# Superheavy Elements and the Strutinsky Prescription*

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(Received 14 February 1974)

Cases are found in which the Strutinsky prescription for single-particle corrections to the liquid-drop model fails. Estimates of the spontaneous-fission lifetimes of superheavy elements are found to be overly sensitive to small changes in the assumed single-particle level spectrum.

With the considerable effort being made at present to produce superheavy elements ($A \approx 300$), it is worthwhile to reconsider the question of their stability. In this work, I examine the problem in the framework of the liquid-drop model (LDM) with single-particle–correction[1,2] (SPC) effects. In terms of a pure LDM, there is no barrier to the spontaneous fission of the superheavy elements. However, because of stabilization effects associated with nonuniformities in the single-particle spectrum, superheavy elements may have finite barriers to spontaneous fission and finite lifetimes. In a recent study of this problem,[3,4] it was concluded that the stabilization effects due to the SPC are rather large and that there are long-lived superheavy elements. However, the single-particle spectra that were used in this study differ in some significant respects from other[5,6] careful extrapolations of single-particle level spacings from the lead region and from the actinides. The points that I wish to consider are (1) how accurate are the extrapolations of single-particle levels to the superheavy-element region and (2) how sensitive are the calculated lifetimes of superheavy elements to the differences in the various extrapolations? This second question has already been considered[7] to some extent. In the course of investigating these questions, a somewhat more basic problem has arisen: (3) How meaningful is the Strutinsky prescription[2,8] for the calculation of SPC effects?

To calculate the SPC arising from nonuniformities in the single-particle spectrum generated from a Woods-Saxon potential, I use the Strutinsky prescription.[2,8] The procedure is first to generate a continuous spectrum from the discrete one by defining

$$G_T(\epsilon,\gamma)=\sum_{i=1}^{N}\sum_{k=1}^{T/2}A_T^{2k}[(\epsilon-\epsilon_i)/\gamma]^{2k}\times\exp\{-[(\epsilon-\epsilon_i)/\gamma]^2\}, \qquad (1)$$

where $\epsilon_i$ is an eigenvalue of the potential, $T$ defines the order of the continuous distribution, $\gamma$ is the smoothing width, and the coefficients $A_T^{2k}$ are well known; take $\epsilon_i$ and $\gamma$ in units of MeV. The eigenvalue spectrum for the Woods-Saxon potential is generated with a harmonic-oscillator basis set. In order to apply the Strutinsky prescription one must extend the summation over eigenvalues $\epsilon_i$ to continuum states. I find that this summation should extend $\sim 5\gamma$ into the continuum. Making use of the function $G_T(\epsilon,\gamma)$, I define $\lambda(M,T,\gamma)$ through the relation

$$M=2\int_{-\infty}^{\lambda(M,T,\gamma)}G_T(\epsilon,\gamma)\,d\epsilon; \qquad (2)$$

i.e., there are $M$ particles in the smooth distribution $G_T(\epsilon,\gamma)$ to the cutoff $\lambda(M,T,\gamma)$. The factor of 2 arises from the fact that the orbitals are doubly degenerate. The Strutinsky prescription for the SPC, denoted as $\Delta E(M,T,\gamma)$, is given by

$$\Delta E(M,T,\gamma)=2\sum_{i=1}^{M/2}\epsilon_i-2\int_{-\infty}^{\lambda(M,T,\gamma)}\epsilon G_T(\epsilon,\gamma)\,d\epsilon. \qquad (3)$$

This quantity is just the difference in energy between the first $M$ particles with the discrete eigenvalues and the first $M$ particles with a continuous distribution of energies. It is this continuous distribution that is presumably being accounted for in the LDM and $\Delta E(M,T,\gamma)$ gives the SPC to the LDM. The crucial feature in the application of the Strutinsky prescription is that the function $\Delta E(M,T,\gamma)$ be constant as a function of both $T$ and $\gamma$ in a large region of $(T,\gamma)$ space. In general, one finds that $\Delta E(M,T,\gamma)$ is in this constant region. If there were no such plateau, the Strutinsky prescription for the SPC would be invalid. There is also a pairing correction associated with the Strutinsky prescription, but it is roughly the same for the various extrapolations considered here, and is ignored.

To get some idea of the uncertainties involved

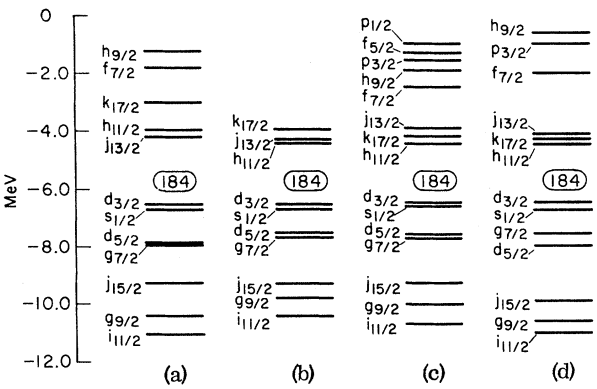

FIG. 1. Extrapolations of neutron single-particle spectra to the superheavy elements: Spectrum $a$ is taken from Ref. 4; spectrum $b$ is taken from Ref. 5; spectrum $c$ is based on the potential of Ref. 6; spectrum $d$ is explained in the text.

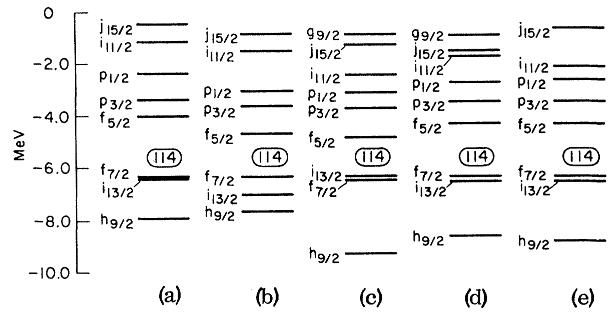

FIG. 2. Extrapolations of proton single-particle spectra to the superheavy elements: Spectrum $a$ is taken from Ref. 4; spectrum $b$ is taken from Ref. 5; spectra $c$ and $d$ are based on the potential of Ref. 6 and discussed in the text; spectrum $e$ is explained in the text.

in the extrapolation of single-particle spectra to the superheavy-element region, it is instructive to consider the results of Rost.[5] He found that it was possible to fit the known single-particle levels in the Pb region with an average deviation of ~200 keV by using a Woods-Saxon potential. A fit of this quality was obtained by introducing a free radius parameter in the spin-orbit term of the potential. With a conventional spin-orbit term, he found that the average deviation is ~400 keV. I emphasize that this is an attempt to fit measured levels. In this context, I feel that it would be foolhardy to claim that one can calculate the unmeasured single-particle level spacing in the superheavy-element region with an accuracy of any better than 500 keV.

Figure 1 shows several extrapolated neutron single-particle spectra for the superheavy elements. The level spacings of Bolsterli *et al.* and Rost were taken from figures in their publications,[3,5] so that there may be some small errors. The spectrum generated from the momentum-dependent potential[6] was obtained with a fifteen-shell oscillator basis ($N_{max}=14$) rather than the fourteen-shell basis used previously. This extrapolation (spectrum $c$, Fig. 1) is based on the hole states in Pb and agrees well with an extrapolation from the actinides. In attempting to apply the Strutinsky prescription to spectra generated from the momentum-dependent potential, I find that the prescription does not work. This failure appears to be related to the extremely high density of levels in the first 20 MeV of the continuum spectrum generated from this potential. In order to circumvent this problem, I went to a momentum-independent Woods-Saxon potential and adjusted a few levels generated from this potential in order to get a spectrum that resembles the spectrum generated from the momentum-dependent potential at the Fermi level. The spectrum ($d$, Fig. 1) is obtained by shifting the $k_{17/2}$ orbital 400 keV and the $j_{13/2}$ orbital 500 keV from a spectrum generated from a conventional Woods-Saxon potential. All of the spectra in Fig. 1 are lined up at $N=184$ to facilitate comparisons. Figure 2 presents extrapolated proton spectra. Two extrapolations of the momentum-dependent potential are given—one based on particle states in the Pb region (spectrum $c$, Fig. 2) and the other based on hole states in the Pb region (spectrum $d$, Fig. 2). Note that an extrapolation from the actinides agrees with extrapolation $d$ in Fig. 2. The final spectrum ($e$, Fig. 2) is a level spectrum generated from a momentum-independent Woods-Saxon potential that corresponds rather well to spectrum $d$ in Fig. 2. The proton spectra are lined up at $Z=114$ to facilitate comparisons. On the whole, the agreement between the various extrapolations of single-particle spectra to the superheavy-element region is quite good, better than one would anticipate the agreement with experimentally determined spectra to be. At $Z=114$, the $f_{7/2}$-$f_{5/2}$ spacing is somewhat larger in spectrum $a$, Fig. 2, than it is in the other spectra. The experimental data from the Pb region[9] and the actinides[10] favor the smaller spacing. At $N=184$, the agreement is remarkable. The $k_{17/2}$ orbital is somewhat higher in energy in spectrum $a$, Fig. 1, than in the other extrapolations. There is no data on this orbital.

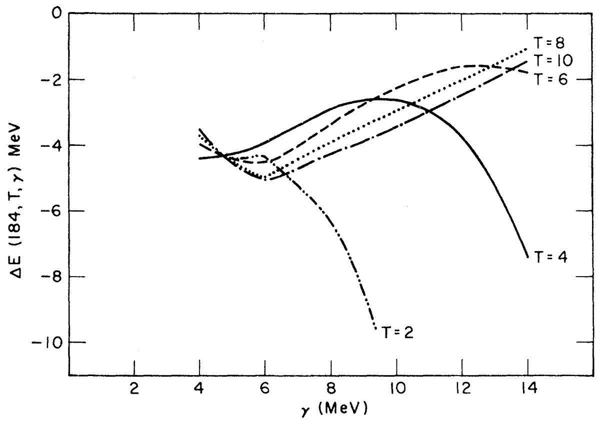


FIG. 3. Calculation of the single-particle correction to the liquid-drop model using the Strutinsky prescription and spectrum $d$, Fig. 1.

Because the Strutinsky prescription is not applicable to spectra generated from the momentum-dependent potential, I carried out calculations with spectrum $d$, Fig. 1 and spectrum $e$, Fig. 2. In carrying out the calculation of the SPC with the Strutinsky prescription, I found a very surprising result. With the use of spectrum $d$, Fig. 1, the Strutinsky prescription breaks down. Figure 3 displays $\Delta E(184, T, \gamma)$ calculated for this set of levels. The calculations were made at 2-MeV intervals in $\gamma$. Inspection of Fig. 3 shows that there is no large plateau region. To orient the reader, I mention that in other cases with different spectra, I have found $\Delta E(M, 8, \gamma)$ quite constant for values of $\gamma$ ranging from 8 to 14 MeV. I note that this breakdown is not confined to $N=184$, but occurs for all mass numbers from 160 to 200. As this result is quite unexpected, I have checked my computer program by comparing my results with those obtained from other programs. Using spectrum $d$, Fig. 1, J. R. Nix has obtained this same breakdown in the Strutinsky prescription for $\Delta E(184, T, \gamma)$. For many choices of $M$ and $\gamma$, B. Wilkins obtains the same values of $\Delta E(M, 2, \gamma)$ that I obtain. I note that I have found other cases in which this breakdown occurs. I emphasize that the breakdown of the Strutinsky prescription that I have found here is not due to any difficulties associated with the treatment of continuum states. It has been noted[3] that using an oscillator basis set, as done here, circumvents difficulties with the continuum. Clearly, this breakdown is associated with the extremely high level density at 184 neutrons. I have carried out calculations of the SPC using the unshifted spectrum from which spectrum $d$, Fig. 1, was obtained. The two spectra differ only in the positions of the bound states $k_{17/2}$ and $j_{13/2}$. With the use of this unshifted spectrum, there is no breakdown in the Strutinsky prescription. The quantity $\Delta E(184, 8, \gamma)$ is quite constant over the interval $6 \le \gamma \le 10$ for the unshifted spectrum. For completeness, I note that it has a value of $-5.9$ MeV. This breakdown of the Strutinsky prescription forces the conclusion that calculations based on this technique are suspect. The consequences of this breakdown are particularly serious in view of the many calculations that have been reported in the literature[11] based on the use of the Strutinsky prescription. There is a need to develop reliable techniques for the calculation of SPC effects for finite potentials. In order to provide a test for any new methods that are developed, I would be glad to provide a listing of the complete set of the eigenvalues in spectrum $d$, Fig. 1.

If we assume that a new method for calculating SPC effects will give results similar to the Strutinsky prescription—in those instances that the Strutinsky prescription does not break down—we can learn something about the sensitivity of calculated spontaneous-fission lifetimes to the assumed single-particle spectrum. Using spectrum $e$, Fig. 2, I find

$$\Delta E(114, 8, \gamma) \approx -3 \text{ MeV}. \tag{4}$$

Lowering the $f_{5/2}$ orbital to 1800 keV above the $f_{7/2}$ orbital, a value midway between spectra $b$, Fig. 2, and $d$, Fig. 2, gives

$$\Delta E(114, 8, \gamma) \approx -2.5 \text{ MeV}. \tag{5}$$

These results should be compared with the value that one obtains[3] with spectrum $a$, Fig. 2,

$$\Delta E(114, T, \gamma) \approx -5 \text{ MeV}. \tag{6}$$

In the superheavy-element region, a change of 1 MeV in the fission-barrier height corresponds[4] to roughly a factor of 100 change in the spontaneous-fission lifetime. These rather small changes in the proton single-particle spectrum amount to a factor of $\sim 10^4$–$10^5$ decrease in the spontaneous-fission lifetime. From Fig. 3, one might guess that

$$0 \ge \Delta E(184, T, \gamma) \ge -5 \text{ MeV} \tag{7}$$

for spectrum $d$, Fig. 1. For spectrum $a$, Fig. 1, one has[3]

$$\Delta E(184, T, \gamma) \approx -7.5 \text{ MeV}. \tag{8}$$

This difference corresponds to at least another factor of $\sim 10^5$ decrease in the spontaneous-fission lifetime. I conclude that rather small, very reasonable, changes in the extrapolated single-particle spectra of superheavy elements give a reduction of $\sim 10^{10}$ in the estimates of spontaneous-fission lifetime relative to previous[4] estimates. The uncertainties in the extrapolated spectrum are sufficiently large that even these lowered estimates might be too large by a factor of $10^{10}$.

The major conclusion that I draw from this work is that the Strutinsky prescription is an extremely questionable technique for calculating single-particle corrections to the liquid-drop model. Until the breakdown of the Strutinsky prescription that has been found here is understood, all results based on the Strutinsky prescription must be regarded as suspect. To the extent that the Strutinsky-prescription results represent those of a correct method of calculating the SPC, I draw a second conclusion. The calculation of spontaneous-fission half-lives of superheavy elements is overly sensitive to the details of the assumed single-particle spectrum. The irreducible uncertainties involved in extrapolating single-particle spectra to the superheavy elements are sufficiently large to vitiate all such calculations.

I thank J. R. Nix for his help in carrying out this work. I thank B. Wilkins for considerable aid in many aspects of this work.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

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# Physical Significance of the Topology of the Bondi-Metzner-Sachs Group

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(Received 22 April 1974)

It has been proposed that the Bondi-Metzner-Sachs group affords a possible explanation for the discreteness of elementary-particle spins. This discreteness result depends on the topology given to the group. A careful analysis of representations in the "nuclear" topology indicates that discrete-spin representations (those subsumed by the original topology) describe bound gravitating systems ("elementary particles") whereas the new continuous-spin representations, if physically realistic, describe scattering states of gravitating systems.

The asymptotic symmetry group of general relativity, the Bondi-Metzner-Sachs (BMS) group[1] $B$, has been of interest for some time as a possible candidate for replacing the Poincare group in a microphysics which takes gravity into account.[2] Recently it was proposed that the group may give an explanation for the discreteness of elementary particle spins.[3] However, this discreteness result, which arises because all little groups of $B$ are compact, depends on the topology with which $B$ is endowed. There is a wide range of choices of "reasonable" topologies, arising from the infinite-dimensional additive supertranslation subgroup $A$ of "arbitrary" real-valued functions on the Riemann sphere $S$. Also, the range of choices available depends on the class of func-