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A_2-A_1 Interference Phase*

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(Received 22 April 1974)

The A_2-A_1 phase difference observed in $\pi^-p \to \pi^+\pi^-\pi^-p$ is compared with the A_2^- phase from a Regge fit to the $d\sigma/dt$ data for A_2^- and the A_1^- phase given by a Reggeized Deck model. The agreement (~ 30°) depends crucially on the contribution of the Reggeized pion propagator to the A_1 phase and requires equal signs for the f and P residues in the A_2 amplitude.

Phases of strong interaction amplitudes—although of obvious interest—are all too seldom measurable. Partial-wave analyses[1,2] of the reaction $\pi^-p \to \pi^+\pi^-\pi^-p$ give not only the magnitudes of A_1 (defined as the state $1^+S \to \rho\pi$) and A_2 production amplitudes but also their relative phase. In this note we use the phase of the A_1^- amplitude from a Reggeized-Deck-model calculation, and the A_2^- phase from a Regge fit to $d\sigma/dt$ for A_2, to predict the A_2^--A_1^- phase and compare the prediction to the data.

Phase of the $\pi^-p \to A_1^-p$ amplitude.—A version of Berger's Reggeized Deck model[3,4] has been shown to agree well with the data for $\pi^-p \to \pi^+\pi^-\pi^-p$ for $M_{3\pi} \leq 1.5$ GeV, excepting the A_2 partial wave (which is nearly absent in the model). In particular, the model predicts correctly the relative phases between different partial waves. It also predicts that the only important amplitude for A_1 production is the nucleon s-channel helicity nonflip amplitude leading to the 1^+ state with $J_z = 0$ (in the t channel). The phase of this amplitude (extracted by partial-wave analysis[5] of the model amplitude) shows only minor dependence on $M_{3\pi}$, in accord with the data.

Typical values of the phase (at $M_{3\pi} = M_{A_2}$) are shown in Table I. We note that the phase exceeds by ~ 60° the value of 90° one would obtain for diffractive production of a stable particle or resonance. The extra phase comes from the signature factor of the Reggeized π propagator and is directly proportional to α_π' ($\alpha_\pi' = 1$ GeV^{-2} was used). This extra phase is required to explain the phase differences observed between different

TABLE I. Deck-model phase for A_1 production at $M_{3\pi} = M_{A_2}$.

P_{lab} (GeV) \ $t - t_{min}$ (GeV^2)	0	−0.1	−0.2	−0.3
6	146°	158°	170°	179°
16	146°	156°	166°	175°
26	146°	156°	165°	173°
36	146°	155°	164°	172°

nonresonant partial waves (e.g., between $1^+S \to \rho\pi$ and $1^+P \to \epsilon\pi$). It turns out also to be necessary to explain the A_2-A_1 phase difference.

$\pi N \to A_2 N$ data.—We review briefly relevant aspects of the data.

(a) Above $p_{\rm lab} \geq 10$ GeV, A_2^- is produced in $\pi^- p \to A_2^-$ in a pure state (a linear combination of the states $|J^P M\rangle = |2^+ \pm 1\rangle$): $|2^+1\rangle + |2^+ -1\rangle$ in the t-channel frame. This result—consistent with dominance of natural parity exchanges—has been obtained in three A_2 decay modes: $A_2^- \to \pi^+\pi^-\pi^-$,[1,2] $A_2^- \to K^0K^-$,[6] and $A_2^- \to \eta^0\pi^-$.[7] In all subsequent discussions we consider only A_2 production in the above state.

(b) The cross section for production of A_2^- in the above state decreases as $p_{\rm lab}^{-0.5}$ from 5 to 40 GeV. This result has been seen in the channel $A_2 \to 3\pi$ and confirmed in the $A_2 \to \eta\pi$ channel. A Regge fit to the observed energy dependence will clearly require inclusion of a trajectory higher than the ρ-f trajectory.

(c) The A_2^- differential cross section, $(\rho_{11} + \rho_{1\,-1})d\sigma/dt$, vanishes in the forward direction. The $A_2^- \to 3\pi$ data can be fitted by $|t - t_{\rm min}|\exp(bt)$, with b increasing from 5 GeV^{-2} at 5 GeV^{-2} to 8.5 GeV^{-2} at 40 GeV.

(d) The A_2^--A_1^- phase has been extracted from the $\pi^- p \to \pi^+\pi^-\pi^- p$ data at several energies ($p_{\rm lab}$ = 5–40 GeV).[1,2] The phase difference comes from the interference density matrix element $\rho_{A_1A_2}$ between the A_2 state ($|2^+1\rangle + |2^+ -1\rangle$) and the A_1 state ($|1^+0\rangle$, $1^+S \to \rho\pi$). As already noted the polarizations refer to t-channel axes. The $M_{3\pi}$ dependence of the observed phase is as expected for a resonant A_2 and an A_1 with slowly varying phase. In the following, A_2 production phases refer to phases obtained by subtracting the phase of the A_2 propagator (90° at $M_{3\pi} = M_{A_2}$) from the observed phases. The s and t dependence of the A_2 production phase is shown in Fig. 3.

(e) The other observed quantity relating to A_2-A_1 interference is the coherence between A_2 and A_1 states, defined as $|\rho_{A_2A_1}|/(\rho_{A_1A_1}\rho_{A_2A_2})^{1/2}$. This factor would be unity if A_2 and A_1 amplitudes had the same dependence on nuclear spin variables. The observed values[8] range from 0.7 ± 0.1 at 5 GeV to 0.9 ± 0.06 at 40 GeV. At energies above 5 GeV the observed coherence is consistent, within errors, with unity. Since the A_1 production (in the model) is nonflip (in the s channel), we conclude that A_2 production is also mainly nonflip, particularly at high energy.

(f) The A_2-A_1 phase differences and the cross sections are nearly equal in the two reactions $\pi^\pm p \to \pi^+\pi^-\pi^\pm p$ at 5 and 7.5 GeV.[2] Since A_1 production is mainly isoscalar exchange, this suggests that isoscalar exchanges dominate.

It is unfortunate that the available $\pi^+ n \to A_2^0$ data[9] are very limited and inadequate to shed further light on this point.

Regge fit to $\pi^- p \to A_2^- p$.—The above discussion suggests a simple model for A_2^- production in which nucleon spin-flip and $I = 1$ exchange contributions are neglected, with f and P trajectories as the main contributors. Some theoretical prejudices are available to support these assumptions. Michael and Ruuskanen[10] discuss the relative contributions of f and ρ trajectories. Using factorization, duality, and π-N scattering data, they predict that the ρ contribution to the nonflip amplitude and the f contribution to the flip amplitude are unimportant; further the ρ contribution to the flip amplitude should be small at small t [it is suppressed by an additional factor $(-t/4m^2)^{1/2}$, where m is the nucleon mass].

We have therefore fitted the data[1,2] using only a nucleon nonflip amplitude with only isoscalar contributions from P and f. We have chosen a simple parametrization of the amplitude:

$$M^{\rm nf}(s,t) = -\sqrt{C}(|t - t_{\rm min}|)^{1/2}\left[\exp(A_P t)\left(\frac{s}{s_0}\right)^{\alpha_P(t)}\exp\left(-\frac{i\pi\alpha_P(t)}{2}\right) + K\exp(A_f t)\left(\frac{s}{s_0}\right)^{\alpha_f(t)}\exp\left(-\frac{i\pi\alpha_f(t)}{2}\right)\right] \quad (1)$$

with C, K, A_P, and A_f as free parameters. We use $s_0 = 1$ GeV and the trajectories $\alpha_P = 1 + 0.36t$, $\alpha_f = 0.56 + 0.86t$.[11]

The differential cross section is related to the amplitude of Eq. (1) by

$$(\rho_{11} + \rho_{1\,-1})d\sigma/dt = |M^{\rm nf}|^2/s^2.$$

We have fitted the $\pi^- p \to A_2^- p$ data from $\pi^- p \to \pi^+\pi^-\pi^- p$ ($p_{\rm lab}$ = 5–40 GeV). A satisfactory fit to the data ($\chi^2/NDF = 10.5/11$) was obtained as shown in Figs. 1 and 2.

The parameters of the fit are $C = 1120 \pm 300$ mb, $A_P = 3.5 \pm 0.5$ GeV^{-2}, $K = 1.5 \pm 0.7$, and $A_f = -0.5 \pm 0.8$ GeV^{-2}.

Three comments are in order:

(a) The fit shown does not include spin-flip contributions. Equally good fits can be made with moderate spin-flip contributions (the size of the flip contribution is limited by the observed A_2-A_1 coherence).

(b) Our fit neglects isovector (ρ) exchange con-

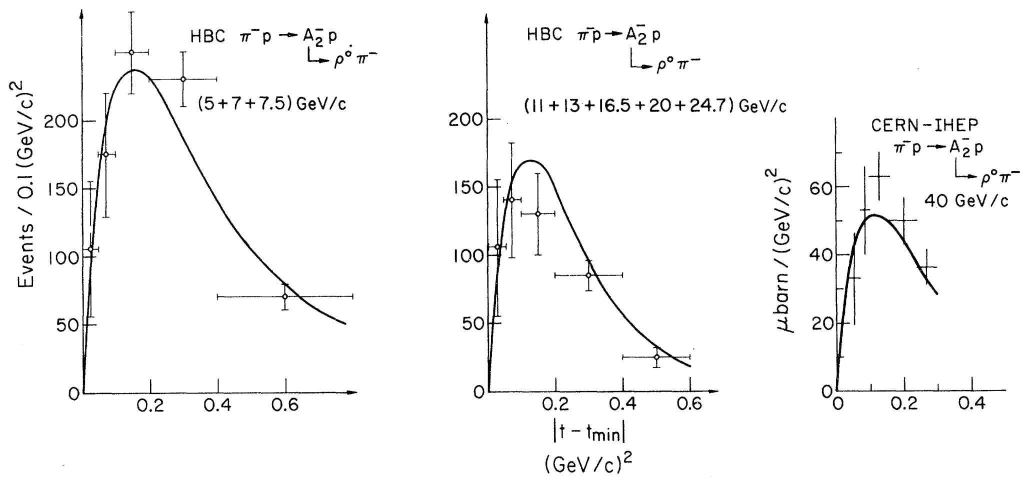

FIG. 1. Comparison of measured to fitted differential cross sections for $\pi^- p \to A_2^- (\to \rho^0 \pi^-) p$.

tributions to the nonflip amplitude. The model A_1 amplitude, in fact, does contain some ρ exchange (at $p_{lab} = 6$ GeV, $t = 0$ the A_1^+ amplitude leads the A_1^- amplitude by 30°). Equality of A_2^+-A_1^+ and A_2^--A_1^- relative phases suggests that some isovector contribution to the nonflip A_2 amplitude is probably present.

The data are not adequate to resolve either question. Furthermore, the resulting predictions about A_2^--A_1^- phase difference are not very sensitive to the answers. We conclude that there is not much reason to push these matters further.

(c) The data on $d\sigma/dt$ show no preference for positive or negative K. Comparable fits can be obtained with f and P contributions with imaginary parts having either the same or opposite signs. The solution given above (equal sign) is preferred since it alone gives a reasonable agreement with the observed A_2^--A_1^- phase differences.

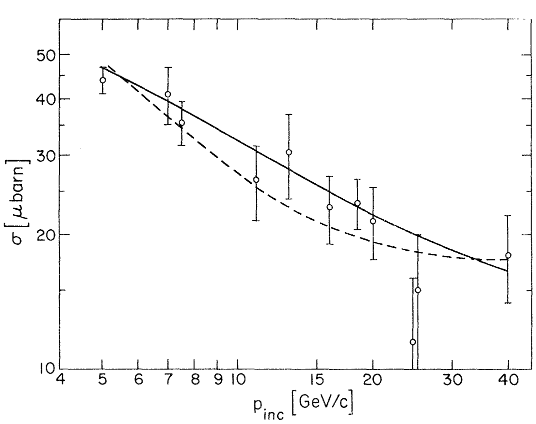

FIG. 2. Measured and fitted integrated cross sections for $\pi^- p \to A_2^- (\to \rho^0 \pi^-) p$ for $1.2 \leq M_{3\pi} \leq 1.4$ GeV and $0 < |t - t_{min}| < 0.7$ (GeV/c)^2, as a function of beam momentum. Solid curve for fit with $K < 0$ and dashed curve for fit with $K < 0$ [see Eq. (1)].

Comparison with observed A_2-A_1 phases.—Figure 3 shows the comparison between the observed A_2^--A_1^- relative phase and the difference between the A_2^- phase from the Regge-model fit [Eq. (1)] and the A_1^- phase from the Deck model (Table I). For all momentum transfer bins and all incident momenta the agreement is within 30°. We note that with a 90° phase for the A_1 amplitude the disagreement would be about 90°.

The estimated uncertainties in the predicted phases associated with uncertainties in various parameters are relatively small: (1) A change in Pomeron slope from 0.36 to 0 changes the predicted A_2 phase by ~5°. (2) A 20% change in the slope of the π trajectory (from $\alpha_\pi' = 1$ GeV^{-2}) changes the predicted A_1 phase by ~10°. (3) The main uncertainties are presumably those associated with the precise form chosen for the Deck model, and with possible contributions of isovector exchange to the A_2^- nucleon nonflip amplitude.

We conclude that in view of the above uncertainties the agreement obtained is satisfactory.

Since the A_2 is a "normal" resonance we expect the amplitude for A_2 production in $\pi N \to A_2 N$ to have the same properties as the production amplitude of a stable 2^+ meson. In particular, the phase of the amplitude (assumed even under s-u interchange) can be inferred from the s dependence (at fixed t) of the imaginary part either via a Regge fit or from a dispersion relation. The same arguments do not apply to the A_1 pro-

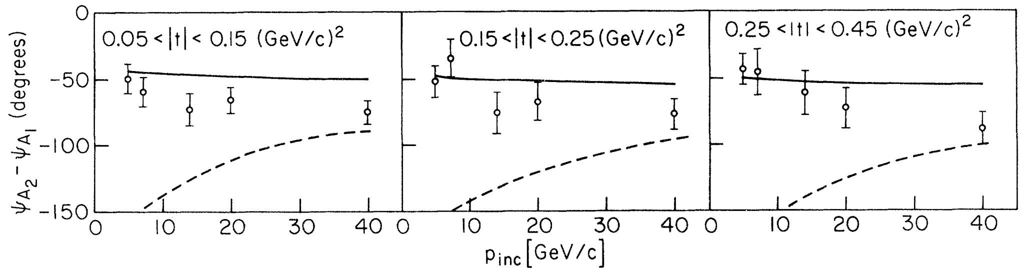

FIG. 3. Comparison of measured A_2-A_1 interference phase to the phase predicted from Regge and Deck model calculation. Dependence on incident momentum is shown for different momentum transfer intervals. [Note: The data at 40 GeV/c extend only to momentum transfer of -0.3 (GeV/c)^2.] Solid (dashed) curve for fit with $K>0$ ($K<0$) in Eq. (1).

duction amplitude. According to the data the A_1 observed in $\pi p \to 3\pi p$ is not a normal resonance but a kinematic accident in the 1^+ projection of the five-point amplitude for $\pi^- p \to \rho^0 \pi p$. It is therefore not surprising to find that the Reggeized Deck model gives an A_1 production phase which is very different from the A_2 production phase, in spite of the almost identical s dependence of the cross sections.

The Regge fit to the A_2 production data confirms the suspicion that a significant Pomeronlike contribution is required to account for the observed s dependence. While this result violates often-stated prejudices[12] regarding Pomeron couplings, it violates—as far as we know—no fundamental principle. In fact, the result seems to be in line with the similarity between Pomeron and f^0 couplings observed in other reactions (in particular in elastic scattering).

With a view to characterizing the couplings in this reaction, we emphasize that the observed A_2 polarization corresponds to a helicity change $|\Delta\lambda| = 1$ in *the t channel* (not s channel) at the π-A_2 vertex. Because the range of t explored limits the nucleon crossing angle to relatively small values, the available data (coherence between A_2 and A_1) do not allow us to distinguish s-channel helicity conservation from t-channel helicity conservation at the nucleon vertex (even with our assumption of s-channel helicity conservation for the A_1).

We note that the data favor equal signs for the f^0 and P coupling in $\pi p \to A_2 p$. Carlitz, Green, and Zee[13] have speculated that $\beta_f(t)/\beta_P(t)$ might be the same in all reactions. Comparison of our ratio of couplings in $\pi N \to A_2 p$ with the corresponding ratio in $\pi N \to \pi N$ and $pp \to pp$ shows rough agreement.

*Work supported in part by the Atomic Energy Commission contract No. AT(11-1)1195 and by the National Science Foundation Contract No. NSF GP 40908X.

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