity interaction is included as

$$H_i=N^{-1}\sum_{kql}U_q\,c_{k+q}^{\dagger}c_k\,e^{-iqR_l}p_l, \qquad (2)$$

where p_l denotes the impurity distribution over the sites R_l. For electrons near the Fermi level in the undistorted chain, the important scatterings are (1) in the forward direction, $q\sim 0$, where U_q is taken as zero since it does not affect the current, and (2) in the backward direction, $q\sim q_0$, where $U_q\simeq U$, a constant.

The impurity scattering is treated in the Born approximation. The self-energy correction of Fig. 1(a) supplies the electron in the uniform chain with a finite lifetime, $\tau=v_F/cU^2$, where v_F is the Fermi speed and c is the impurity concentration. All other self-energy terms within the Born approximation, such as shown in Fig. 1(b), vanish. We could include in the lifetime the effects of the electron scattering by "ordinary" phonons, with wave vectors away from

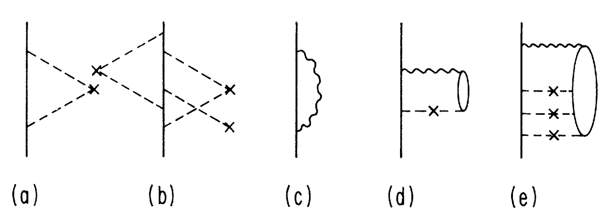

FIG. 1. Electron self-energy terms. Solid lines denote electrons, wavy lines phonons, and dashed lines impurity.

the neighborhood of q_0, i.e., not belonging to those soft phonons just above T_P which constitute the fluctuations.

The self-energy of a phonon with wave vector q close to q_0 is calculated in the mean-field approximation, represented by Fig. 2(a). The electron lines are understood to include self-energy corrections due to impurity scatterings such as shown in Fig. 2(b). Phonon energies, temperature, and the reciprocal electron lifetime $1/\tau$ are all assumed to be much smaller than $\mathcal{E}_F$, the Fermi energy. The extreme incommensurability is used in the sense that the process of an electron excited by the q_0 phonon from the neighborhood of $-k_F$ to k_F is included, but not the process from the neighborhood of k_F to k_F+q_0 which is smaller by a factor of $\max(\omega, 1/\tau, T)/\mathcal{E}_F$. Then, the phonon Green's function is given by

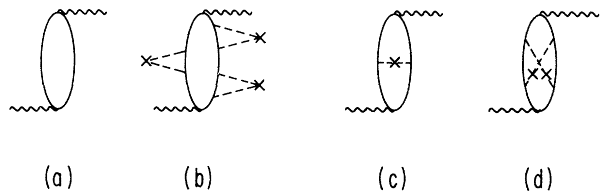

FIG. 2. Phonon self-energy terms.

$$1/D(q_0+p, i\omega_l) = \omega_l^2 - \lambda\Omega^2\{\ln(T_{P0}/T) + \Psi(\tfrac{1}{2}) - \tfrac{1}{2}\Psi(\tfrac{1}{2}+\alpha[1-i\tau(|\omega_l|+v_F p)]) - \tfrac{1}{2}\Psi(\tfrac{1}{2}+\alpha[1-i\tau(|\omega_l|-v_F p)])\}, \qquad (3)$$

where λ is the usual dimensionless electron-phonon coupling constant, Ω is the unrenormalized phonon frequency at q_0, Ψ is the digamma function, $\alpha = 1/4\pi\tau T$ is the electron-hole pair-breaking parameter, and T_{P0} is the transition temperature in the absence of impurities.

From Eq. (3) it follows that the mean-field transition temperature T_P is given by

$$\ln(T_{P0}/T_P) = \Psi(\tfrac{1}{2} + 1/4\pi\tau T_P) - \Psi(\tfrac{1}{2}). \qquad (4)$$

Ordinary impurities serve to depress the transition temperature in the same manner as for the excitonic insulator[5] and as magnetic impurities do in superconductors,[6] as has also been noted by Schuster.[7]

For small ω and p, the soft phonon is given by

$$1/D(q_0+p, i\omega_l) = \omega_l^2 + \omega_p^2 + \Gamma|\omega_l|, \qquad (5)$$

with

$$\omega_p^2 = \lambda\Omega^2(\epsilon + \xi^2 p^2), \qquad (6)$$

$$\epsilon = \ln(T/T_{P0}) + \Psi(\tfrac{1}{2}+\alpha) - \Psi(\tfrac{1}{2}), \qquad (7)$$

$$\xi^2 = -\Psi''(\tfrac{1}{2}+\alpha)v_F^2/32\pi^2T^2, \qquad (8)$$

$$\Gamma = \lambda\Omega^2\Psi'(\tfrac{1}{2}+\alpha)/4\pi T. \qquad (9)$$

ϵ is a measure of the temperature difference $t = (T-T_P)/T_P$. For small t, ϵ is proportional to t.

We collect in Table I the limiting values of these quantities in two cases: (1) the clean limit, $T_P \gg 1/\tau$; and (2) the dirty limit $\mathcal{E}_F \gg 1/\tau \gg T_P$, provided that τ does not drop below its critical value $\tau_{cr} = \gamma/\pi T_{P0}$, where T_P vanishes.

We include only those impurity dressings in the phonon self-energy which are consistent with the self-energy diagram of Fig. 1(a). These are

TABLE I. Summary of expressions in the clean and dirty limits. Conductivity is given in units of $e^2/\hbar$ per unit cross-sectional area of a chain with $l = v_F\tau$ and $\zeta = 7\zeta(3)/\pi^2$.

	Clean limit ($\alpha \ll 1$)	Dirty limit ($\alpha \gg 1$)
ϵ	t	$t/12\alpha^2$
t_c	$(\zeta\pi^2)^{1/3}/4$	$3(\alpha^4/2)^{1/3}$
ξ	$\zeta^{1/2}v_F/4T$	$l/\sqrt{2}$
Γ	$\lambda\Omega^2\pi/8T$	$\lambda\Omega^2\tau$
σ_1	$-(1-2\zeta\alpha)(1+8\alpha/\lambda)\pi l^2/8\zeta t^{1/2}$	$-(1+8\alpha/\lambda)3\sqrt{6}\,l/4\pi t^{1/2}$
σ_2	$(1+4\zeta\alpha)\pi l^2/8\zeta t^{1/2}$	$2\sqrt{6}\,l/\pi t^{1/2}$
$\sigma_1+\sigma_2$	$(1-4/3\zeta\lambda)3\zeta^{1/2}l/4t^{1/2}$	$(1-24\alpha/5\lambda)5\sqrt{6}\,l/4\pi t^{1/2}$

either like those of Fig. 2(b) which contribute to Eq. (3) or of Fig. 2(c) which are negligible. Corrections such as in Fig. 2(d), which have not been kept, have been found to be negligible in the clean limit and to modify the dirty-limit results, though not in an essential way.

As in our earlier work,[2] we calculate the fluctuation conductivity in leading powers of $(\epsilon_c/\epsilon)^{1/2}$, where ϵ_c is the Ginzburg critical value[8] (Table I). Interpreted within the one-dimensional model, the specific-heat measurements[9] at 55 K would imply an order of magnitude $\epsilon_c \sim 0.1$, although the transition there may actually be three-dimensional. In the dirty limit, since $t_c \gg 1$, there is no region of interest in which the expansion in $(\epsilon_c/\epsilon)^{1/2}$ is valid. Nevertheless, it indicates the nature of the corrections to the normal-state conductivity. The leading terms of fluctuation conductivity in $(\epsilon_c/\epsilon)^{1/2}$ are given in Fig. 3. Again, impurity dressing of the electron propagator is understood. The current vertex contains the ladder sum of impurity lines. These terms are consistent with the self-energy terms of Figs. 1(a), 1(c), and 1(d).

Figures 3(a) and 3(b) are analogous to the conductivity due to the electron-phonon scattering. However, the soft phonon now is a long-wavelength fluctuation. It has the effect of creating a fluctuating energy gap in the electron band, giving the electron self-energy

$$\Sigma(k, iE_n) = \Delta^2/(iE_n + \mathcal{E}_k + i/2\tau + iv_F c^{1/2}/\xi), \quad (10)$$

where $\mathcal{E}_k$ is the electron energy measured from the Fermi level and the gap fluctuation is given by[10]

$$\Delta^2 = \pi v_F T/2\xi\epsilon^{1/2}. \quad (11)$$

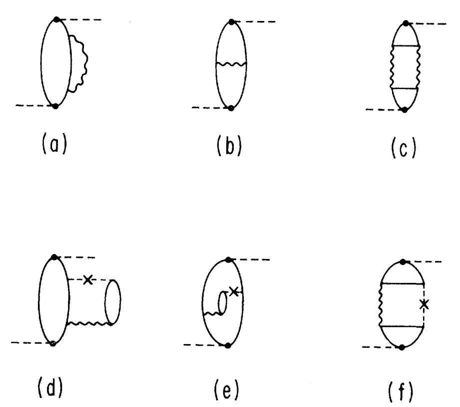

FIG. 3. Fluctuation contributions to conductivity. The circle denotes the current vertex dressed by impurities.

The net effect on the conductivity through the processes in Figs. 3(a) and 3(b) is to make the contribution insulatorlike as in the commensurate case, giving

$$\sigma_1 = -\pi l_1^2/\xi\epsilon^{1/2}, \quad (12)$$

where

$$l_1 = [\Psi'(\tfrac{1}{2}+\alpha) - \tfrac{1}{2}\alpha\Psi''(\tfrac{1}{2}+\alpha)]^{1/2}\, l/2\pi \quad (13)$$

and $l = v_F\tau$ is the scattering length.

Figure 3(c) is analogous to the ordinary phonon-drag term. In the half-filled case this term vanishes,[2] while in the extreme incommensurate case it is finite. In the former case, the ground state is doubly degenerate and the low-temperature phase is insulating. In the latter, the ground state is continuously degenerate and a metastable current-carrying state can be constructed.[3] The phonon-drag contribution above T_P corresponds to the collective-mode contribution[11,12] below T_P. Figure 3(c) contributes to the conductivity[13]

$$\sigma_2 = \pi l_2^2/\xi\epsilon^{1/2}, \quad (14)$$

with

$$l_2^2 = \Lambda^2/2\Psi'(\tfrac{1}{2}+\alpha), \quad (15)$$

where Λ comes from the triangular vertex, given by

$$\Lambda = [\Psi'(\tfrac{1}{2}+\alpha) - \alpha\Psi''(\tfrac{1}{2}+\alpha)]\, l/\pi. \quad (16)$$

Figures 3(d) and 3(e) correspond to an impurity-induced lattice distortion which modifies the gap fluctuation (11) to $\Delta^2(1+8\alpha/\lambda)$ and thus increases σ_1 in magnitude. Correction due to the sum of terms such as in Fig. 1(e) is of the same order of magnitude. The impurity phonon-drag term of Fig. 3(f) is analogous to the pinning of the collective mode below T_P. It gives no contribution to the dc conductivity above T_P since there is no phonon-drag current associated with a static impurity.

The total fluctuation conductivity is given by the sum of Eqs. (12) and (14) with σ_1 modified by the factor $(1+8\alpha/\lambda)$. In Table I the limiting values are given. For sufficiently strong electron-phonon interaction, the phonon-drag contribution is larger and the fluctuation conductivity is positive. In all regions the conductivity is impurity limited and proportional to $t^{-1/2}$. In the clean limit the leading term in τ^2 cancels. This cancelation follows from the fact that the coefficients of the τ^2 terms are proportional to the corresponding diagrammatic contributions in the completely pure case where Peierls's theorem[1] implies the infinite conductivity of the free-electron gas.

In the phenomenological theory of Allender, Bray, and Bardeen,[4] the electron is assumed to relax quickly to the running lattice wave. In the absence of impurity scattering, such relaxation could arise from higher-order electron-phonon scattering.[14] In the present calculation, the electron lifetime τ may arise from both impurity scattering and non-q_0 phonon scattering as discussed above. Our expression for the phonon-drag conductivity σ_2 has the same form as the result of Ref. 4 if we identify their ξ_0 with the coherence length ξ in the *dirty limit.*

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Observation of Thermal Fluctuations in Superconducting Microbridges

F. J. Rachford and C. Y. Huang
Department of Physics, Case Western Reserve University, Cleveland, Ohio 44106

and

M. Nisenoff and S. Wolf
Naval Research Laboratory, Washington, D. C. 20375
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Thermal fluctuations of the type described by Langer and Ambegaokar and by McCumber and Halperin have been observed in superconducting thin-film rings containing microbridge sections. A peak in the fluctuation rate was found near the transition temperature. The temperature dependence of the observed rates is in agreement with the McCumber-Halperin fluctuation-rate equations and the deduced parameters are consistent with the theory.

In the course of a series of experiments investigating the properties of superconducting quantum interference devices operating at microwave frequencies, current-induced steps have been observed in the effective impedance of the tin microbridge sections of our samples which are very similar to the voltage steps found in the dc-current-driven transition to the normal state of one-dimensional tin microbridges and whiskers.[1-3] These steps appear to be caused by the generation of localized centers of Ohmic dissipation in the microbridge sections of our samples. The properties of these steps and their relation to the dc-current observations will be discussed in another publication. Data are presented on thermal-fluctuation effects associated with the production of these steps in a temperature region just below the transition temperature of our films.