actual state of μ^+ in liquid helium.

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Injection and Control of Intense Relativistic Electron Beams in a Torus

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Efficient injection (> 80%) of an intense relativistic electron beam into a torus has been achieved using curvature drift injection. Drift was stabilized with a supplemental dipole magnetic field but beam loss in the radial direction due to the defocusing effect of the dipole field limited containment time. We propose a system for beam trapping and confinement which employs drift injection and beam energy losses in a toroidal-plus-betatron-type field.

Intense toroidal relativistic electron beams have been proposed as a means of heating toroidal reactors, as well as a means of producing the confining field for the plasma.[1,2] We recently proposed drift injection as a way of injecting externally generated intense beams into toroidal fields when the injection duration is longer than the time of one toroidal circuit (multiturn injection).[3] This Letter reports an experiment which demonstrates drift injection and the control of the circulating beam using a supplemental magnetic field. We also describe the conceptual extension of the drift-injection technique to give stable toroidal containment.

Beam injection using the curvature drift demonstrated previously[3,4] works by drifting the beam along the direction of the major axis, away from the injector insert, during the first toroidal circuit. Drift velocity is adjusted so as to clear the injector by judicious choice of the toroidal field strength B_φ and/or beam relativistic factor γ. Drift continues until the beam reaches a chamber surface unless additional forces are applied to stabilize the motion at a suitable distance from the injector. For stabilization we have used a supplemental magnetic field because this gives greater control of beam trajectory than does conducting-wall stabilization. Since the curvature drift is actually an $\vec{F}_c \times \vec{B}_\varphi$ drift, with $\vec{F}_c$ the centrifugal force, we need only apply a counteracting radial force to stop the drift. This force must be small near the injector and increase along the direction of drift so that beam drift will be little reduced at first, and be stopped only after the displacement needed to miss the injector is attained. By applying an external field with predominant component B_z parallel to the major axis, beam electrons will experience a radial force $F_r = (e/c)V_\varphi B_z$, where V_φ is the toroidal velocity of the beam electrons, e is the electron charge, and c is the speed of light. The net drift velocity is then

$$V_z = -(c/eB_\varphi)(F_c - F_r). \tag{1}$$

If B_z, and therefore F_r, increases with distance from the injector, V_z will decrease as the beam drifts until $F_c = F_r$, halting the drift. The critical B_z field, B_{zc}, for which the drift velocity is zero is $B_{zc} = \gamma m_0 c^2/eR_b$, where R_b is the major radius of the electron beam.

By extending the 180° sector used previously,[3,4] a cast-epoxy torus was constructed with major radius 20.5 cm and interior dimensions 10.5 cm (radial) by 16.5 cm (axial).

Figure 1 shows the experimental arrangement. Some of the toroidal field lines are diverted to the beam-generating diode through the injector and return to the torus through free space. The injector section was 47 cm long, and the injector field equalled the toroidal field at the major radius. The drift-control field $B_z(z)$ was generated by dipole field coils (Fig. 1) 1 cm above the floor plane of the torus. The beam generator was a Physics International Pulserad 312 with 1-in.-diam carbon cathode. Injection of two different beams was investigated: a 40-nsec pulse at either 10 kA, 1.75 MeV or 10 kA, 0.96 MeV (peak values). The system was filled with neutral nitrogen at 1000 Torr to provide rapid (few nanoseconds) charge neutralization by primary electron-atom collisions. Drifted distance was determined from the discoloration of a sheet of Cinemoid[5] No. 48 in the r-z plane as witness plate. This gave accuracy of ±0.3 cm in drifted-beam edge position. Cinemoid position varied from $\varphi = 180°$ to 540°, with $\varphi = 0°$ defined as the injector-torus junction. A Rogowski coil at $\varphi = 120°$ measured net toroidal current $I_n(t)$. On some pulses a Faraday cup at $\varphi = 0°$ or 180° gave beam current. Relativistic electrons striking walls were detected by a scintillator-photodiode combination which time resolved the bremsstrahlung. The central results of the experiment are these:

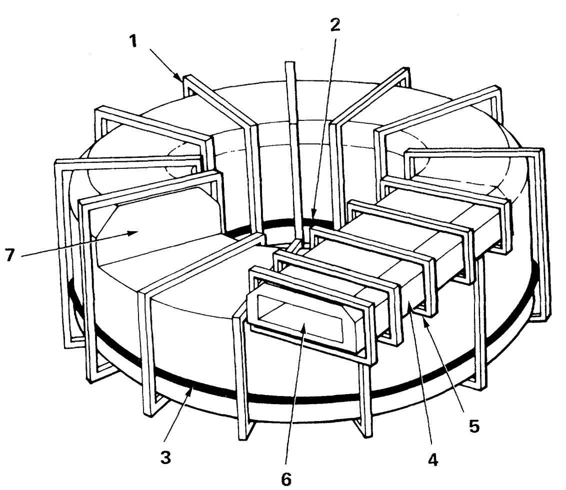

FIG. 1. Schematic of apparatus. (1) Toroidal field coil (12), (2) inner dipole-field coil, (3) outer dipole-field coil, (4) injector, (5) injector field coil, (6) beam injection aperture, and (7) abutment wall at $\varphi = 240°$, the injector "insert."

Beam injection.—Integration of the wave forms from the Faraday cup at 180° indicates charge injection and transport efficiency of 80%. This figure should be interpreted as a lower limit because of suspected reflection from the carbon Faraday-cup collector due to magnetic mirroring (line-tying effect in the carbon, diamagnetism in the beam channel). With Cinemoid across the lower region of the chamber, the drift of the 0.96-MeV beam in a 4-kG toroidal field was followed to $1\frac{1}{2}$ turns, when the beam reached the floor.

Drift control by dipole field.—Figure 2 shows the good agreement obtained between theory and measurement of drifted distances in the toroidal-plus-dipole field. Here the calculation includes, in addition to Eq. (1), a toroidal-self-field effect which causes an additional small radially outward force [see Eq. (2) below]. The data correspond to drift velocities V_z of $0.05c$ to $0.12c$. The agreement shown in Fig. 2 is significant because it lends confidence to further predictions of beam motion in special toroidal configurations, such as the one proposed below for toroidal containment.

Stopping of vertical beam drift by dipole field.—To study the effect of the critical field B_{zc} in the freely circulating case (no Cinemoid or Faraday cup), we analyze the x-ray wave form from the photodiode, which was collimated using lead to view only the 0–120° sector of the torus. The wave-form amplitude is a measure of instantaneous electron-loss flux in that sector. Using the 1.75-MeV beam with $B_\varphi = 2.35$ kG, the dipole field strength was varied up to 888 G at its spatial maximum B_{zm}. For this case, $B_{zc} = 400$ G.

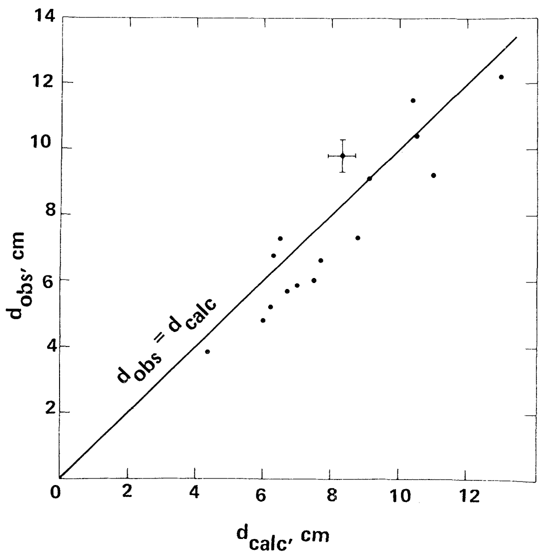

FIG. 2. Comparison of calculated and observed drifted distances in toroidal plus dipole field. Dipole-field spatial maximum, B_{zm}, ranges from -510 to $+800$ G in these measurements; toroidal field B_φ ranges from 1.5 to 4.0 kG. In all cases here the beam was intercepted before drifting to the position (if any) where $B_z = B_{zc}$.

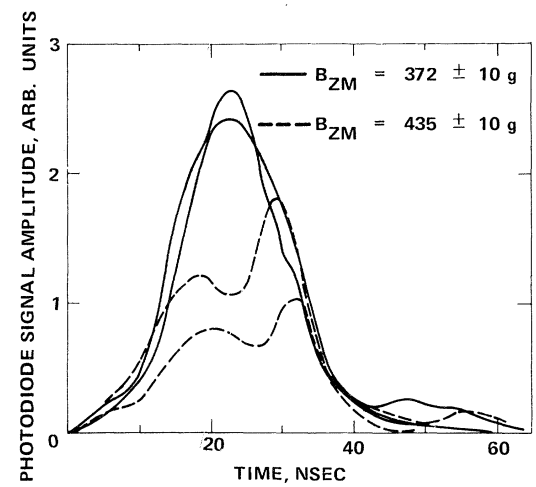

FIG. 3. Photodiode wave forms, showing change in photodiode-signal structure when dipole-field maximum B_{zm} exceeded critical value $B_{zc} = 400$ G. A third replication of each type is not shown.

Thus for $B_{zm} < 400$ G the beam should reach the floor, but when $B_{zm} > 400$ G the beam should be held off the floor and the drift stopped.

The results of these measurements are complicated by the fact that the dipole field's horizontally (i.e., radially) diverging components would in a few nanoseconds bring the beam to the sidewalls even if the vertical (i.e., axial) drift stopped when the beam reached the $B_z = B_{zc}$ position[6]; and by the fact that the detector did not discriminate between floor x rays and wall x rays. However, we observed that the shape and amplitude of the x-ray wave form varied with B_{zm}, and that this variation was very gradual except in a narrow range of B_{zm} around $B_{zm} = B_{zc}$: The transition from $B_{zm} < B_{zc}$ to $B_{zm} > B_{zc}$ gave an abrupt change in both the structure and amplitude of the x-ray wave form. This is shown in Fig. 3, and can be reasonably taken as the signature of a qualitative difference in the mode of electron loss when the critical field was applied. Note in Fig. 3 that the effect occurs in midpulse, when the injected electron γ is highest; i.e., the highest-γ electrons are the ones involved in the change. These are the only electrons in the pulse that would still reach the floor when B_{zm} was just less than B_{zc}, because $B_{zc} \propto \gamma$ (and because $B_{zc} = 400$ G was computed for peak γ). It is therefore reasonable to deduce that electron loss to the floor was dramatically reduced by making $B_{zm} > B_{zc}$, i.e., the axial drift was effectively stopped. This conclusion is clearly inferential and is supplementary to Fig. 2.

The above results show the viability of drift injection as a multiturn toroidal injection scheme. Beams were efficiently injected into the torus and a simple dipole field gave drift control but caused radial loss of the beam. The remaining obstacle to the creation of a toroidal beam of useful duration is a magnetic-containment geometry compatible with drift injection.

The method we propose uses toroidal-plus-betatron–type field geometry, with containment resulting from partial beam energy loss by self-magnetic-field generation and/or by heating of plasma. The z and r components of the betatron-type field are $B_z(r, z) = B_0(R_0/r)^n$ and $B_r(r, z) = -(n/r)zB_0(R_0/r)^n$, respectively, where B_0 and R_0 are constants. A single-particle analysis (no self-field) shows that for $B_z \ll B_\varphi$, the projection of the trajectory in the r-z plane is an ellipse (with Larmor gyration superimposed) if $0 < n < 1$.[7,8] We have analyzed the guiding-center equations including the beam's poloidal self-magnetic-field; in this case the ellipse's center (which always lies on the symmetry plane of the betatron field) is at radius

$$R_e = \frac{\gamma m_0 c^2 \beta_{\parallel}}{eB_0(1-n)}\left[1+\frac{I_n}{I_A}\left(\ln\frac{4\pi R_b}{a}-1.45\right)\right] - \frac{nR_0}{(1-n)}, \quad (2)$$

where I_A is the Alfvén current, a is the beam radius, and it is assumed that $B_0 \ll B_\varphi$, $\beta_\perp \ll B_\parallel$ (cold beam), and $(R_e - R_0)/R_0 \ll 1$.

If γ and I_n vary with time suitably, the consequent time variation of R_e can be used to trap the beam away from the injector, which lies on the initial elliptical trajectory. Two important effects can decrease γ and thereby modify the trajectory: (1) Part of the kinetic energy goes into self-magnetic-field energy (inductive trapping), and (2) plasma heating extracts beam energy. In the case where γ decreases due to the back emf induced by the buildup of toroidal net current I_n, differentiation of Eq. (2) (taking account of the coupled time dependence of I_n, $\beta_\parallel$, γ, and I_A) shows that R_e decreases as long as I_n increases:

$$\frac{dR_e}{dt} = -\frac{dI_n/dt}{B_0 c(1-n)}\left[\ln\frac{4\pi R_b}{a} - 3.45\right]. \quad (3)$$

We have developed a computer code that calculates the trajectory of a test-beam electron, including self-field effects (hoop force and induced electric fields), given $I_n(t)$. Figure 4 shows one such case, illustrating toroidal containment by the effect described in Eq. (3).

In conclusion, we have shown that drift injection and an external magnetic field can be used to circulate and control an intense relativistic electron beam in a torus. We propose and have numerically shown that trapping can be achieved using toroidal-plus-betatron-type fields and beam energy losses. Reactor application of this technique, with substantially larger currents and fields, is under study.

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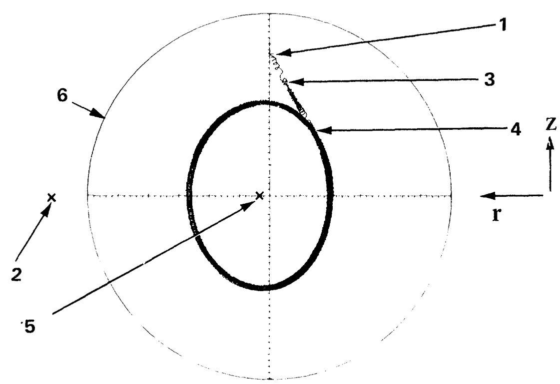

FIG. 4. Computer calculation of inductive trapping of test-beam electron, showing trajectory projected into r-z plane. Parameters in this case are $I_n(t) = I_0(1-e^{-t/\tau})$, $I_0 = 5$ kA, $\tau = 50$ nsec, $\gamma = 4.0$ (injected), torus major radius 60 cm, minor radius 15 cm, 4 cm beam diameter, $B_\varphi = 5$ kG, and $B_z = 90$ G at minor axis, $n = 0.5$. (1) Test electron injected at $t = 0$ at beam center, 12 cm from minor axis, $\beta_\perp/\beta_\parallel = 0.18$. (2) Initial instantaneous center of ellipse (i.e., corresponding to position 1). (3) Electron drift during period of decreasing γ. Center of instantaneous ellipse continuously moves inward radially. (Trajectory appears irregular because only a sampling of increments is plotted.) After 1 turn the electron has drifted down 1.8 cm, radially inward 0.7 cm, nominally clearing the injector structure. (4) $d\gamma/dt \simeq 0$ and $I_n \approx I_0$ at $t \gtrsim 100$ nsec (7.4 turns); thus the ellipse stops changing and is traced out by subsequent motion. The trajectory is now on a closed surface entirely contained and away from the injector. (5) Final center of elliptical motion. (6) Toroidal chamber.

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