^{84}Kr + ^{82}Se have at least 5 discrete γ rays below 0.6 MeV which are known to carry off $10\hbar$. In the odd-mass cases ($3n$ and $5n$ reactions) even more angular momentum is carried off by the discrete lines. Thus the $\bar{N}_\gamma$ values are consistent with the rms l values estimated from Fig. 1 provided the continuum γ rays are predominantly of the stretched ($I \rightarrow I-2$) $E2$ type. It is not yet clear whether the angular distributions are consistent with this requirement. The situation is somewhat relieved since the neutrons may carry off a few units of angular momentum, and there may be a few continuum γ rays below 0.6 MeV.

We have found large variations in the number of γ rays emitted following (HI, $xn\gamma$) reactions, especially for different products of the same target-projectile system. These variations have been shown to be mainly an angular-momentum effect, and support a simple model which gives considerable insight into the effects of angular momentum in compound-nucleus de-excitation. The question as to how much information the continuum γ rays can give beyond these angular-momentum effects is an interesting one. Some of our preliminary results for average energies and angular distributions of the continuum γ rays suggest that different de-excitation modes in a given product nucleus can be distinguished experimentally.

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‡On leave from University of Munich, Germany.

§On partial leave from Stanford University, California.

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Average Multiplicity of Unresolved Photon Cascades in the De-excitation of Highly Excited Compound Nuclei*

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Heavy ion (HI, xN) reactions leading to ^{158}Dy, ^{160,162,164}Er, and ^{170,172}Hf were studied in order to determine the average multiplicity of the unresolved photons underlying discrete spectra of rotational band transitions observed in rare-earth even-even nuclei. The average multiplicity is found to be small (6), independent (to the accuracy of this experiment) of the ground-state rotational band level in which the cascade terminates.

The basic features of the processes by which compound nuclei of high excitations and large angular momenta divest themselves of excess energy and spin have been known for some time.[1] A clear and detailed treatment of this topic has been given in a series of papers by Grover and collaborators.[2] It is reported that compound nuclei with angular momenta up to $(40-50)\hbar$ have been formed by heavy ion (HI) reactions.[3] De-excitation proceeds through successive particle emission followed by γ-ray cascades. This process is shown schematically in Fig. 1 in which the nuclear levels of compound, daughter, and residual nuclei are plotted on an angular momen-

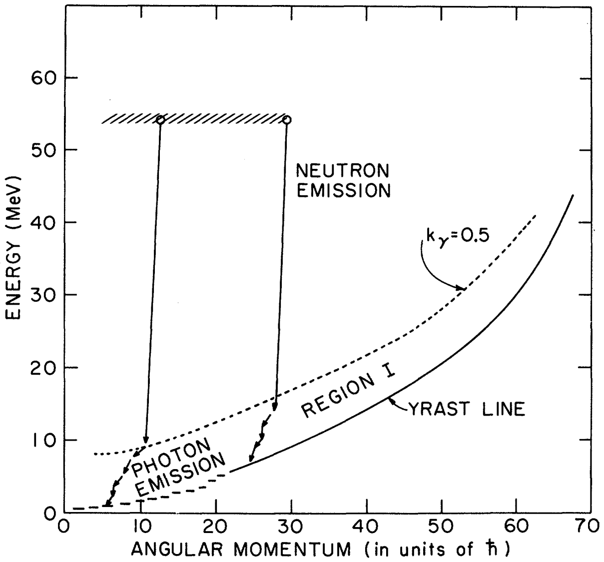

FIG. 1. Grover-type diagram of the de-excitation of highly excited nuclei. The example illustrates a (HI, xN) reaction in the rare-earth region. The compound nucleus (open circle) decays by emitting neutrons until it assumes a state close to the dotted line, about 1 neutron binding energy above the yrast line, where photon emission becomes as probable as neutron emission ($k_\gamma = 0.5$). The yrast line joins the lowest energy states with given J's. The decay proceeds in region I by photon emission until the yrast line is reached, which is then followed to the ground state. The individual levels in the initial portion of the yrast line represent the levels in the ground-state rotational band.

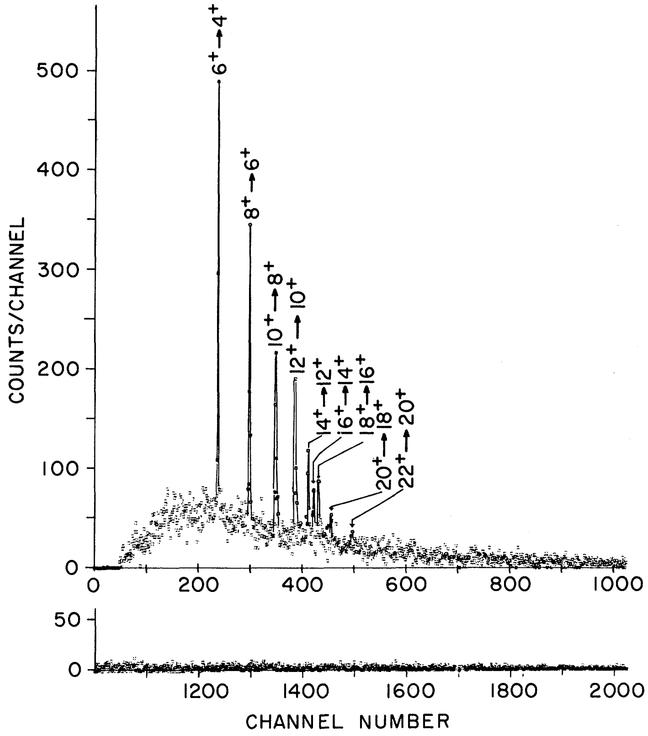

FIG. 2. Spectrum of γ rays in coincidence with the $4^+ \rightarrow 2^+$ transition of ^{158}Dy.

tum versus excitation energy diagram.[2] The heavy solid line represents for the residual nucleus the locus of states of lowest energy as a function of the angular momentum J. Grover has named this line the yrast line and the states represented by this line, the yrast levels. The shaded region represents possible initial states of the compound nucleus formed in a HI reaction. In the rare-earth region studied individual states (open circles) decay by neutron emission until levels of the residual nucleus at excitations of less than 1 neutron binding energy above the yrast line are formed. The dotted line appearing about 1 neutron binding energy above the yrast line in Fig. 1 indicates where the probability of photon emission is 0.5. At this point in the de-excitation process the nucleus has lost little angular momentum. The nucleus now de-excites by a variety of photon cascades to the yrast line. These cascades produce a continuum composed of a large number of unresolved γ rays. A discrete spectrum results from the final cascade along the yrast line to the ground state. The purpose of this investigation is to determine N_γ, the average number of γ gays in the "continuum" per disintegration for compound nuclei formed in the rare-earth region. This average multiplicity $N_\gamma = (\sum_n n\sigma_n)/\sigma_{\text{tot}}$, where n is the multiplicity of a cascade, σ_n the cross section associated with multiplicity n, and σ_{tot} the cross section for all cascades.

Analyses have been made on ^{158}Dy, ^{160,162,164}Er, and ^{170,172}Hf populated by (^{12}C, 4n) and (^{18}O, 4n) reactions. The ^{158}Dy results have been reported earlier.[4] Thick metallic rare-earth targets of 8 to 16 mg/cm^2 were used. γ-γ coincidences were observed with two Ge(Li) detectors placed at various distances and angles around the target. Singles data were stored in core while 4096 ×4096 channel coincidences were recorded in "event mode" on tape. Background-subtracted spectra of γ rays in coincidence with the transitions in the ground-state rotational band were obtained by analyzing these data.

The spectra consist of discrete peaks corresponding to the transitions in the rotational band and a "continuum" which extends beyond the last of the observed discrete lines (Fig. 2). By measuring the counts in each discrete peak, correcting for total as against peak counts, summing the discrete counts, and subtracting from the total number of counts in the spectrum, we obtained the number of counts in the continuum. Since more than 90% of all decays in these nuclei proceed through the $6^+ \to 4^+$ and the $4^+ \to 2^+$ transitions, the number of counts in these transitions were taken as a measure of σ_{tot} in all cases and N_γ was taken to be the number of counts in the continuum divided by the number of counts in one or the other of these transitions. The ^{158}Dy experiment was repeated with a 6.3-cm-high lead cone in front of the detector in order, through selective absorption, to suppress the detection of photons originating at the target. In this way 10% of the continuum was found to be due to neutrons and scattering effects. Addition effects were accounted for where necessary. No correction for total-efficiency variation with energy was applied, since the energy distribution of the continuum is unknown. In the energy range of interest, this variation and its associated error do not exceed 30%.

All results are summarized in Table I where the numbers are the N_γ's for cascades (region I in Fig. 1) feeding into the ground-state rotational band of the indicated nucleus at all levels above the transition used as a gate. Therefore, the N_γ for the $2^+ \to 0^+$ transition is averaged over all type-I cascades while the N_γ for the $10^+ \to 8^+$ transition is averaged over the cascades which feed into rotational levels at the 10^+ or higher levels only. Nevertheless, it is possible to infer some of the general, average properties of N_γ. Only the ^{158}Dy data were corrected for neutron events and the other effects. It was assumed that the correction in the other nuclei is the same (10%). The average of all N_γ (with the exception of one) is 6.0. Statistical errors have not been given because it was felt that inherent systematic errors could play a larger role than statistics, and a better indication of the precision of the measurements is obtained from a comparison of the three independent ^{160}Er runs. It is therefore reasonable to take 1.5 as the error and to state that N_γ for all these nuclei is 6 ± 1.5. The spread in the numbers of any particular row is usually less than this. It is interesting to compare this number with other determinations of N_γ appearing in the literature. Karnaukhov and Oganesyan[5] irradiated Sn with 78-MeV ^{12}C ions and estimated the average number of γ rays in the total (including yrast transitions) cascade to be "not less than 10." In the ^{158}Dy run reported in the present work, the average number of discrete γ rays per cascade is about 6.5 and thus the average number of photons per cascade is about 12.5. Mollenauer[6] reported about 14 photons per reaction in an experiment in which ^{165}Ho was bombarded by 103.5–115-MeV ^{12}C ions. Williamson *et al.*[7] report a value for N_γ of 5.4 in $(\alpha, 3n)$ and $(\alpha, 4n)$ reactions on ^{167}Er at 40.5 MeV. Finally, Degnan *et al.*[8] reported a multiplicity of about 4 or less in the complete γ cascade for compound-nucleus formation in light nuclei with (d,p) and (p,p') reactions at 12 and 17 MeV. The agreement between this measurement and the HI reactions reported in

TABLE I. Average multiplicity of photons in the continuum.

Nucleus	Proj.	Beam Energy (MeV)	Transition Used as Gate								
			$2^+\to0^+$	$4^+\to2^+$	$6^+\to4^+$	$8^+\to6^+$	$10^+\to8^+$	$12^+\to10^+$	$14^+\to12^+$	$16^+\to14^+$	$18^+\to16$
^{158}Dy^a	^{12}C	65		5.5	4.8	5.7	5.9	5.3			
^{160}Er	^{12}C	70		8.3	8.4	7.9	7.1	7.3			
"	^{12}C	71.3		7.2	6.4	6.9	6.3	8.0	5.6		7.8
"	^{12}C	73.5		8.0	6.9	7.0	6.5	6.3			
" (average)^b				7.8	7.2	7.3	6.6	7.2			
^{162}Er	^{12}C	66			6.3	4.4	3.4	4.6	4.0		
^{164}Er	^{18}O	75	8.9	9.5	8.4	9.3	7.0	6.2			
^{170}Hf	^{12}C	70			8.9	7.6	7.6	7.3	7.6	8.2	
^{172}Hf	^{12}C	70		11.3	6.2	6.4	6.5	5.2	5.4	7.9	

[a] These data for ^{158}Dy have been corrected for neutron and addition effects which contribute to the background continuum.

[b] Average of the three ^{160}Er runs.

the above referenced papers is interesting since coincidence measurements were used in this work, and singles in the others. This suggests that most of the reactions studied in the earlier works were remarkably clean so that the results were not affected by the presence of accompanying reactions.

It is interesting that N_γ does not change with position in the rotational band of the transition being fed by the cascade. Barring the suggestion of a trend to higher values for the few lowest gates in Table I, N_γ remains remarkably constant for gates varying from the $6^+ \rightarrow 4^+$ transition to (in one case) the $18^+ \rightarrow 16^+$ transition. Thus, when the three ^{160}Er runs are averaged, the N_γ obtained is close to 7 (uncorrected) from the $6^+ \rightarrow 4^+$ to the $12^+ \rightarrow 10^+$ transitions inclusive.

If it is accepted that the number of γ rays in the "continuum" cascade is independent of the spin of the rotational state at which the cascade terminates, one may offer a simple picture of the de-excitation process with the help of Fig. 1. In the Grover picture the energy available to cascades in region I is the same regardless of J, since γ-ray emission becomes prominent at an excitation of 1 binding energy of the neutron above the yrast level. If it is now assumed that at a given ΔE above the yrast level the distribution of states available to the intermediate cascade and the population distribution[2] of these states in energy after neutron emission are roughly independent of J, one would account for the constancy of N_γ as a function of entry point in the ground-state rotational band. The effect of a thick target is to introduce a variation in the average excitation energy with angular momentum not unlike that associated with the yrast line in the region of the ground-state band. Thus the available energy for neutron and γ emission to the yrast line is roughly independent of angular momentum.

This assumption of the independence of the distribution of states above the yrast level as a function of J is not totally unfounded. According to Ginocchio[9] the energy dependence of shell-model states with a definite angular momentum is close to Gaussian for nuclei with a large number of particles in a large but finite set of shell-model orbits. His approximate expressions for the energy centroid and width of the distribution of shell-model states agree with exact shell-model calculations for ^{20}Ne and ^{62}Ni, indicating that they are already large enough systems to satisfy the required initial conditions. It follows from his analysis of the distribution of intrinsic states as a function of both energy and angular momentum that

$$\rho_{J'+\Delta J}(\Delta E + E_0(J')) \approx \rho_{J+\Delta J}(\Delta E + E_0(J)),$$

where $E_0(J)$ is the yrast energy for angular momentum J and $\rho_{J+\Delta J}(\Delta E + E_0(J))$ is the level density for angular momentum $J+\Delta J$ at an excitation of ΔE above the yrast level. For example, 1 MeV above the yrast level for $J=10$, the density of $J=11$ states is the same as the density of the $J=19$ states 1 MeV above the $J=18$ yrast level. Thus, it is plausible that the intermediate cascade may find distributions of states and relative properties [e.g., $B(E1)$, $B(E2)$] sufficiently similar so that N_γ may to first approximation be independent of the path of the cascade in J space.

It should be noted that the value of 6 for N_γ requires a relatively high average energy for the γ rays in this intermediate cascade ($\gtrsim 1.0$ MeV) in Grover's scheme. This is consistent with estimates of the average energy of the photons in the continuum of 1 MeV or greater indicated by our data. In any case, these values for N_γ and the average energy of the photons in the cascade are in agreement with Grover's calculations, indicating that photon emission becomes important at an excitation of 1 neutron binding energy above the yrast level.

Studies of the de-excitation process following $(\alpha, 3n)$[8] and ^{40}Ar-induced[10] reactions have led to a more complex description of this process. In these investigations it was thought that rotational transitions of softer γ rays were needed to help carry angular momentum away from the nucleus during the intermediate cascade, and a detailed scheme involving rotational bands has been made.[10] These studies, however, were conducted under different conditions from the present one. The experimental results of the present paper ($N_\gamma = 6$ independent of the rotational level terminating the type-I cascade) are not incompatible with the conclusions of these other investigations.

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Nucleus-Nucleus Optical Potential Using a Density-Dependent Two-Body Interaction

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The nucleus-nucleus optical potential is calculated by using a density-dependent two-body effective interaction where the local density takes into account the contribution of both the projectile and the target densities. The results indicate that it is important to use such an interaction when the densities of the colliding nuclei overlap significantly.

The nucleus-nucleus optical potential has been derived recently by several authors by folding in a phenomenological nucleon-nucleus optical potential with the projectile density.[1] In this procedure each nucleon of the projectile is treated essentially as free and so the perturbation felt by each nucleon in the projectile as a result of the field of all other nucleons is ignored. The most important consequence of such a scheme is the implicit neglect of the saturation properties of the two-body interaction, which prevents the nuclear density from increasing beyond a certain magnitude. The optical potential calculated in this way is therefore overestimated when the centers of mass of the colliding nuclei are at short distances from each other and the densities of the two nuclei overlap significantly. It is, however, expected to be of the right order of magnitude at or near the touching radius of the interacting nuclei when the overlap is not so significant.

Galin *et al.*,[2] on the other hand, have computed the energy of the nucleus-nucleus interaction by using the phenomenological energy-density function of Brueckner *et al.*[3] The structure of each nucleus in this calculation is assumed to be conserved entirely during the contact and the nuclear-matter densities are assumed to overlap in a reversible process without any rearrangements. Although the method of Galin *et al.*[2] is an improvement over other works,[1] the Brueckner "sudden approximation" described above cannot be expected to yield realistic results after the nuclei have fused. On the other hand, if one of the colliding nuclei is light, for example, a ^{4}He or ^{3}He, the concept of a phenomenological energy-density functional cannot be used with any confidence.

In this Letter I propose to use a density-dependent two-body effective interaction to construct the nucleus-nucleus optical potential—the density-dependent part, as is well known,[4] takes into account approximately the saturation properties of the two-body interaction. It will be shown that the nucleus-nucleus potential, in this way, can be calculated for any kind of projectile, light or heavy, without any loss of generality. First, the nucleon-nucleus optical potential is calculated by folding in this interaction with the target density[5]; the nucleon-nucleus potential is then folded in with the projectile density. The importance of using an effective two-body interaction such as a density-dependent interaction has been amply demonstrated recently[5] for the nucleon-nucleus optical potential. In a nucleus-nucleus interaction, however, each nucleon in the projectile is embedded in a nuclear medium, thus departing from a free-nucleon case. *The local density in the two-body interaction therefore should be computed taking into account the contribution of the density from both the target and*