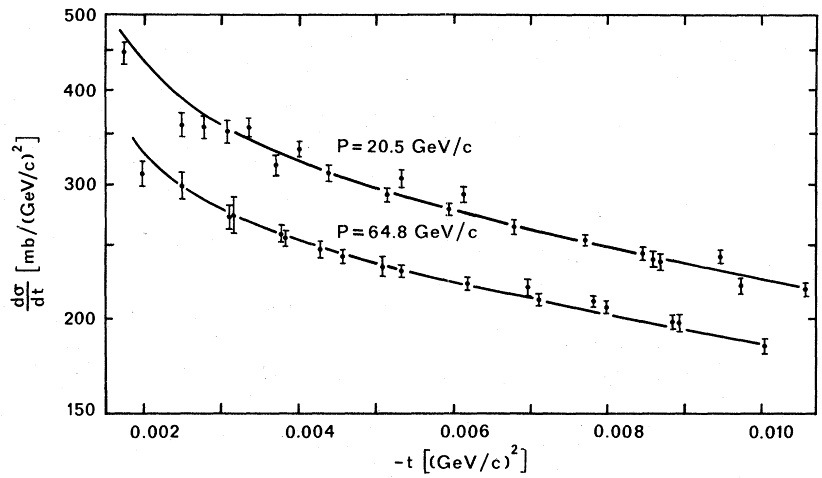

FIG. 1. Differential cross sections for elastic *pd* scattering at 20.5 and 64.8 GeV/*c*. The data are from Ref. 1. The curves correspond to Eq. (12).

average, 0.17 less than those obtained in Ref. 5, and we conclude that ρ_n does not appear to approach zero as rapidly as previously indicated.

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A Picture of Multiple-Meson Production*

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High-energy hadron-hadron collisions are viewed as proceeding by a strong short-range "quark-quark" interaction, generating a "spark" in the form of a self-interacting meson field, which yields observable mesons only after the incident particles have passed. This picture leads to the possibility of rare events with high multiplicity. There is a definite contradiction to Gottfried's recent model of multiple production *in nuclei,* but little practical distinction for beams below 1 TeV.

Multiperipheral models make a number of predictions which are in reasonable agreement with high-energy data on multiple-meson production, including the existence of a central rapidity plateau for the single-particle inclusive cross section. I consider here an alternative scheme, which gives almost indistinguishable results for hadron-hadron collisions, but which generates a simple space-time picture of the collision process, permitting semiquantitative deductions about collisions with nuclei as targets.[1]

The picture I adopt assumes some small constituents of the hadron which I shall call quarks for definiteness. In a high-energy collision, normally only one quark in the projectile intercepts one quark in the target. This is required by the success of the additive quark model for total cross sections.[2] When the two quarks collide, they provide a source for a mesonic "spark," just as an electrical discharge acts as a source of light. Let us suppose for simplicity that the associated meson field is scalar, and self-interacting. If we look in any Lorentz frame in which the projectile is moving rapidly to the right, while the target moves rapidly to the left ("rapidly" means with high rapidity, i.e., speed imperceptibly infraluminal), then the colliding quarks will have negligible thickness in their direction of motion, and the mesonic source will be effectively a noncircular disk: a delta function in time and in the longitudinal direction, but spread by a typical quark-quark interaction radius in the transverse direction. If the quarks are small and totally black, then that radius is about $\frac{1}{3}$ fm for nonstrange quarks, less for strange.

Clearly, from case to case, the source strength will vary, but I suppose that the self-interaction of the meson field will make the number of produced mesons insensitive to the exact value of this strength, once it surpasses a certain threshold. Consequently, the inclusive rapidity distribution will be flat in the central region of rapidity, with a multiplicity per unit rapidity interval, or rapidity density, independent of the nature of projectile and target. This is because in each Lorentz frame in the central region, the meson source will flash, fast hadronic matter will depart rapidly to right and left, and slow mesons will materialize in a time of about 1 fm/c, knowing nothing about the precise nature of their source.

Diffraction can certainly be incorporated in the standard Good-Walker[3] way; it is simply a quantum-wave "leftover" of the dynamic interaction which produces sparks.

The main formal distinctions between this scheme and one with multiperipheral pion exchange are that the exchange line which radiates pions (or clusters of pions) carries a large mass, of the order of 1 GeV (to account for the small quark-quark interaction range), and that many exchange lines cross independently from projectile to target, each producing a single forward meson line. This contrasts with the multiperipheral case, where a single exchange line shakes off many mesons. Also, the associated diagram would be a "screened" graph, since the radiated mesons interact strongly with their near-rapidity neighbors. The result is a form similar to the "inside-outside" graph discussed by Bjorken, and Casher, Kogut, and Susskind.[4,5]

The main conclusions from the mesonic-spark picture of production processes are the same as from many other models,[6] and so we turn to the differences. If this picture is right, then occasionally (perhaps 1% of the time) in a proton-proton collision, two distinct quark collisions will occur, separated in impact parameter by a sufficient distance to produce distinct sparks. (If the collisions were very close together, they would simply provide a single effective source for the meson spark, of twice the usual strength. Because of the assumed insensitivity of the spark to source strength, the resulting spark would be indistinguishable from that of a single quark-quark collision.) For these events the mean multiplicity would be roughly twice that in the typical event. Such an effect might be detected as a correlation between multiplicities in the left and the right hemispheres for very high multiplicities in either hemisphere.

Now let us apply the same assumptions to the collision of a proton with a nucleus. Using the known p-p inelastic cross section, we may deduce ν, the expected number of collisions with different nucleons of a proton passing through a given target nucleus on a straight-line trajectory (neglecting any effect of a particular collision on the probability of further collisions). The multiplicity of slow mesons in the target rest frame will be ν times as great as that for a hydrogen target. This is simply because ν different target nucleons are independently fragmented, and is a result which follows in many different pictures. There is one qualification—secondary interactions of the fragments may somewhat increase their number while degrading their energy, but these should not be drastic effects. A 1-GeV π^+ (rapidity 2.7) incident on a proton yields only one charged relativistic particle, more than 90% of the time. Furthermore, as I have argued before,[7] a produced particle with rapidity greater than this is not likely to develop its full power of interaction with a target nucleon before leaving the nucleus.

The spark picture provides a natural cutoff to the rapidity interval for which mean multiplicity is multiplied by ν. Let L be the mean length in the lab frame between successive sparks produced by a very high-rapidity projectile. These sparks are then separated by the lightlike interval

$$\Delta t = L, \quad \Delta z = L. \tag{1}$$

In a frame that moves in the projectile direction with a lab rapidity y, we have

$$\Delta t' = e^{-y}L, \ \Delta z' = e^{-y}L. \tag{2}$$

We have already discussed the assumption that two simultaneous sparks can develop independently only if their sources are separated by a critical distance, call it d. The existence of a d greater than the quark-quark interaction range ($\frac{1}{3}$ fm) is necessary if the picture is to be compatible with experiment. Otherwise the two ends of a single spark could both generate mesons throughout the rapidity plateau, leading to strong, long-range rapidity correlations in the two-particle inclusive distribution. Clearly, d cannot be much greater than 1 fm, the maximum range of known strong interactions. For definiteness, I adopt the value $d = \frac{2}{3}$ fm. We shall see later the consequences of changing this value.

What if two sources act at the same point, but different times? Since the spark from the first source cannot fly out faster than c, two independent sparks could not be generated unless the time interval were greater than d/c. It is possible that an even longer interval might be needed, but for simplicity we take the same criterion in time as in space.

Combining the two cases, we arrive at a criterion that two sources separated by the Lorentz interval $(\Delta T, \Delta X)$ in a given inertial frame will independently generate slow mesons in that frame:

$$(\Delta T)^2 + (\Delta X)^2 > d^2. \tag{3}$$

Substituting $\Delta t'$, $\Delta z'$ from Eq. (2), and taking $L \approx 3$ fm[8] we get a cutoff rapidity $Y_c \approx 2$. Below Y_c, the rapidity density is amplified by a factor ν. How sharp is the cutoff at Y_c? Two factors enter here. First, collisions can occur with a larger separation in the lab. This could give one more unit in rapidity. Secondly, the essence of our technique is to estimate the production of slow particles in a particular Lorentz frame. "Slow" means with rapidity of order 1 in that frame. Hence, the cutoff should become completely effective at $y \approx 4$, and so this will be the application in what follows:

The rapidity density is multiplied by ν for

$$y \lesssim Y_c = 4. \tag{4}$$

Above Y_c, only those collisions in which independent sparks are separated transversely by a distance greater than d will exhibit a doubled rapidity density. This should be roughly 10% of all collisions, estimated as follows. The proton has a radius of about $\frac{2}{3}$ fm. If two sparks are to be separated by $\frac{2}{3}$ fm, then each must be in the outer half of the proton cross section [probability $\approx (\frac{1}{2})^2$], and they must be more or less on opposite hemidisks (probability $\approx \frac{1}{2}$). This gives a probability of sufficient separation

$$P \approx \tfrac{2}{3} \times \tfrac{1}{2} \times \tfrac{1}{2} \times \tfrac{1}{2} \approx 0.1, \tag{5}$$

where the first factor is the probability that two distinct projectile quarks interact in the successive collisions. Clearly, the order of magnitude is not changed if there are 3 or 4 collisions. Also, the estimate is not better than an order of magnitude. The main point is that upward fluctuations of rapidity density in the entire plateau would be an order of magnitude more likely with a nuclear target than with a nucleon target.

While the cutoff Y_c in Eq. (4) is only weakly sensitive to the value of d, the probability P in Eq. (5) is very sensitive: For $d = \frac{4}{3}$, P vanishes. Thus, the prediction of enhanced multiplicity at low laboratory rapidity is much less sensitive to detailed assumptions than is the prediction of a 10% excess at higher rapidity.

At present, there are not data of sufficient quality at sufficiently high energy to detect the proposed 10% increase in mean multiplicity in the plateau region. Gottfried[1] has proposed a different model which leads to a 70% increase in multiplicity in emulsion, independent of energy, and associated entirely with the lower third of the rapidity plot. His result is in excellent agreement with data from 70 to 10 000 GeV, or an interval from 5 to 10 in projectile rapidity.[9] There are two observations to be made on Gottfried's model. The first is aesthetic. An observer watching a very high-rapidity hadron collide with a very high-rapidity nucleus could tell whether the nucleus-to-hadron rapidity ratio in his frame was greater or less than $\frac{1}{2}$, simply by counting the number of slow produced particles he sees. This violates a "generalized Lorentz invariance principle", that only absolute differences of rapidity should have measurable consequences, and not fixed ratios of rapidities which become arbitrarily large. The second comment is practical. The lower part of the energy range studied is clearly preasymptotic, while Gottfried's is an asymptotic formula. If only the 1- and 10-TeV data are taken as asymptotic, it becomes very hard to distinguish between the multiplicity formula of Gottfried,

$$n(\text{emulsion}) \approx n(\text{hydrogen})(1+\tfrac{2}{3}), \tag{6}$$

and that suggested here,

$$n(\text{emulsion}) \approx 1.1n(\text{hydrogen}) + (\approx 3)(\nu - 1), \quad \nu \approx 3. \tag{7}$$

Unfortunately, the small samples of cosmic-ray data at these higher energies do not permit a sufficiently detailed study of rapidity distributions to help make the distinction. Even with more data, one is hamstrung by the fact that the lower third of the rapidity plot is contained within Y_c, even at 10 TeV.

What about nucleus-nucleus collisions? At energies like 1 TeV per nucleon, our picture implies

$$n(\text{nuc-nuc}) \sim \zeta n(p\text{-}p), \tag{8}$$

where ζ is the mean number of independent columns of projectile nucleons which pass through columns of target nucleons. This is distinct from the multiperipheral-model guess of Gribov.[10] He takes a very different approach in which only surface nucleons can act as anchors of a multiperipheral chain, implying that, for fantastic energies, nucleus-nucleus cross sections go like $A^{4/3}$ (where A is mass number), but the $(A\text{-}A)/(p\text{-}p)$ multiplicity ratio goes to 1.

It seems likely that multiperipheral-*cum*-parton models would agree with the expectation of excess particles with finite rapidity in the laboratory following collision with a nuclear target of a high-rapidity hadron. However, these models could hardly generate the asymptotic Gottfried rapidity distribution, nor the $\approx 10\%$ uniform excess predicted here. A 500-GeV exposure of a variety of nuclear targets might at least settle this point, and could bury the spark picture.

The essential feature of this, as well as many other schemes, is that no large transfers of momentum or energy occur in short times.[11] This immediately implies a great similarity of multiparticle production processes for collisions of particles with widely different baryon numbers. In particular, our strongest single prediction is an asymptotically flat rapidity density in the central region of rapidity. The magnitude of that density is less certain, but in our picture would be about 10% higher for hadron-nucleus than for hadron-hadron collisions, and much higher for nucleus-nucleus collisions. As usual, at either end of the rapidity plot will be fragmentation regions, with nuclear fragmentation having a substantially higher multiplicity than hadron fragmentation.

Despite the difficulty of distinguishing experimentally between this and other models, even the present nuclear data enforce the view that high-energy strong processes are really rather slow and not so strong.

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[5]If the exchange lines are identified as the well-known vector mesons, one obtains a qualitative explanation of the smaller cross section of strange quarks; it comes from the shorter range and weaker coupling to quarks of the φ (coupled to strange q) compared to the ω (coupled to nonstrange q). One could not demand a quantitative result, since the relative source strengths for meson production of overlapping ω fields, of overlapping ω and φ fields, or of overlapping φ fields are not known. Although this qualitative correlation of quark cross sections with vector-meson masses is quite striking, it does not appear prominently in the literature.

[6]This is not quite true. The assumption of a fixed number of quarks, together with the natural use of longitudinal phase space, leads to a total cross section rising asymptotically as $[\ln(\ln s)]^2$, but even the highest foreseeable energies would scarcely serve to distinguish this from $[\ln s]^2$, since $\ln s$ would not be very large.

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[8]L is set equal to the mean free path of the incident hadron between inelastic collisions in the nucleus, treated as a dilute gas of nucleons. Gottfried (Ref. 1) uses a smaller value of L, obtained from a uniform nucleus with a sharp edge. Note that the dependence of Y_c deduced below on L and d is weak, $Y_c \approx 2.5 + \ln(L/d)$.

[9]A recent analysis demonstrates that most, if not all, excess particles come at low rapidities ($\lesssim 4$) in 67 and 200 GeV emulsion exposures. R. Hołyński, S. Krazywdziński, and K. Zalewski, University of Cracow Report No. INP 856/PH (to be published).

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[11]A point at the heart of Ref. 7.

ERRATUM

PHOTOELASTIC TENSOR OF SILICON AND THE VOLUME DEPENDENCE OF THE AVERAGE GAP. David K. Biegelsen [Phys. Rev. Lett. 32, 1196 (1974)].

It should be noted that Cardona, Paul, and Brooks (Ref. 11) also measured the pressure dependence of the radio frequency dielectric constant of silicon. They obtained $d\ln\epsilon/dP = (-4 \pm 1) \times 10^{-7}\ \mathrm{kg}^{-1}\ \mathrm{cm}^2$. This implies a value of $\bar{p} = -0.098 \pm 0.025$, in somewhat better agreement with our measurement of $\bar{p} = -0.059 \pm 0.003$ than their optical measurements. It is assumed that in their samples impurity and free-carrier effects on the rf dielectric constant were negligible.