

from an earlier analysis of this experiment. The zero at  $t = -1.3 \text{ GeV}^2$  may be associated with the NSWSZ for  $f^0$  which has been observed here from the present analysis.

We thank F. Buhl and Group-A Lawrence Berkeley Laboratory for letting us use their  $A_2^+$  data, and K. Robinson, W. Fickinger, A. Engler, and R. Kraemer for showing us their  $A_2^0$  data before publication.<sup>2</sup> A discussion with L. Trueman was useful and one of us (K.-W.L.) enjoyed an unusual conversation with G. Fox.

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<sup>1</sup>D. J. Crennell *et al.*, Phys. Lett. **35B**, 185 (1971). K.-W. Lai, in *Phenomenology in Particle Physics—1971*, edited by C. B. Chin, G. C. Fox, and A. J. G. Hey (California Institute of Technology Press, Pasadena, Calif., 1971), p. 257.

<sup>2</sup>J. Diaz *et al.*, Phys. Rev. Lett. **32**, 260 (1974).

<sup>3</sup>F. Buhl *et al.*, private communication.

<sup>4</sup>The  $\rho^0\pi^\pm$  events in the  $J^P = 2^+$  ( $D$  wave) state was estimated in the mass interval 1.2–1.44 GeV. In the fit, density-matrix elements ( $\rho_{00}, \rho_{1-1}, \rho_{11}$ ) were treated as free parameters but all subjected to the positivity constraints. Corrections have been made for background  $2^+$  events and for the mass interval cut. See G. Ascoli *et al.*, Phys. Rev. Lett. **25**, 962 (1970), for details of the method.

<sup>5</sup>We used the value  $A_2 \rightarrow \rho\pi/A_2 \rightarrow \text{all} = 72.4 \rightarrow 2.1\%$  [T. A. Lasinski *et al.*, Rev. Mod. Phys., Suppl. **45**, S1 (1973)] which has been affected the least by discoveries of other new but minor modes of  $A_2$ . See Ref. 2.

<sup>6</sup>For  $A_2^+$  we used also the shape of  $d\sigma/dt$  of the  $\eta\pi^+$  mode which is also characterized by a small background (< 15%). See M. Alston-Garnjost *et al.*, Phys. Lett. **33B**, 607 (1970). Furthermore we observe no detectable differences in shapes of  $d\sigma/dt$  at the incident momenta 4.5, 6, and 7.1 GeV/c at the present level of statistics; thus, we assume the shape of  $d\sigma/dt$  for  $A_2^+$  at 6 GeV/c by using 7.1 GeV/c data.

<sup>7</sup> $\Lambda(1520)$  is defined from 1.5 to 1.54 GeV and does not overlap with the  $A_2^-$  region at our incident momenta.

<sup>8</sup>Ref. 3 and K. J. Foley *et al.*, Phys. Rev. D **6**, 747 (1972).

<sup>9</sup>See G. C. Fox and A. J. G. Hey, Nucl. Phys. **B56**, 366 (1973), for details.

<sup>10</sup>D. J. Crennell *et al.*, Phys. Rev. Lett. **27**, 1674 (1971); H. A. Gordon *et al.*, Phys. Rev. D **8**, 779 (1973).

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## Exotic Interactions of Charged Leptons\*

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We consider the information available about nonelectromagnetic lepton interactions in experiments which are relatively insensitive to the loop structure of the underlying theory. Further experiments which would clarify the role of exotic interactions are proposed and discussed.

Recent theoretical and experimental developments have once again focused attention on the question: What is the precise nature of the interaction between electrons, muons, and hadrons? Almost all gauge theories<sup>1</sup> augment the conventional electromagnetic interaction between these particles with additional interactions. Furthermore, recent experimental results indicating (a) a nearly constant cross section in  $e^+e^-$  annihilation into hadrons<sup>2</sup> and (b) deviations from Bjorken scaling in deep inelastic muon-hadron scattering<sup>3</sup> have invited the speculation<sup>4</sup> that exotic (i.e., nonelectromagnetic) interactions may be coming into play at newly available energies.

In this note we report on a systematic study of the information available about exotic lepton interactions in "tree experiments." (By "tree experiments" we mean experiments which are insensitive to the loop structure of the underlying theory.) We restrict ourselves to energies well below the masses of all intermediate vector bosons; this permits us to encompass a variety of theoretical proposals in a compact phenomenological interaction. We determine the consequences of this interaction for (a) atomic spectroscopy, (b)  $e^+e^-$  annihilation, and (c) deep inelastic lepton-hadron scattering. While our analysis leads us to tilt towards the view that exotic lepton interactions are unlikely to play a significant

role in (b) and (c) at present energies or to provide a safe avenue of escape for the notion of "preco-cious asymptotic freedom," its confrontation with recent experiments does not allow us to make any categorical statements. Novel and precise experiments, of the type discussed below, must be carried out before the matter is fully resolved.

*Phenomenological parametrization.*—We postulate the effective  $(V,A)$ -type interaction

$$H_{\text{eff}} = (G_F/2\pi\alpha\sqrt{2}) \sum_{i,j} (\epsilon_{ij}^{VV} \bar{\psi}_i \gamma^\rho \psi_i \bar{\psi}_j \gamma_\rho \psi_j + \epsilon_{ij}^{VA} \bar{\psi}_i \gamma^\rho \psi_i \bar{\psi}_j \gamma_\rho \gamma_5 \psi_j + \epsilon_{ij}^{AA} \bar{\psi}_i \gamma^\rho \gamma_5 \psi_i \bar{\psi}_j \gamma_\rho \gamma_5 \psi_j), \quad (1)$$

the summation being over  $e, \mu$  and all the quarks required to meet the needs of weak-interaction theory and hadron spectroscopy.  $G_F$  and  $\alpha$  are the Fermi and the fine-structure constants, respectively.

The theoretical values of the parameters  $\epsilon$  can be quickly read off in any gauge model, from the expression for the neutral current. Thus, in the popular quartet charm model<sup>1, 5, 6</sup>

$$\epsilon_{ij}^{VV} = \zeta(\tau_3 - 4Q \sin^2 \xi)_i (\tau_3 - 4Q \sin^2 \xi)_j, \quad \epsilon_{ij}^{VA} = -\zeta(\tau_3 - 4Q \sin^2 \xi)_i (\tau_3)_j, \quad \epsilon_{ij}^{AA} = \zeta(\tau_3)_i (\tau_3)_j, \quad (2)$$

where  $Q$  is the electric charge,  $\xi$  is the Salam-Ward-Weinberg angle,<sup>7</sup> and  $\tau_3 = +1$  for  $p$  and  $p'$ ,  $-1$  for  $n, \lambda, e$ , and  $\mu$ . Also

$$\zeta = \pi\alpha(m_w^2/m_z^2 \cos^2 \xi) = \pi\alpha \text{ (for one Higgs doublet)}. \quad (3)$$

In the "purely elastic" variant of a triplet charm model,<sup>1, 6, 8</sup>

$$\epsilon_{ij}^{VV} = 36\pi\alpha R_2; \quad \epsilon_{ij}^{VA} = \epsilon_{ij}^{AA} = 0. \quad (4)$$

Here  $R_2 = [(1 - 2 \sin^2 \xi)m_w/m_z \cos \xi]^2$  can be bounded experimentally<sup>6</sup>:  $R_2 < 0.48$ .

In relating the parameters  $\epsilon$  to physically observed quantities, we shall make free use of the naive—but highly successful—quark model<sup>9</sup> for protons and neutrons as well as the quark-parton picture of high-energy dissociation.<sup>10</sup> This is within the spirit of one of the objectives of this study, namely, to see if exotic lepton-hadron interactions can rescue asymptotic freedom, since the quark-parton picture provides a pedestrian handle on asymptotic freedom, which is adequate modulo logarithmic corrections.<sup>11</sup>

*Atomic spectroscopy.*<sup>12</sup>—The derivation of atomic level shifts, resulting from the interaction of Eq. (1), is quite straightforward. We merely state the results.<sup>13</sup>

(i) Hyperfine splitting in muonium:

$$\Delta E(^3S_1) - \Delta E(^1S_0) = -64\lambda \epsilon_{e\mu}^{AA} \alpha^4 m_e^3 / M_N^2, \quad (5)$$

where

$$\lambda = G_F M_N^2 / 16\pi^2 \alpha^2 \sqrt{2} \simeq 10^{-3}. \quad (6)$$

Equation (5) corresponds to

$$\Delta\nu = -0.005 \epsilon_{e\mu}^{AA} \text{ MHz}. \quad (7)$$

If we require that this  $\Delta\nu$  be less than the limits of disagreement between conventional theory [quantum electrodynamics (QED)] and experiment, we obtain the bound

$$|\epsilon_{e\mu}^{AA}| < 10. \quad (8)$$

(ii) Shift of  $nS$  levels in hydrogenic atoms:

$$\Delta E(nS) = (16Z^3/n^3)\lambda [Z(2\epsilon_{ep}^{VV} + \epsilon_{en}^{VV}) + (A - Z)(\epsilon_{ep}^{VV} + 2\epsilon_{en}^{VV})] \alpha^4 m_e^3 / M_N^2. \quad (9)$$

Most important is the shift of the  $2S$  level in atomic hydrogen,

$$\Delta\nu(2S) \simeq 1.6 \times 10^{-4} (2\epsilon_{ep}^{VV} + \epsilon_{en}^{VV}) \text{ MHz}. \quad (10)$$

The agreement between conventional theory and experiment will tolerate a  $\Delta\nu < 10^{-1}$  MHz. Hence

$$|\frac{1}{3}(2\epsilon_{ep}^{VV} + \epsilon_{en}^{VV})| < 200. \quad (11)$$

(iii) Hyperfine splitting in hydrogen:

$$\Delta\nu_{\text{hfs}}/\nu_{\text{hfs}} = 12\lambda(m_e/M)[\epsilon_{ep}^{AA}(1 + g_A) + \epsilon_{en}^{AA}(1 - g_A)]/(1 + \kappa_p). \quad (12)$$

Here  $g_A$  is the axial-vector-to-vector ratio in neutron  $\beta$  decay and  $\kappa_p = 1.79$ . Requiring that  $\Delta\nu_{\text{hfs}}$  be less than the theoretical uncertainty in the conventional calculation<sup>12</sup> of  $\nu_{\text{hfs}}$  (the experiment is good to 1 part in  $10^{12}$ , so there is essentially no experimental error), we obtain

$$|\epsilon_{ep}^{AA} - \frac{1}{9} \epsilon_{en}^{AA}| < 1, \quad (13)$$

a remarkably stringent bound.

(iv) Muonic atoms: While the level shifts in S states of muonic atoms due to exotic interactions are much larger than in ordinary atoms, the major contributions to the observed shifts depend on poorly known parameters such as the distribution of charge and magnetic moment in the nucleus. Therefore, one cannot learn much about  $\epsilon_{\mu h}^{VV}$  and  $\epsilon_{\mu h}^{AA}$  from muonic atoms. However, by comparing nuclear charge radii as determined from muonic atoms and from electron scattering, we can conclude that

$$|\epsilon_{ep}^{VV} - \epsilon_{\mu p}^{VV} + \epsilon_{en}^{VV} - \epsilon_{\mu n}^{VV}| < 40. \quad (14)$$

The circular polarization of muonic x rays, or other pseudoscalar correlations involving such x rays, appears to be a feasible atomic probe of  $\epsilon_{\mu h}^{VA}$  and  $\epsilon_{\mu h}^{AV}$ . Measurement of the circular polarization of the photon in the decay  $2S_{1/2} \rightarrow 1S_{1/2} +$  (one photon) in muonic atoms gives information<sup>14</sup> about  $\epsilon_{\mu h}^{AV}$ , whereas a measurement of the photon circular polarization in the muonic hyperfine transition ( $n_1S, F = I + \frac{1}{2}$  to  $n_2S, F = I - \frac{1}{2}$ ) gives information<sup>15</sup> about  $\epsilon_{\mu h}^{VA}$ . Detection of these transitions, and measurement of the relevant correlations, may be feasible with the intense muon sources now being developed. Similar experiments have been proposed for heavy electronic ions<sup>14</sup> and atoms<sup>16</sup> to measure  $\epsilon_{eh}^{AV}$ .

*$e^+e^-$  annihilation.*—The cross sections for muon-pair and hadronic final states, in the presence of the interaction of Eq. (1), take the form

$$\sigma(e^+e^- \rightarrow \mu^+\mu^-)/\sigma_\gamma(e^+e^- \rightarrow \mu^+\mu^-) = 1 - 4(\lambda s/M^2)\epsilon_{e\mu}^{VV} + 4(\lambda s/M^2)^2 \sum_{m,n=V,A} (\epsilon_{e\mu}^{mn})^2, \quad (15)$$

$$\sigma(e^+e^- \rightarrow \text{hadrons})/\sigma_\gamma(e^+e^- \rightarrow \text{hadrons}) = 1 + 4(\lambda s/M^2)A + 4(\lambda s/M^2)^2B. \quad (16)$$

Here  $s$  is the square of the total c.m. energy and

$$A = \sum_i Q_i \epsilon_{ei}^{VV} / \sum_i Q_i^2, \quad B = \sum_i \sum_{mn} (\epsilon_{ei}^{mn})^2 / \sum_i Q_i^2, \quad (17)$$

$Q_i$  being the quark charges. Also, throughout this paper,  $\sigma_\gamma$  refers to one-photon exchange cross sections:

$$\sigma_\gamma(e^+e^- \rightarrow \text{hadrons})/\sigma_\gamma(e^+e^- \rightarrow \mu^+\mu^-) = \sum_i Q_i^2, \quad (18)$$

$$\sigma_\gamma(e^+e^- \rightarrow \mu^+\mu^-) = 4\pi\alpha^2/3s. \quad (19)$$

Careful measurements of  $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$  at SPEAR will permit us to put more stringent bounds on the  $\epsilon_{e\mu}$  than was possible, for example, in Eq. (8).

Despite the presence in  $\sigma(e^+e^- \rightarrow \text{hadrons})$  of a term falling as  $1/s$ , a constant term, and a term rising as  $s$ , one can choose  $A$  and  $B$  such that this cross section is within 20% of 25 nb in the interval  $8M^2 < s < 28M^2$ . Values of  $\epsilon$  as high as 30 or so are needed.

*Deep inelastic scattering.*—Let  $p$  be the momentum of the target hadron and let  $k$  and  $k - q$  be the momenta of the incident and outgoing leptons, respectively. In terms of the usual scaling variables [ $x = |q^2|/2q \cdot p$ ,  $y = q \cdot p/k \cdot p$ ] the double differential cross sections may be displayed as follows:

$$\frac{1}{2}[\sigma(l_R^- N) + \sigma(l_L^+ N)]_{xy} / \sigma_\gamma(l^- N)_{xy} = 1 - 4(\lambda|q^2|/M^2)A' + 4(\lambda|q^2|/M^2)^2B', \quad (20)$$

$$\frac{1}{2}[\sigma(l_R^- N) - \sigma(l_L^+ N)]_{xy} / \sigma_\gamma(l^- N)_{xy} = -[4(2y - y^2)/(2 - 2y + y^2)](\lambda|q^2|/M^2)[C' - 2(\lambda|q^2|/M^2)D']. \quad (21)$$

Here  $\sigma_{xy} \equiv \partial^2\sigma/\partial x \partial y$  and  $R$  and  $L$  denote incident-lepton helicities.<sup>17</sup> The coefficients  $A'$ ,  $B'$ ,  $C'$ , and  $D'$  are, in general, functions of  $x$  and their precise form is not terribly enlightening. In specific situations and/or models, they can become  $x$  independent. For example, if we assume that nucleons contain only  $p$ - and  $n$ -type quarks, we find that

$$A' = \frac{3}{5}[2(\epsilon_{ip}^{VV} + \epsilon_{ip}^{AV}) - (\epsilon_{in}^{VV} + \epsilon_{in}^{AV})]; \quad B' = \frac{9}{5}[(\epsilon_{ip}^{VV} + \epsilon_{ip}^{AV})^2 + (\epsilon_{ip}^{AA} + \epsilon_{ip}^{VA})^2 + (\text{terms with } p \rightarrow n)]; \quad (22)$$

$$C' = \frac{3}{5}[2(\epsilon_{ip}^{AA} + \epsilon_{ip}^{VA}) - (\epsilon_{in}^{AA} + \epsilon_{in}^{VA})]; \quad D' = \frac{9}{5}[(\epsilon_{ip}^{VV} + \epsilon_{ip}^{AV})(\epsilon_{ip}^{AA} + \epsilon_{ip}^{VA}) + (\text{terms with } p \rightarrow n)].$$

Here again, despite the presence of terms linear and quadratic in  $|q^2|$  in Eq. (20) one can accommodate a nearly constant deviation of about 30% from the scaling limit, in the interval  $20M^2 < |q^2| < 40M^2$ , by judicious ascription of numerical values to the  $\epsilon$ 's, in the range 5–10.

*Remarks.*—(i) We have followed the path suggested by gauge theories and have parametrized exotic lepton-hadron couplings in terms of quark fields and restricted ourselves to  $V$  and  $A$  interactions;  $S$  interactions stemming from Higgs fields have been discussed in the literature<sup>18</sup> and we have little to add.

Note that, so far as atomic spectroscopy is concerned, we could equally well have parametrized the interaction of Eq. (1) in terms of nucleon fields and bounded the relevant parameters without appeal to the quark model. The bounds obtained for  $\epsilon_{ep}$  follow by identifying the appropriate  $\epsilon_{ep}$  with the left-hand side of Eq. (11) or Eq. (13). We could also have included  $S$ -,  $P$ -, and  $T$ -type covariants which, however, would be indistinguishable from  $V$  and  $A$  in the static limit.

(ii) It is evident that exotic lepton couplings of the sort expected in some of the existing gauge models have almost negligible effects at present on atomic spectroscopy, and on  $e^+e^-$  annihilation and deep inelastic scattering at present-day energies. However, improvements in spectroscopic measurements and in QED calculations, and the availability of higher energies, will certainly bring the effects of these couplings into the realm of measurability. In the meantime, one can still draw some useful inferences from Eq. (13) in the context of present-day models. For example, if the Higgs sector of the quartet charm model is enlarged so that  $m_W > m_Z \cos \xi$ , rather than being equal to it, the usual lower bound on  $m_Z$  ( $> 76$  GeV) is no longer valid; Eq. (13) can be used to yield the modest bound

$$m_Z > 6 \text{ GeV.}$$

(iii) Theories which attempt to explain the SPEAR  $e^+e^-$  data and the, rather preliminary, muon-production data from the National Accelerator Laboratory in terms of exotic couplings require that some  $\epsilon$ 's be on the order of 10 or so. If these theories are constrained<sup>4</sup> in such a way that  $\epsilon^{AA}$  is of the same order of magnitude as the other  $\epsilon$ 's, Eq. (13) implies that they must be rejected. The speculation that exotic lepton couplings are at the root of the difficulties faced by the notion of precocious scaling can not be dismissed, however, until better bounds are avail-

able on all the  $\epsilon$ 's. The  $\epsilon^{VA}$  and  $\epsilon^{AV}$  are particularly tricky; two independent handles are available, however, and worthy of experimental investigation: (a) the circular polarization of muonic x rays, mentioned above, and (b) the difference between  $l^+$  and  $l^-$  deep inelastic cross sections calculated in Eq. (21).<sup>19</sup>

(iv) An interesting difference occurs between the effects of exotic interactions in deep inelastic scattering, as given in Eqs. (20) and (21), and the uncalculated effects of two-photon exchange which might also be large in some kinematic regions. While the former may interfere with one-photon exchange in both  $\sigma(l_R^- N) \pm \sigma(l_L^+ N)$ , the latter can interfere with one-photon exchange only in  $\sigma(l_R^- N) - \sigma(l_L^+ N)$ , essentially by the charge-conjugation properties of the current. Measurement of both quantities therefore would go far to distinguish these two possibilities.

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<sup>1</sup>For a recent review see M. A. B. Bég and A. Sirlin, Rockefeller University Report No. COO-2232B-47, 1974 (to be published).

<sup>2</sup>B. Richter, *Bull. Amer. Phys. Soc.* **19**, 100(T) (1974), and in *Proceedings of the Conference on Lepton Induced Reactions*, Irvine, California, 7–8 December 1973 (to be published).

<sup>3</sup>K. W. Chen, *Bull. Amer. Phys. Soc.* **19**, 100 (1974). (The possibility that deviations from  $\mu$ - $e$  universality are manifesting themselves in the National Accelerator Laboratory experiment can not be discounted, but need not be taken seriously at this time.)

<sup>4</sup>A. Salam, remark at the Conference on Lepton Induced Reactions, Irvine, California, 7–8 December 1973. J. C. Pati and A. Salam, to be published.

<sup>5</sup>S. Weinberg, *Phys. Rev. D* **5**, 1412 (1972); C. Bouchiat, J. Iliopoulos, and Ph. Meyer, *Phys. Rev. Lett.* **38B**, 519 (1972).

<sup>6</sup>M. A. B. Bég, to be published.

<sup>7</sup>A. Salam and J. C. Ward, *Phys. Lett.* **13**, 168 (1964); S. Weinberg, *Phys. Rev. Lett.* **19**, 1264 (1967).

<sup>8</sup>M. A. B. Bég and A. Zee, *Phys. Rev. Lett.* **30**, 675 (1973).

<sup>9</sup>M. A. B. Bég, B. W. Lee, and A. Pais, *Phys. Rev. Lett.* **13**, 514 (1964); for a review see G. Morpurgo, in *Proceedings of the XIV International Conference on High Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN Scientific Information

Service, Geneva, 1968).

<sup>10</sup>J. D. Bjorken and E. A. Paschos, Phys. Rev. **185**, 1975 (1969), and Phys. Rev. D **1**, 3151 (1970). See also R. Budny, Phys. Lett. **39B**, 553 (1972); R. Budny and A. McDonald, Oxford University Report No. 37/73, 1973 (to be published).

<sup>11</sup>These corrections arise in deep inelastic scattering [cf. H. Georgi and H. D. Politzer, Phys. Rev. D **9**, 416 (1974)] but not in  $e^+e^-$  annihilation [cf. A. Zee, Phys. Rev. D **8**, 4038 (1973)].

<sup>12</sup>For a recent review of the experimental situation and the conventional theory (QED) see B. E. Lautrup, A. Peterman, and E. deRafael, Phys. Rep. **3C**, 193 (1972).

<sup>13</sup>We restrict the discussion to effects which yield somewhat reasonable bounds on the  $\epsilon$ 's. [Consideration of the ortho-para splitting in positronium, for ex-

ample, yields the (absurd) bound  $|\epsilon_{ee}^{VV} - 3\epsilon_{ee}^{AA}| < 1.5 \times 10^5$  and, therefore, does not warrant inclusion in the text.]

<sup>14</sup>G. Feinberg and M. Y. Chen, Phys. Rev. D **10**, 190 (1974).

<sup>15</sup>G. Feinberg, to be published.

<sup>16</sup>M. A. Bouchiat and C. C. Bouchiat, Phys. Lett. **48B**, 111 (1974).

<sup>17</sup>The charge-helicity combinations used here correspond to the situation in the National Accelerator Laboratory experiment. (We thank Professor K. W. Chen for a discussion on this point.) Similar formulas can be derived for other combinations.

<sup>18</sup>For example, R. Jackiw and S. Weinberg, Phys. Rev. D **5**, 2396 (1972).

<sup>19</sup>See also M. Suzuki, Nucl. Phys. **B70**, 154 (1974), and earlier references cited therein.

## $A_2$ - $A_1$ Interference Phase\*

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The  $A_2$ - $A_1$  phase difference observed in  $\pi^-p \rightarrow \pi^+\pi^-\pi^-p$  is compared with the  $A_2^-$  phase from a Regge fit to the  $d\sigma/dt$  data for  $A_2^-$  and the  $A_1^-$  phase given by a Reggeized Deck model. The agreement ( $\sim 30^\circ$ ) depends crucially on the contribution of the Reggeized pion propagator to the  $A_1$  phase and requires equal signs for the  $f$  and  $P$  residues in the  $A_2$  amplitude.

Phases of strong interaction amplitudes—although of obvious interest—are all too seldom measurable. Partial-wave analyses<sup>1,2</sup> of the reaction  $\pi^-p \rightarrow \pi^+\pi^-\pi^-p$  give not only the magnitudes of  $A_1$  (defined as the state  $1^+S-\rho\pi$ ) and  $A_2$  production amplitudes but also their relative phase. In this note we use the phase of the  $A_1^-$  amplitude from a Reggeized-Deck-model calculation, and the  $A_2^-$  phase from a Regge fit to  $d\sigma/dt$  for  $A_2$ , to predict the  $A_2^-$ - $A_1^-$  phase and compare the prediction to the data.

*Phase of the  $\pi^-p \rightarrow A_1^-p$  amplitude.*—A version of Berger's Reggeized Deck model<sup>3,4</sup> has been shown to agree well with the data for  $\pi^-p \rightarrow \pi^+\pi^-\pi^-p$  for  $M_{3\pi} \leq 1.5$  GeV, excepting the  $A_2$  partial wave (which is nearly absent in the model). In particular, the model predicts correctly the relative phases between different partial waves. It also predicts that the only important amplitude for  $A_1$  production is the nucleon  $s$ -channel helicity non-flip amplitude leading to the  $1^+$  state with  $J_z=0$  (in the  $t$  channel). The phase of this amplitude (extracted by partial-wave analysis<sup>5</sup> of the model amplitude) shows only minor dependence on  $M_{3\pi}$ ,

in accord with the data.

Typical values of the phase (at  $M_{3\pi}=M_{A_2}$ ) are shown in Table I. We note that the phase exceeds by  $\sim 60^\circ$  the value of  $90^\circ$  one would obtain for diffractive production of a stable particle or resonance. The extra phase comes from the signature factor of the Reggeized  $\pi$  propagator and is directly proportional to  $\alpha_{\pi'}$  ( $\alpha_{\pi'}=1$  GeV<sup>-2</sup> was used). This extra phase is required to explain the phase differences observed between different

TABLE I. Deck-model phase for  $A_1$  production at  $M_{3\pi}=M_{A_2}$ .

$t-t_{\min}$ (GeV <sup>2</sup> )	0	-0.1	-0.2	-0.3
$P_{1ab}$ (GeV)				
6	146°	158°	170°	179°
16	146°	156°	166°	175°
26	146°	156°	165°	173°
36	146°	155°	164°	172°