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## Anisotropy in the *B* Phase of He<sup>3</sup>

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The magnetic anisotropy of the *B* phase (assumed to be a Balian-Werthamer phase) is investigated within the Landau-Ginzburg theory. The anisotropy is determined by the action of the dipolar energy and the magnetic field on a single vector  $\vec{n}$  and it is shown that walls will induce variations in  $\vec{n}$  on a scale of  $R_C \sim 0.1$  cm. In the presence of a field  $H$  this characteristic length becomes  $R_C H_B / H$ , where  $H_B$  is of the order of 100 Oe.

The discovery<sup>1</sup> of the new superfluid phases of He<sup>3</sup> has generated considerable excitement because of the anisotropic nature of the states involved. These phases are undoubtedly BCS-type states made up of Cooper pairs with the orbital and spin angular momenta equal to 1.<sup>2</sup> The *A* phase has attracted most attention because in it each pair has  $m_L = 1$  along some axis  $\vec{l}$  and  $m_S = 0$  along some other axis  $\vec{k}$ .<sup>3-5</sup> These axes, along with the phase and magnitude of the order parameter, characterize the state of the *A* phase. It has been shown that  $\vec{l}$  acts as an anisotropy axis for propagation of the various types of sound waves<sup>4</sup> and superfluid flow.<sup>3,5</sup> It has also been shown<sup>6</sup> that  $\vec{l}$  will be oriented perpendicular to the surface of any walls so that the *A* phase should exhibit textures, that is, gradual variations of  $\vec{l}$  which are determined by the shape of the sample, the applied fields, and any imposed currents. In this sense this phase is similar to a liquid crystal. In the same context little attention has been

paid to the *B* phase because it is generally believed to be isotropic. Although in many respects this statement is true it is not so for the magnetic properties of the *B* phase,<sup>7,8</sup> and we shall show that there are a number of theoretical reasons which make the *B* phase attractive from the point of view of studying textures and singularities of the order parameter with nuclear magnetic resonance techniques.

The *B* phase is believed to be the Balian-Werthamer state<sup>9</sup> in which each pair has  $\vec{J} = \vec{L} + \vec{S} = 0$ , but with the spin variables rotated (arbitrarily) relative to the orbital variables. The state is therefore specified by a rotation matrix which we write in terms of the axis of rotation  $\vec{n}$  and the angle of rotation  $\theta$ . Thus introducing the notation used previously<sup>10</sup> to specify the order parameter, namely that

$$\langle a_{\vec{k}\beta\alpha} - \vec{k}_\gamma \rangle \propto \sum_{\alpha i} d_{\alpha i} \hat{k}_\alpha (\sigma^i \sigma^Y)_{\beta\gamma}, \quad (1)$$

we write

$$d_{\alpha i} = (\Delta/\sqrt{3}) [\delta_{\alpha i} \cos\theta + n_\alpha n_i (1 - \cos\theta) + \epsilon_{\alpha i j} n_j \sin\theta]. \quad (2)$$

Here  $|\Delta|$  is the isotropic gap. For an infinite system the value of the angle  $\theta$  is determined by minimizing the dipolar energy<sup>7</sup> and it has been found to be  $\theta = \cos^{-1}(-\frac{1}{4})$ . We will find that even in the presence of surfaces this angle remains essentially unchanged. The dipolar energy does not orient  $\vec{n}$  but in the presence of a magnetic field  $\vec{n}$  tends to orient itself along the field because of a combination of the depairing effect of the field and the dipolar energy.<sup>7,8</sup> For a finite system the fact that the surface acts to depair the  $m_l=0$  component along its normal must be taken into account. To do this we introduce a trial form for  $d_{\alpha i}(\vec{r})$  which incorporates this boundary condition:

$$d_{\alpha i}(\vec{r}) = (\Delta/\sqrt{3})[\delta_{\alpha\beta} - f(r_{\perp})s_{\alpha}s_{\beta}]R_{\beta i}(\vec{r}). \quad (3)$$

The  $R_{\alpha i}(\vec{r})$  is the rotation matrix specified by  $\vec{n}(\vec{r})$  and  $\theta$ ,  $\vec{s}$  is a vector normal to the surface, and

$f(r_{\perp})$  is assumed to be nonzero only when the perpendicular distance from the surface  $r_{\perp}$  is of the order of the coherence length  $\xi(T)$  ( $\sim 200\text{\AA}$ ). With  $f(0)=1$  this function satisfies the boundary conditions imposed by the depairing of the  $m_l=0$  component of the triplet. There are actually additional depairing effects<sup>6</sup> due to the undoubtedly non-specular nature of reflection from the walls and due to the curvature of the walls. These will tend to reduce the tangential components of the order parameter but should not change the dominant anisotropies at the surface induced by the strong depairing of the component normal to the surface. We ensure that there are no spin currents flowing into the surface by imposing the additional boundary condition that  $(\vec{s} \cdot \nabla)\vec{n} = 0$ . By using Eq. (3) and the general expression for the gradient terms in the Landau-Ginzburg theory of these phases<sup>3,11</sup> we can calculate the energy required to bend  $\vec{n}$  as a function of position. It is found to be

$$F_B^B = \frac{c}{13} \int_V d^3r \{ 16|\nabla \times \vec{n}|^2 + 13(\nabla \cdot \vec{n})^2 - 2\sqrt{15}(\nabla \cdot \vec{n})(\vec{n} \cdot \nabla \times \vec{n}) - 5(\vec{n} \cdot \nabla \times \vec{n})^2 + 16\nabla \cdot [(\vec{n} \cdot \nabla)\vec{n} - \vec{n}(\nabla \cdot \vec{n})] \}, \quad (4)$$

where

$$c = \frac{\hbar^2 \rho}{4m^*} \left( 1 - \frac{T}{T_c} \right) \left( \frac{13}{8} \right). \quad (5)$$

The last term in (4) is a surface term but is generally nonzero. There are other terms involving  $f$  but these can be shown to be small or independent of  $\vec{n}$ . We have set  $\theta = \cos^{-1}(-\frac{1}{4})$ . Equation (4) along with the orientation energy in an external field constitute the bulk energies that depend on  $\vec{n}$ . This orientation energy is

$$F_H^B = -a \int d^3r (\vec{H} \cdot \vec{n})^2, \quad (6)$$

where  $a$  is given in terms of the dipole energy  $E_D$  and the gyromagnetic factor  $\gamma$ ,<sup>2,8</sup>  $a \approx \hbar^2 \gamma^2 E_D / [T_c(T_c - T)]$ . Using (3) we find two different surface energies. The first comes from inserting (3) into the dipolar energy<sup>8</sup> and is of the form

$$F_D^S = -b \int_S d^2r [(\vec{s} \cdot \vec{n})^2 - \frac{5}{18}(\vec{s} \cdot \vec{n})^4], \quad (7)$$

where  $b \approx \xi E_D$ , that is, the coherence length times the bulk dipole energy. This term tends to align  $\vec{n}$  perpendicular to the surface. The second contribution arises because near the surface the order parameter is not symmetric and consequently the susceptibility is no longer isotropic.<sup>12</sup> This induces a field-dependent surface energy of the form

$$F_H^S = -d \int d^2r (\vec{s} \cdot \vec{R} \cdot \vec{H})^2, \quad (8)$$

where  $\vec{R}$  is the rotation matrix in (3) and  $d$  is of order  $\xi(\chi_N - \chi_B)$ , i.e., the coherence length times the difference in susceptibility between the normal state and the Balian-Werthamer state. This term is minimized if  $R$  is such as to rotate  $\vec{H}$  into  $\pm \vec{s}$ . If  $\theta$  is greater than the angle between  $\vec{s}$  and  $\vec{H}$  a direction for  $\vec{n}$  can always be found such as to minimize  $F_H^S$ . Since the equilibrium value of  $\theta$  is  $> 90^\circ$  one can always minimize this term by varying the direction of  $\vec{n}$ . Also, since the surface dipolar term cannot compete with the bulk dipole energy,  $\theta$  does not vary appreciably near the walls, so that we are allowed to concentrate on the variation of  $\vec{n}$ . Inserting the expression for  $\vec{R}$  in terms of  $\vec{n}$  we find for  $\vec{s} \perp \vec{H}$

$$F_H^S = \frac{25d}{16} \int_S d^2r [(\vec{s} \cdot \vec{n})(\vec{n} \cdot \vec{H}) + (\frac{2}{3})^{1/2} \vec{n} \cdot (\vec{s} \times \vec{H})]^2. \quad (9)$$

Combining these results we find that the textures of  $\vec{n}$  for various shaped samples are characterized by two fields  $H_S$  and  $H_B$  and a characteristic length  $R_c$ ,

$$R_c \equiv c/b \approx 0.1(1 - T/T_c)^{1/2} \text{ cm},$$

$$H_S \equiv (b/d)^{1/2},$$

$$H_B \equiv (b^2/ac)^{1/2}.$$

Both  $H_S$  and  $H_B$  are of order 10–100 G and are

relatively temperature independent. That the length  $R_c$  goes to zero is a reflection of the softness of the bending energy as  $T \rightarrow T_c$ . Very near  $T_c$  the surface terms become small and the bulk anisotropy dominates so that  $\vec{n}$  becomes everywhere parallel to  $\vec{H}$ . This fact may help explain the recent experiments of Webb, Kleinberg, and Wheatley<sup>13</sup> and Ahonen, Haikala, and Krusius.<sup>14</sup>

The most relevant situation to study is a cylindrical sample with  $\vec{H}$  parallel to the sample axis. In zero field there are two competing textures (see Fig. 1), one in which  $\vec{n}$  points along the cylinder axis and simply flares out as a function of the distance from the center (texture I) and the other where  $\vec{n}$  lies in the plane perpendicular to the cylinder axis and exhibits an in-plane structure in which there are two disgyration lines on opposite sides of the cylinder (texture II). In Fig. 1 we have drawn these textures as they appear at zero field. A disgyration here is defined in a fashion similar to that used by de Gennes for the A phase, that is, a line about which  $\vec{n}$  rotates always pointing in a tangential direction. We find that the in-plane structure is stable for the radius of the cylinder  $R < 74R_c$ . As  $R$  decreases the disgyrations are gradually excluded and the texture becomes uniform with  $\vec{n}$  along an arbitrary direction in the plane. In an external field the texture I is more favored, and one finds that if one starts with texture II there is a first-order transition at a critical field to texture I.

Turning now to the question of NMR line shapes, since  $\theta$  is fixed the NMR resonance frequency

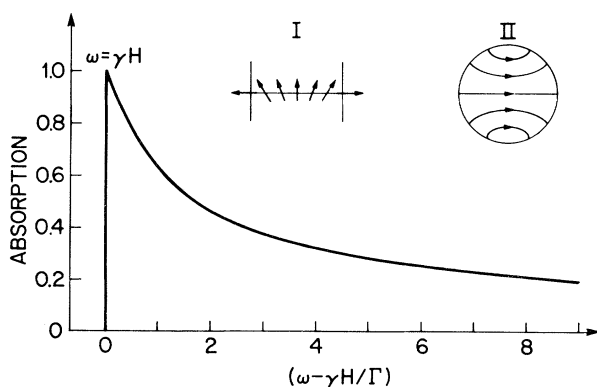


FIG. 1. A plot of the high-field NMR line shape for a cylindrical sample. The two textures that we have considered are illustrated in the upper part of the figure. The first is a side view of the cylinder and the arrows represent the direction of  $\vec{n}$ . To illustrate the second texture we take a view along the axis of the cylinder and the lines represent tangents to the vector field.

only depends on the angle between  $\vec{n}$  and the external field. At large fields ( $\gamma H \gg \Omega_L$ ) the transverse resonance frequency  $\omega_t$  is<sup>7</sup>

$$\omega_t^2 = (\gamma H)^2 + \Omega_L^2 \sin^2 \chi(\vec{r}), \quad (10)$$

where  $\chi$  is the angle between  $\vec{n}$  and  $\vec{H}$  and  $\Omega_L$  is the longitudinal resonance frequency.

The shift in frequency gives rise to a line shape  $P(\omega)$ , which we calculate assuming that each region of the sample resonates at its local frequency  $\omega_t$ . It is

$$P(\omega) = V^{-1} \int d^3r \delta[\omega - \omega_t(\vec{r})], \quad (11)$$

where  $V$  is the cylinder volume.

In texture I where  $\vec{n}$  has a component along  $\vec{H}$  we may find  $P(\omega)$  corresponding to the case in which  $R \gg R_c H_B / H$ , since the variation of  $\vec{n}$  is then described by

$$\chi(\vec{r}) = A \sinh(rH/R_c H_B), \quad (12)$$

where  $r$  measures the distance from the cylinder axis.  $A$  is a constant given by  $A = A_0 / \sinh(RH/R_c H_B)$ , where  $A_0$  is the angle between  $\vec{n}$  and the magnetic field at the cylinder surface. Since over most of the sample  $\vec{n}$  is nearly parallel to  $\vec{H}$ , we may approximate  $\sin \chi$  by its argument. We get

$$P(\omega) = 2(H_B R_c / HR)^2 (1/\Gamma) f(2[(\omega - \gamma H)/\Gamma]), \quad (13)$$

$$\omega \geq \gamma H,$$

where the characteristic width is  $\Gamma = (\Omega_L^2 / \gamma H) A^2$  and

$$f(x) = \ln[x^{1/2} + (x+1)^{1/2}] / (x+x^2)^{1/2}. \quad (14)$$

This function is exhibited in Fig. 1 in which  $P(\omega)$  is plotted versus frequency.

Experimentally<sup>15</sup> what is observed is not the ideal line shape  $P(\omega)$ , but an average of it corresponding to the variation in magnetic field strength over the sample caused by the presence of a finite field gradient. Consider such an average for the transverse line shape:

$$N(\Delta\omega) = \int_{\gamma H}^{\gamma H + \Delta\omega} P(\omega) d\omega \quad (15)$$

$$\approx 1 - \frac{H_B R_c}{HR} \ln \left( \frac{\Omega_L^2}{2\Delta\omega \gamma H} A_0^2 \right), \quad (16)$$

which is valid when  $N$  is close to unity. The main dependence on magnetic field comes from the factor  $H_B R_c / HR$ , since the logarithm is slowly varying. The average represents the amount of unshifted signal which is observed,<sup>15</sup> as a function of magnetic field. If we plot the maximum ab-

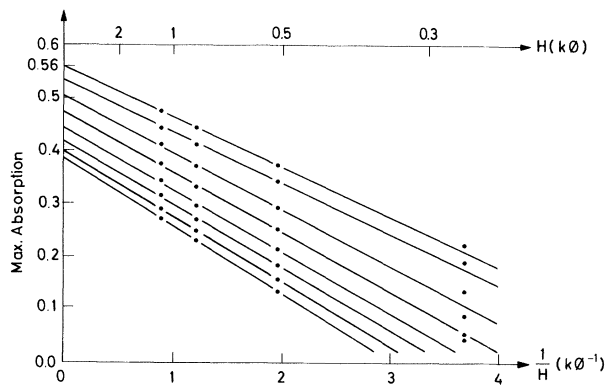


FIG. 2. The maximum absorption data of Ref. 13 plotted versus  $1/H$ . The slope gives an estimate of  $H_B R_C \approx 10-20$  cm Oe. The curves from top to bottom are increasing pressures (decreasing temperature) relative to the  $A$  transition; from the top down the pressures relative to  $A$  are 21.0, 23.5, 27.1, 30.7, 34.3, 38.0, 41.6, and 45.2 mbar, respectively.

sorption data of Ref. 13 versus the inverse magnetic field (Fig. 2), this dependence on  $1/H$  shows up very clearly. Because of the uncertainty in the estimates of  $H_B$  and  $R_C$ , as well as  $\Delta\omega$ , a direct numerical comparison between the measured and calculated slopes cannot be made. We wish to point out, though, that the experimentally determined slope of the curves in Fig. 2 is reproduced for reasonable values of the parameters, namely,  $R_C H_B \sim 10$  to 20 cm Oe, assuming the logarithm to be about 2 to 4.

It should also be mentioned that these results only depend on  $\vec{n}$  not being along  $\vec{H}$  at the surface and consequently are not dependent on the detailed knowledge of surface energies.

In conclusion, we emphasize that there are several reasons that NMR studies of textures may be very rewarding in the  $B$  phase.

(1) There is only a single vector involved,  $\vec{n}$ , and the local resonance frequency is determined

by its angle relative to the external field. In the  $A$  phase the directions of both  $\vec{k}$  and  $\vec{l}$  play a role in determining the resonance frequency.

(2) The lengths in the  $B$  phase are longer,  $R_C = [(F_N - F_S)/E_D] \xi = 10^6 \xi \sim 0.1$  cm, so that textures should be easily observable and, indeed, are clearly evident in the data in Fig. 2. In other unpublished NMR data rather sharp dips occurred which may be due to a point singularity in the order parameter. These will be discussed elsewhere.

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