Angular-Momentum Effects on Continuum Gamma Rays Following Heavy-Ion Reactions*

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The average number of continuum γ rays above 0.6 MeV has been measured for each of several final products in the reactions $p+{}^{165}\mathrm{Ho}$, ${}^{16}\mathrm{O}+{}^{150}\mathrm{Sm}$, and ${}^{84}\mathrm{Kr}+\mathrm{Se}$. This number ranges from 2 to 20 and is shown to correlate mainly with the average angular momentum leading to each product.

The γ-ray spectra following heavy-ion (HI, $xn\gamma$) reactions have two main features: discrete lines from heavily populated low-lying levels of the final-product nuclei, and a continuum which presumably represents all the higher transitions, none of which has sufficient population to be resolved with present techniques. Most recent work has focused on the discrete lines, producing systematic information on the low-lying high-spin ($\lesssim 20\hbar$) states of nuclei. A few attempts have been made to study the continuum region. Mollenauer[1] studied ($\alpha, xn\gamma$) and (^{12}C, $xn\gamma$) reactions without specifying the outgoing reaction channel. He measured the average number of continuum γ rays, $\bar{N}_\gamma$, their average energy, and their angular distribution. Sunyar and co-workers[2] have reported some preliminary results on $\bar{N}_\gamma$ for the reaction ${}^{150}\mathrm{Nd}({}^{13}\mathrm{C}, 5n\gamma){}^{158}\mathrm{Dy}$. The present work, using p, ^{16}O, and ^{84}Kr projectiles, shows that large variations occur in the number of γ rays in the continuum region depending mainly on the outgoing channel, and that this variation may be understood in terms of a simple model.

In our work it was essential to specify the reaction channel because many channels occur simultaneously in the reactions and a composite continuum of γ rays would be difficult to interpret. Thus we measured the continuum in coincidence with known discrete transitions that specified particular channels, as was done by Sunyar and co-workers.[2] For the discrete lines we used a 40-cm^3 Ge(Li) detector at an angle of 90° to the incident beam direction and about 5 cm from the target. The continuum γ rays were detected in a 7.5 cm × 7.5 cm NaI crystal, 15 cm from the target, and at angles of 0°, 45°, and 90°. An absorber, consisting of 0.32-cm Pb and 0.32-cm Cu, was placed in front of the NaI detector, resulting in an overall detection efficiency which is very nearly constant for any γ ray above 0.5 MeV. Since there are known discrete lines in the spectra of interest up to about 0.6 MeV, and since there are not likely to be many continuum γ rays below this energy,[1,2] we took 0.6 MeV as the lower limit for measurements of the continuum. The detection probability for 1.1-, 1.4-, and 2.8-MeV γ rays in the coincidence arrangement was found to be the same within about 20% and equal to 0.43%. Thus, even for multiplicities as high as 20, the probability of two γ rays being detected simultaneously was less than 10% and represented a small correction. The arrangement was further tested with a ^{252}Cf fission source, by using fission fragments as the coincidence triggers, and agreement with previous results[3] for the number of γ rays per fission was obtained. In these calibrations,[3] as well as during the in-beam experiments, a background spectrum was taken with a 5-cm-thick Pb cone between the NaI detector and the source (target). We have shown that a simple subtraction of the spectrum with the Pb cone from that with no cone is very unlikely to cause an error in $\bar{N}_\gamma$ exceeding 10%.

Beams of 347-MeV ^{84}Kr provided by the Berkeley SuperHILAC were used to bombard targets of ^{82}Se (1.3 mg/cm^2) enriched to 97%. This produced the compound nucleus ^{166}Yb, and various discrete lines in the final nuclei ^{163}Yb ($3n$), ^{162}Yb ($4n$), and ^{161}Yb ($5n$) could be used as coincidence gates. A target of 99% enriched ^{150}Sm (1.4 mg/cm^2) was also bombarded at the Berkeley 88-in. cyclotron with 88-MeV ^{16}O to produce the same compound nucleus and products. Both targets were evaporated onto Pb backings about 25 μm thick, which stopped the beam and recoiling nuclei with no appreciable background. We also bombarded a ^{165}Ho target (220 mg/cm^2) with 25-MeV protons at the cyclotron in order to produce the compound nucleus ^{166}Er, and product nuclei ^{163}Er ($3n$) and ^{164}Er ($2n$). The average numbers of γ rays, $\bar{N}_\gamma$, associated with each discrete line are given in Table I. The uncertainty attached to each $\bar{N}_\gamma$ value is estimated to be ±20%,

TABLE I. Average number of continuum γ rays above 0.6 MeV.

E (keV)	$I_i \rightarrow I_f$	$\bar{N}_\gamma$ ^{84}Kr + ^{82}Se	$\bar{N}_\gamma$ ^{16}O + ^{150}Sm
	^{162}Yb($4n$)		
166	$2 \rightarrow 0$	11	8
320	$4 \rightarrow 2$	14	10
437	$6 \rightarrow 4$	12	9
521	$8 \rightarrow 6$	12	9
579	$10 \rightarrow 8$		9
	^{163}Yb($3n$)		
202	$17/2^+ \rightarrow 13/2^+$	20	14
345	$21/2^+ \rightarrow 17/2^+$	19	9
	^{161}Yb($5n$)		
232	$17/2^+ \rightarrow 13/2^+$	10	5

E (keV)	$I_i \rightarrow I_f$	p + ^{165}Ho
	^{163}Er($3n$)	
127	$13/2^+ \rightarrow 9/2^+$	1.2
165	$15/2^+ \rightarrow 13/2^+$	2.2
171	$13/2^- \rightarrow 11/2^-$	2.3
190	$9/2^- \rightarrow 5/7^-$	1.3
213	$15/2^+ \rightarrow 11/2^+$	1.3
218	$17/2^+ \rightarrow 13/2^+$	1.7
236	$11/2^- \rightarrow 7/2^-$	1.6
	^{164}Er($2n$)	
208	$4 \rightarrow 2$	3.7
314	$6 \rightarrow 4$	3.3
410	$8 \rightarrow 6$	3.2

except for the 232-keV line in the ^{84}Kr + ^{82}Se data. This line appeared broad in both the singles and the coincident spectra, and thus the uncertainty toward lower values has been doubled to 40%. The Se target thickness causes a ± 8-MeV spread (c.m.) about the average value of the excitation energy of the compound system; the $\bar{N}_\gamma$ values listed in Table I for the Se target are thus average values over this range of excitation energy.

The variations of $\bar{N}_\gamma$ in Table I are more than a facter of 10 overall, and nearly a factor of 3 in various ^{16}O + ^{150}Sm reactions alone. Such large variations are not likely to be caused simply by the difference in excitation energy resulting from different numbers of neutrons evaporated. This is shown by the p + ^{165}Ho reaction, where a difference of one neutron causes a change of less than two γ rays. It seems more likely that these large variations in $\bar{N}_\gamma$ result from angular-momentum effects. This can be tested by application of a simple semiclassical model.[4] We picked a compound nucleus as neutron-rich as possible in order to be able to neglect charged-particle competition, and indeed, the (HI, $\alpha x n \gamma$) products that could be detected in our spectra had individual cross sections less than 10 mb. The number of neutrons emitted from a compound system depends on the energy available, which is roughly the difference between the excitation energy and the minimum energy E_l required to carry the angular momentum $l\hbar$ of the system. Since an evaporated neutron cannot change l by more than 1 or 2 units, the energy E_l must be left in the residual nucleus. If for example, l is such that E_l is equal to the sum E_n of the neutron binding and kinetic energies, then it is apparent that one fewer neutron can be emitted than would be the case for $l = 0$ ($E_0 = 0$). We can estimate that $E_l \approx \hbar^2 l^2 / 2\mathcal{I}$, where $\mathcal{I}$ is a moment of inertia near the rigid-body value. Since E_n is about 10 MeV for the nuclei considered here, and $\hbar^2/2\mathcal{I}$ is approximately 9 keV, this gives $\Delta(l^2)/\Delta x = 1100$ (x is the number of neutrons emitted). Thus if a large amount of angular momentum is brought into a compound nucleus in this region, the total cross section will be divided into "bins," having widths $\Delta(l^2) = 1100$, each of which corresponds to a particular number of neutrons being evaporated. For example, if a bin starts at $l = 0$, it can hold all angular momenta up to 33, and the bin starting at $l = 33$ corresponds to the evaporation of one less neutron. It is important to note that at a given bombarding energy the maximum cross section for each bin is approximately constant. Using a relation given previously[5]—$l_{\text{upper}}^2 = 1.5\sigma\mu E_{\text{c.m.}}$, where σ is in barns, μ is the reduced mass in mass units, and $E_{\text{c.m.}}$ is the center-of-mass energy in MeV—we find

$$\frac{\Delta\sigma}{\Delta x} = \frac{[\Delta(l^2)/\Delta x]}{1.5\mu E_{\text{c.m.}}} \approx \frac{740}{\mu E_{\text{c.m.}}}. \qquad (1)$$

For our ^{16}O + ^{150}Sm case, with $\mu = 14.5$ and $\bar{E}_{\text{c.m.}} = 78$, Eq. (1) gives $\Delta\sigma = 650$ mb, compared with our measured $\Delta\sigma$ for the $4n$ reaction of 540 ± 100 mb. For the ^{84}Kr + ^{82}Se case, Eq. (1) gives $\Delta\sigma = 110$ mb, compared with our measured $4n$ cross section of 105 ± 25 mb. For protons, the calculated $\Delta\sigma$ is around 30 b, indicating a bin size much larger than the reaction cross section. Thus Eq. (1) seems reasonably well borne out in our cases.

Some of the expectations derived from this simple model for our three projectile-target systems are illustrated in Fig. 1. First the yrast, or lowest-energy, states for each l are plotted by

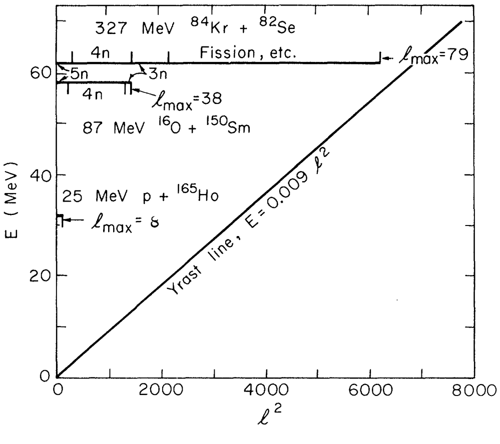

FIG. 1. Plot of excitation energy versus angular momentum squared for a nucleus having $A \approx 160$. The yrast line is shown, together with the excitation energies and angular-momentum ranges for the three reactions studied. The projectile energies shown have been corrected for target thickness.

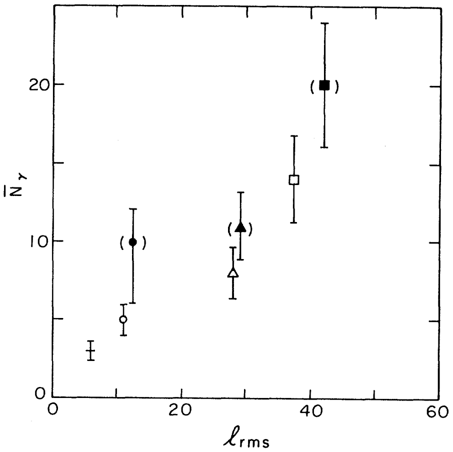

FIG. 2. Average number of continuum γ rays ($E_\gamma > 0.6$ MeV) coincident with the lowest observed discrete transition ($2 \rightarrow 0$ or $\frac{17}{2} \rightarrow \frac{13}{2}$) versus the input l_{rms} values estimated from Fig. 1. The cross is for the reaction $^{165}\mathrm{Ho}(p,xn\gamma)^{163,164}\mathrm{Er}$ and the circles, triangles, and squares are for the $5n$, $4n$, and $3n$ reactions from $^{16}\mathrm{O}+{}^{150}\mathrm{Sm}$ (open) and $^{84}\mathrm{K}+{}^{82}\mathrm{Se}$ (solid). The parentheses on the $^{84}\mathrm{Kr}+{}^{82}\mathrm{Se}$ data indicate that considerable uncertainty in the l_{rms} values is introduced by the target thickness in this case.

using the $\hbar^2/2\mathscr{I}$ value given above. For the $^{16}\mathrm{O}+{}^{152}\mathrm{Sm}$ case we take the $4n$ cross section of 540 mb to represent a "full" bin. The measured $5n$ cross section then represents 0.22 of a bin or a $\Delta(l^2)$ of 240, and the $3n$ cross section represents 0.09 of a bin or $\Delta(l^2) = 100$. Thus the $5n$-$4n$ boundary can be located at $l^2 = 240$, the $4n$-$3n$ boundary at $l^2 = 1340$, and the upper limit of the $3n$ reaction at $l^2 = 1440$. This indicates $l_{\max}$ for this reaction to be 38, in accord with the estimate

$$l_{\max} = 0.219R[\mu(E_{\mathrm{c.m.}} - E_{\mathrm{CB}})]^{1/2}, \qquad (2)$$

where E_{CB} is the Coulomb barrier energy in MeV and R is the sum of the nuclear radii in femtometers. For the $^{84}\mathrm{Kr}+{}^{82}\mathrm{Se}$ case, we again take our $4n$ cross section as a full bin, and find 0.27 and 0.65 bins for the $5n$ and $3n$ reactions, respectively. This makes the $5n$-$4n$ boundary at $l^2 = 304$, the $4n$-$3n$ boundary at $l^2 = 1410$, and the upper limit of the $3n$ reaction at $l^2 = 2140$. This indicates an $l_{\max}$ of 46, whereas Eq. (2) gives $l_{\max} = 79$. At first this might seem surprising since the $3n$ is not a full bin according to our measurements and we find the $2n$ cross section to be only ~5 mb. However, it is well known that above a certain angular momentum the compound nucleus fissions[6] or does not form,[7] and the above value of ~$50\hbar$ represents an experimental estimate of that angular momentum. For the $p+{}^{165}\mathrm{Ho}$ reaction, Eq. (2) gives 8, so that essentially no angular-momentum effects should occur.

For each full or partial bin in Fig. 1, an rms value of l can be estimated. These are 11, 28, and 37 for the $5n$, $4n$, and $3n$ reactions in the $^{16}\mathrm{O}+{}^{150}\mathrm{Sm}$ case; 12, 29, and 42 for the $^{84}\mathrm{Kr}+{}^{82}\mathrm{Se}$ case; and 6 for the $p+{}^{165}\mathrm{Ho}$ case. To test whether the $\bar{N}_\gamma$ values from Table I correlate with these l_{rms} values, we have plotted these quantities against each other in Fig. 2. For the $p+{}^{165}\mathrm{Ho}$ case, we took an average (weighted by the cross sections) for the $2n$ and $3n$ reactions of $\bar{N}_\gamma = 3$. It is quite apparent that a correlation exists in Fig. 2, although the relationship is not so good as to exclude completely the possibility of some other effects. For example there is some suggestion that the $^{84}\mathrm{Kr}+{}^{82}\mathrm{Se}$ reaction produces slightly more γ rays than does the $^{16}\mathrm{O}+{}^{150}\mathrm{Sm}$ reaction. Nevertheless, we conclude that most of the variation of $\bar{N}_\gamma$ is due to variation of the input angular momentum. The correlation in Fig. 2 also supports the ideas embodied in Fig. 1, and indicates that the lines separating the different bins cannot be too fuzzy.

It is also interesting to consider whether there are enough γ rays to carry off the rms l values obtained from Fig. 1. Figure 2 shows that an input angular momentum of $30\hbar$ gives about ten γ rays. Since a multipolarity higher than $E2$ is not likely, this accounts for a maximum of $20\hbar$. However, the $4n$ reactions from both $^{16}\mathrm{O}+{}^{150}\mathrm{Sm}$ and

^{84}Kr + ^{82}Se have at least 5 discrete γ rays below 0.6 MeV which are known to carry off 10$\hbar$. In the odd-mass cases (3n and 5n reactions) even more angular momentum is carried off by the discrete lines. Thus the $\bar{N}_\gamma$ values are consistent with the rms l values estimated from Fig. 1 provided the continuum γ rays are predominantly of the stretched ($I \rightarrow I-2$) $E2$ type. It is not yet clear whether the angular distributions are consistent with this requirement. The situation is somewhat relieved since the neutrons may carry off a few units of angular momentum, and there may be a few continuum γ rays below 0.6 MeV.

We have found large variations in the number of γ rays emitted following (HI, $xn\gamma$) reactions, especially for different products of the same target-projectile system. These variations have been shown to be mainly an angular-momentum effect, and support a simple model which gives considerable insight into the effects of angular momentum in compound-nucleus de-excitation. The question as to how much information the continuum γ rays can give beyond these angular-momentum effects is an interesting one. Some of our preliminary results for average energies and angular distributions of the continuum γ rays suggest that different de-excitation modes in a given product nucleus can be distinguished experimentally.

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[1] J. F. Mollenauer, Phys. Rev. 127, 867 (1962).

[2] E. der Mateosian, S. Cochavi, O. C. Kistner, A. W. Sunyar, and P. Thieberger, Bull. Amer. Phys. Soc. 17, 530 (1972); A. Sunyar, in *Proceedings of the Heavy-Ion Summer Study*, edited by S. T. Thornton, CONF-720669 (Oak Ridge National Laboratory, Oak Ridge, Tenn., 1972).

[3] A. B. Smith, P. R. Fields, and A. M. Friedman, Phys. Rev. 104, 699 (1956).

[4] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (Wiley, New York, 1952).

[5] D. Ward, F. S. Stephens, and J. O. Newton, Phys. Rev. Lett. 19, 1247 (1967).

[6] S. Cohen, F. Plasil, and W. J. Swiatecki, in *Proceedings of the Third Conference on Reactions between Complex Nuclei*, edited by A. Ghiorso, R. M. Diamond, and H. E. Conzett (University of California Press, Berkeley, Calif., 1963), p. 325.

[7] J. Wilczynski, Nucl. Phys. A216, 386 (1973).

Average Multiplicity of Unresolved Photon Cascades in the De-excitation of Highly Excited Compound Nuclei*

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Heavy ion (HI, xN) reactions leading to ^{158}Dy, ^{160,162,164}Er, and ^{170,172}Hf were studied in order to determine the average multiplicity of the unresolved photons underlying discrete spectra of rotational band transitions observed in rare-earth even-even nuclei. The average multiplicity is found to be small (6), independent (to the accuracy of this experiment) of the ground-state rotational band level in which the cascade terminates.

The basic features of the processes by which compound nuclei of high excitations and large angular momenta divest themselves of excess energy and spin have been known for some time.[1] A clear and detailed treatment of this topic has been given in a series of papers by Grover and collaborators.[2] It is reported that compound nuclei with angular momenta up to (40–50)$\hbar$ have been formed by heavy ion (HI) reactions.[3] De-excitation proceeds through successive particle emission followed by γ-ray cascades. This process is shown schematically in Fig. 1 in which the nuclear levels of compound, daughter, and residual nuclei are plotted on an angular momen-