Charge and Particle Conservation in Black-Hole Decay

B. Carter
Observatoire de Paris-Meudon, 92190 Meudon, France
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The qualitative consequences of Hawking's prediction of thermal emission from black holes are considered. Whereas mass and angular momentum loss proceeds on cosmologically long time scales, on the other hand electric discharge will take place extremely rapidly. Hypothetical mechanisms for baryon and lepton conservation are discussed.

In classical (i.e., unquantized) general relativity theory any black hole in equilibrium has an angular velocity Ω and an electromagnetic potential Φ, which are necessarily *constant* over the horizon.[1-3] When an incoming wave corresponding to a boson state with energy E, axial angular momentum $n\hbar$, and electric charge e is scattered by the black hole, the absorption probability Γ will be *negative*[4,5] (amplification), whenever $E - n\hbar\Omega - \mu$ is negative, where the *chemical potential* μ is given by

$$\mu = e\Phi. \tag{1}$$

This "superradiance" effect led Zel'dovich[6] to predict *spontaneous emission*, which will occur in the corresponding (time and axial angle reversed) outgoing states. Hawking[7] has recently predicted that spontaneous emission will occur much more generally at just the rate required to maintain thermal equilibrium with angular velocity Ω and chemical potential μ at a temperature Θ given by

$$\Theta = (\hbar/2\pi k)\kappa, \tag{2}$$

where κ, which is also necessarily[3,8] constant over the surface of the black hole, determines the asymptotic exponential relation

$$\tau_0 - \tau \propto \exp\{-\kappa t_{\rm ret}\} \tag{3}$$

between the proper time τ along the world line of a particle crossing the horizon at τ_0 and the ignorable outgoing null-time coordinate that would most conveniently be used by a distant observer. (This decay constant also represents the limiting "gravitational acceleration" of a co-rotating particle, meaning a certain "red-shifted" acceleration, *not* the locally measured acceleration which would tend to infinity on the horizon; the inverse κ^{-1} represents the characteristic time scale for exponential decay of normal modes, and hence also for formation of the black hole.)

By the "first law" of black hole mechanics,[3,8] the prediction (2) also implies that a black hole should be thought of as having an entropy S given by

$$S = (kc^3/\hbar G)\sigma_0 \tag{4}$$

(in striking confirmation of an order-of-magnitude prediction by Bekenstein[9]) where σ_0 is the "irreducible cross section" of the hole which is given in terms of the surface area A of the horizon by

$$\sigma_0 = \pi r_0^{\,2} = \tfrac{1}{4}A, \tag{5}$$

the irreducible Schwarzschild radius r_0 being related to the irreducible mass M_0, as introduced by Christodoulou,[10] by

$$r_0 = 2GM_0/c^2 \sim GM/c^2, \tag{6}$$

where M is the mass of the black hole.

By introducing the dimensionless form parameter

$$\epsilon = 1 - 2\Omega J/Mc^2 - Q\Phi/Mc^2, \tag{7}$$

where J and Q are, respectively, the angular momentum and electric charge, and by using the generalized Smarr formula,[11,8,3] one obtains the estimate

$$\Theta = \frac{c^2 M}{2S}\epsilon \sim \frac{c^3\hbar}{kG}\frac{\epsilon}{M}. \tag{8}$$

Using the much more obvious order-of-magnitude estimates

$$\Omega \sim J/Mr_0^{\,2} \sim c^4 J/G^2M^2, \tag{9}$$

$$\Phi \sim Q/r_0 \sim c^2Q/GM, \tag{10}$$

one is led (by considering the limits to accretion that would arise, respectively, from centrifugal and electrostatic repulsion) to guess that Ω and Φ should be limited by conditions of the form

$$(cJ/GM^2)^2 \sim (GM\Omega/c^3)^2 \lesssim 1, \tag{11}$$

$$Q^2/GM^2 \sim G\Phi^2/c^4 \sim G\mu^2/c^4e^2 \lesssim 1. \tag{12}$$

It is clear from (7) that the same order-of-magnitude limits are necessary for ϵ to remain positive, and that for fixed M the temperature Θ will tend to zero as these limits are approached.

[The "no-bifurcation theorems"[12,3,13] to the effect that continuous secular changes in an isolated black hole equilibrium state can only depend on the changes in M, J, and Q, in conjunction with the conclusion[14-18] that the only possible nonrotating limits are spherical, make it virtually certain that the only isolated stable equilibrium states are described by the Kerr solution[19] and its charged generalization,[20] for which one has the explicit precise formula $\epsilon = (1 - c^2J^2/G^2M^4 - Q^2/GM^2)^{1/2}$. However in order to obtain generalizable conclusions we wish to avoid relying on special properties of the Kerr solutions.]

Although the special Kerr separability properties of the scalar[21] and higher-spin[22] wave equations simplify Hawking's discussion,[7] they are not essential for the outcome which can be expressed as a prediction that there will be particle emission within the energy range dE in any outgoing state with well-defined axial quantum number n and (asymptotic) total quantum number j at a rate given by

$$-d\dot{N} = \frac{\Gamma dE}{2\pi\hbar}\left\{\exp\left(\frac{E - n\hbar\Omega - \mu}{k\Theta}\right) \pm 1\right\}^{-1}, \tag{13}$$

where the sign is positive for fermions and negative for bosons. By the usual principles of scattering theory we can expect for uncharged particles that Γ will be very small compared with unity unless

$$j\hbar \lesssim p r_0, \tag{14}$$

where p is defined in terms of the particle rest mass m by $E^2 = p^2c^2 + m^2c^4$. If we can ignore the electric charge on the black hole (which, as shown below, will always be justifiable in practice except perhaps for supermassive black holes), we are led to predict mass and angular momentum loss rates

$$-\dot{M} \sim \hbar c^4/G^2M^2, \quad -\dot{J} \sim \hbar\Omega, \tag{15}$$

provided that r_0 is not less than the proton Compton wavelength; i.e., provided that

$$M \gtrsim \hbar c/Gm_p, \tag{16}$$

so that only a limited number of kinds of particles [those with zero rest mass, and also when the limit (16) is approached, electrons with mass m_e] can contribute. The present theory is incapable of dealing reliably with the situation when M drops below the limit (16) since an unknown number of meson and baryon states come into operation, possibly[23] so many that the mass loss would become explosive, but above this limit (about 10^8 tons) the characteristic mass-decay time scale τ_M will be given by

$$\frac{1}{\tau_M} = -\frac{\dot{M}}{M} \sim -\frac{\dot{J}}{J} \sim \frac{\hbar c^4}{G^2M^3}, \tag{17}$$

so that it will always be at least as long as the present age of the universe which is given[24,25] by the characteristic stellar evolution lifetime

$$\tau \sim \hbar^2/cGm_p^3. \tag{18}$$

It is to be noted that for given M, the loss rate $-\dot{M}$ is roughly independent of Θ. If ϵ is very small the dominant superradiant contribution will ensure that it increases so that the black hole will tend asymptotically towards a state of conformal contraction with a fixed limiting form parameter $\epsilon_c > 0$.

Let us now consider what will happen when there is a nonzero electric charge quantum number $N_e = Q/e$. Wherever $E - mc^2 \lesssim \mu$ the centrifugal barrier-penetration condition (14) must be supplemented by the electrostatic barrier-tunnelling condition $Ep \gtrsim (eQ/\hbar c)m^2c^3$ so that, as Gibbons[26] has pointed out, superradiant discharge modes, i.e., those with $E \leq \mu$, can be effective only if

$$\frac{\hbar}{c^3}\mu \gtrsim m_e^2 r_0, \tag{19}$$

since there is no charged particle lighter than the electron. Hence, using (13) to estimate the discharge rate $-\dot{N}_e$, we find that there are three essentially distinct regimes: (a) If

$$|\mu| \lesssim (Gc/\hbar)m_e^2M, \quad 1 \lesssim GMm_e/\hbar c, \tag{20}$$

there will be essentially no discharge at all; (b) in the *thermal discharge* regime, with

$$GM\mu/\hbar c^3 \lesssim 1, \quad GMm_e/\hbar c \lesssim 1, \tag{21}$$

we shall have

$$-\dot{N}_e \sim \hbar\mu; \tag{22}$$

(c) and finally, in the *superradiant discharge regime*, i.e., when

$$(Gc/\hbar)m_e^2M \lesssim |\mu|, \quad 1 \lesssim GM\mu/\hbar c^3, \tag{23}$$

we shall have

$$-\dot{N}_e \sim (G^2/\hbar^3c^6)M^2\mu^3. \tag{24}$$

It follows (since $Gm_e^2 \sim 10^{-42} \ll e^2/\hbar c \sim 1/137 < 1$) that unless conditions (20) are satisfied, the characteristic discharge time $\tau_Q = -N_e/\dot{N}_e$ will always be extremely short in comparison with τ_M; in fact τ_Q will actually be shorter than the characteristic time $\kappa^{-1} \gtrsim r_0/c$ for formation of the black hole—which means that the charge could never have been there at all—unless M is within a factor $(\hbar c)^{1/2} m_p/em_e$ (which works out to be $\sim 10^2$, using $\hbar c/e^2 \sim 137$, $m_p/m_e \sim 1800$) of the mass limit (16). Hence for all practical purposes we may take it that

$$\frac{Q^2}{GM^2} \lesssim \frac{Gm_e^2}{e^2}\left(\frac{Gm_e M}{\hbar c}\right)^2, \tag{25}$$

which is small compared with the limit (12) unless M is greater than the typical stellar mass $M \sim (\hbar c/G)^{3/2} M_p^{-2}$ by a factor $(m_p/m_e)^2(e^2/\hbar c)^{1/2} \sim 10^5$. For black holes below this mass, the electromagnetic field will never have a perceptible effect on the space-time geometry. [For black holes above this mass, it is conceivable that the charge could be great enough to invalidate (15); it would be possible for the factor ϵ to tend to zero, bringing the decay asymptotically to a halt.]

In any case, whenever a black hole decays below the mass limit $M \sim \hbar c/Gm_e$, and hence before reaching the critical mass (16), it will neutralize itself *completely*.

Even if it does not explode immediately (i.e., even if there is not an absolute upper limit,[23] $k\Theta \lesssim m_p c^2$, to physically attainable temperatures), a black hole which falls below the critical mass (16) will certainly dissipate all its remaining mass on a time scale shorter than the present age (18) of the universe (the last stage being inevitably explosive as Hawking has pointed out[7]). According to previous ideas, the fact that baryons, leptons, etc. could be hidden in a black hole was interpreted as "transcending" the corresponding conservation laws, but if the black holes themselves can disappear without trace it must be recognized frankly that for practical purposes these conservation laws are violated. In order to avoid this conclusion Wheeler[27] has suggested, though without discussing the possible mechanism, that the chemical potential μ in the emission formula (13) should contain additional contributions due to baryons, etc., in addition to the electric contribution (1). It is difficult[28,29] to see how this could come about except by the action of repulsive classical force fields extending outside the horizon and proportional to the numbers of particles inside (such fields being generated by zero-rest-mass, odd-integer-spin bosons).

Let us consider the case of baryons, for which, as Dicke has pointed out,[30] the corresponding coupling constant $e_B^2/\hbar c$ (the analog of the electric "fine structure" coupling constant) must be extremely small, $e_B^2/Gm_p^2 \lesssim 10^{-7}$, in order to have escaped detection by the Eotvos experiment. Assuming that the preceding order-of-magnitude formulas apply also to baryon discharge, with e_B, m_p, and baryon number N_B in place of e, m_e, and N_e, we see there is a striking difference from the electric case, for which the analogous ratio is $e^2/Gm_e^2 \gg 1$. This means that superradiant discharge can never become effective and that a decaying black hole will retain an effectively *constant* baryon number N_B, and so will charge right up to the limit corresponding to (12) at which stage Θ would tend to zero so that further decay would be prevented. The black hole would settle down at a *finite* ground state with *residual mass* M_* given by

$$M_* \sim G^{-1/2} e_B N_B. \tag{26}$$

It is conceivable that e_B could be sufficiently large that an "ordinary" black hole[31,32] formed by the collapse of a star with $N_B \sim (\hbar c/G)^{3/2} m_p^{-3}$ could leave a residual black hole above the limit (16).

The Dicke-Eotvos experiment also places an upper limit on any hypothetical electron-leptonic charge (though not on a muon-leptonic charge). However since there are no known lower limits on neutrino masses, the corresponding charge/mass ratios might be *either* large (leading to relatively rapid leptonic discharge) *or* small (leading to leptonic contributions to the residual mass).

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Extraction of $R=\sigma_L/\sigma_T$ from Deep Inelastic *e-p* and *e-d* Cross Sections*

E. M. Riordan, A. Bodek, M. Breidenbach,† D. L. Dubin, J. E. Elias,‡ J. I. Friedman, H. W. Kendall, J. S. Poucher, and M. R. Sogard§
Physics Department and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

and

D. H. Coward
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 16 May 1974)

The quantity $R=\sigma_L/\sigma_T$ is extracted for the proton, deuteron, and neutron from deep inelastic e-p and e-d scattering cross sections measured in recent experiments at Stanford Linear Accelerator Center. For $\omega \leq 5$ the kinematic behavior of νR_p is consistent with scaling, indicative of spin-$\frac{1}{2}$ constituents in a parton model of the proton. We also find that within large statistical errors, R_d and R_n are consistent with being equal to R_p.

We have extracted longitudinal and transverse virtual photoabsorption cross sections σ_L and σ_T from deep inelastic electron-proton (e-p) and electron-deuteron (e-d) scattering cross sections that were measured in two experiments[1,2] at the Stanford Linear Accelerator Center (SLAC). Values of $R=\sigma_L/\sigma_T$ for the proton (R_p) are presented and compared with current theoretical predictions. In an earlier experiment,[3] R_p was found to be consistent with the constant value 0.18 ±0.10. This small value of R_p suggested spin-$\frac{1}{2}$ constituents[4] of the proton, but full verification of this hypothesis requires a detailed knowledge of its kinematic variation.[5] In the present work R_p is determined over a larger kinematic range and its accuracy is sufficiently improved to allow examination of its kinematic variation. The first determinations of R for the deuteron and neutron, R_d and R_n, are also reported.

The inelastic scattering of an electron of incident energy E to final energy E' through an angle θ is described in the first Born approximation by the exchange of a virtual photon of energy $\nu=E-E'$ and invariant mass squared $q^2=-4EE'\sin^2(\theta/2)=-Q^2$. The differential cross section is related to σ_L and σ_T as follows[6]:

$$\frac{d^2\sigma}{d\Omega\, dE'}(E,E',\theta)=\Gamma[\sigma_T(\nu,Q^2)+\epsilon\sigma_L(\nu,Q^2)],$$

where Γ is the flux of transverse virtual photons