

³We are really measuring the sum of the cross sections $\nu p \rightarrow \nu n \pi^+ l \pi^0$ and $\nu p \rightarrow \nu p \pi^0 l \pi^0$, $l \geq 0$, but since our neutrino spectrum peaks at 500 MeV/c and is down by an order of magnitude by 1500 MeV/c, we expect the contribution of the final states with additional π^0 's to be very small.

⁴J. Campbell *et al.*, Phys. Rev. Lett. **30**, 335 (1973).

⁵More details of the experiment are given by S. J. Barish, Argonne National Laboratory Report No. ANL/HEP 7418 (unpublished).

⁶Y. Cho *et al.*, in Proceedings of the Sixteenth International Conference on High Energy Physics, The University of Chicago and National Accelerator Labora-

tory, 1972 (unpublished), paper 473.

⁷In doing this we are implicitly assuming that the characteristics of our neutral- and charged-current events are the same. This is true on the Salam-Weinberg model but may not be true in general. For the charged-current events, we measure the ratio $N(\nu p \rightarrow \mu^- p \pi^+ \pi^0)/N(\nu p \rightarrow \mu^- p \pi^+) = 0.1 \pm 0.05$ and, therefore, we reduce the observed $\nu N \pi/\mu^- p \pi^+$ ratios by 10%. In addition, for the one-prong $+\gamma$ events, a small contribution from the reaction $\nu d \rightarrow \nu n \pi^0(p_s)$ has been subtracted.

⁸S. Adler, private communication.

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Hierarchy of Interactions in Unified Gauge Theories*

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We present a general formalism for calculating the renormalization effects which make strong interactions strong in simple gauge theories of strong, electromagnetic, and weak interactions. In an SU(5) model the superheavy gauge bosons arising in the spontaneous breakdown to observed interactions have mass perhaps as large as 10^{17} GeV, almost the Planck mass. Mixing-angle predictions are substantially modified.

The scaling observed in deep inelastic electron scattering suggests that what are usually called the strong interactions are not so strong at high energies. Asymptotically free gauge theories of the strong interactions¹ provide a possible explanation: The gluon coupling constant $g(\mu)$ (defined as the value of a three-gluon or gluon-fermion-fermion vertex with momenta characterized by a mass μ) is small when μ is several GeV or larger, but becomes large when μ is small, through the piling up of the logarithms encountered in perturbation theory. In one recent calculation² a fit was found for a gauge coupling [in a color SU(3) model]³ with $g^2(\mu)/4\pi \simeq 0.1$ when $\mu \simeq 2$ GeV.

If $g(\mu)$ is small when μ is large, then perhaps the strong gauge coupling at some large fundamental mass is of the same order as the couplings in gauge theories of the weak and electromagnetic interactions.⁴ Georgi and Glashow⁵ have recently gone one step farther, and proposed a model based on the *simple* gauge group SU(5), in which there naturally appears only one free gauge coupling. In their model, SU(5) suffers a spontaneous breakdown to the gauge subgroups SU(3) and SU(2)⊗U(1), which are associated respectively with the strong³ and the weak and

electromagnetic⁶ interactions. In order to suppress unobserved interactions, Georgi and Glashow made the necessary assumption⁷ that some vector bosons are superheavy.

We find the notion of a simple gauge group uniting strong, weak, and electromagnetic interactions extraordinarily attractive. However, as emphasized by Georgi and Glashow, the success of any such scheme hinges on an understanding of the effects which produce the obvious disparity in strength between the strong and the weak and electromagnetic interactions at ordinary energies. We therefore wish to present in this paper a general formalism for the calculation of such effects. This will lead us to an estimate of the mass of the superheavy gauge bosons. Where a specific model of the gauge groups of the observed interactions is needed as an example, we shall assume that the strong and the weak and electromagnetic interactions are described by color SU(3)³ and by SU(2)⊗U(1), respectively, and where a specific example of a unifying simple gauge group is needed, we shall use SU(5).

If we neglect all renormalization effects, the embedding of the gauge groups G_i of the observed interactions in a larger simple group G imposes a relation among their coupling constants. We

normalize the generators T_α of G so that in any representation D of G we have

$$\text{Tr}(T_\alpha T_\beta) = N_D \delta_{\alpha\beta}, \quad (1)$$

where N_D may depend on the representation but not on α and β . We use the same normalization conventions for the gauge groups of the observed interactions. Then invariance under G implies that the coupling constants g_G , g_3 , g_2 , and g_1 associated with the group G and the subgroups $SU(3)$, $SU(2)$, and $U(1)$, respectively, are equal. The usual $SU(2)$ and $U(1)$ coupling constants⁵ may be identified as

$$g = g_2, \quad g' = g_1/C, \quad (2)$$

where C is a constant entering the relation between the charge Q and the $SU(2)$ and $U(1)$ generators \bar{T} and T_0 , normalized according to Eq. (1):

$$Q = T_3 - CT_0. \quad (3)$$

The weak mixing angle⁶ is then given by

$$\sin^2 \theta = e^2/g^2 = g'^2/(g^2 + g'^2) = (1 + C^2)^{-1}. \quad (4)$$

In any representation of G , reducible or irreducible,

$$\text{Tr}(Q^2) = (1 + C^2)\text{Tr}(T_3^2). \quad (5)$$

If we take our representation to consist of the left-handed states of three quartets of colored quarks, three antiquark quartets, and $\nu_e, \nu_\mu, e^-, e^+, \mu^-, \mu^+$, then there are eight $SU(2)$ doublets, and so $\text{Tr}(T_3^2) = 4$, while $\text{Tr}(Q^2) = \frac{32}{3}$, so that

$$C^2 = \frac{5}{3}, \quad \sin^2 \theta = \frac{3}{8}. \quad (6)$$

This is the case for the $SU(5)$ model.⁵ We shall leave C arbitrary in what follows, and will find that the choice of the simple unifying group G enters the calculation only through the single parameter C .

Now let us see how to take renormalization effects into account. The gauge couplings are functions of the momentum scale μ , and the above relations among gauge couplings really only apply when μ is much larger than the superheavy boson masses, where the breaking of G may be neglected. However, the observed values of the gauge couplings refer to much smaller values of μ , of the order of the W and Z masses, or even smaller. The problem is to bridge the gap between superlarge values of μ , where G imposes relations among the gauge couplings, and ordinary values of μ , where the gauge couplings are observed.

In order to accomplish this, we make use of the theorem⁸ that all matrix elements involving only "ordinary" external particles with momenta and masses much less than all superheavy masses may be calculated in an effective renormalizable field theory, which is just the original field theory with all superheavy particles omitted, but with coupling constants that may depend on the superheavy masses. All other effects of the superheavy particles are suppressed by factors of an ordinary mass divided by a superheavy mass.

When μ is large compared with all ordinary masses but small compared with all superheavy masses, the μ dependence of the couplings is governed by a renormalization-group equation,⁹

$$\mu \frac{d}{d\mu} g_i(\mu) = \beta_i(g_i(\mu)), \quad (7)$$

with β_i calculated in the effective field theory based on the "observed" gauge group G_0 . If all $g(\mu)$ are small, then β_i depends only on g_i , with¹⁰

$$\beta_i(g(\mu)) \approx b_i g_i^3(\mu) \text{ for } |g_i| \ll 1, \quad (8)$$

so that

$$g_i^{-2}(\mu) \approx \text{const} - 2b_i \ln \mu. \quad (9)$$

The integration constants are determined by the underlying simple group G . Specifically, if we suppose that all superheavy gauge bosons have masses of the order of some typical superheavy mass M , and if we take μ to be of the order of but somewhat smaller than M , then the $g_i(\mu)$ may be calculated perturbatively in the simple gauge theory based on G , and as long as $g_i(M)$ is sufficiently small, each gauge coupling will be essentially given by its group-theoretic value g_G neglecting all renormalizations. Thus Eq. (9) gives

$$g_i^{-2}(\mu) = g_G^{-2}(M) + 2b_i \ln(M/\mu) \quad (10)$$

for $\mu \leq M$.

The gauge coupling constants g_{i_0} observed in present experiments are essentially given by the values of the $g_i(\mu)$ when μ is some "ordinary" mass m , of the order of 10 GeV. Since all these couplings are small and therefore slowly varying in the range of interest, our result is not particularly sensitive to the value of the "ordinary" mass at which we choose to study the couplings.

Let us now specifically assume that the "observed" gauge group is $SU(3) \otimes SU(2) \otimes U(1)$, but for the moment leave open the choice of the group G . Choosing convenient linear combinations of

(10), we have

$$\frac{C^2}{g_{1_0}^2} + \frac{1}{g_{2_0}^2} - \frac{(1+C^2)}{g_{3_0}^2} = \frac{1}{e^2} - \frac{(1+C^2)}{g_{3_0}^2}$$

$$= 2 \ln\left(\frac{M}{m}\right) [b_1 C^2 + b_2 - b_3(1+C^2)], \quad (11)$$

$$C^2(g_{1_0}^{-2} - g_{2_0}^{-2}) = e^{-2} [1 - (1+C^2) \sin^2\theta]$$

$$= 2C^2(b_1 - b_2) \ln(M/m). \quad (12)$$

To calculate the b 's, we note that any multiplet of particles forming a representation of G does not contribute at all to the b 's if *all* particles are superheavy, while it contributes equally to all b 's if *no* particles in the multiplet are superheavy, and therefore in either case has no effect in Eqs. (11) and (12). We shall assume that the only multiplet which contains *both* ordinary and superheavy particles is the gauge multiplet itself, in which case¹

$$b_2 = -\frac{22}{3}(4\pi)^{-2} + b_1, \quad b_3 = -11(4\pi)^{-2} + b_1; \quad (13)$$

so (11) and (12) give

$$\ln\left(\frac{M}{m}\right) = \frac{3(4\pi)^2}{22(1+3C^2)} \left(\frac{1}{e^2} - \frac{(1+C^2)}{g_{3_0}^2} \right), \quad (14)$$

$$\sin^2\theta = (1+3C^2)^{-1} (1+2C^2 e^2/g_{3_0}^2). \quad (15)$$

For $C^2 = \frac{5}{3}$ [the SU(5) value], $m = 10$ GeV, and reasonable values of $g_{3_0}^2/4\pi$, we obtain the results displayed in the following table:

$g_{3_0}^2/4\pi$	M (GeV)	$\sin^2\theta$
0.5	2×10^{17}	0.175
0.2	2×10^{16}	0.187
0.1	5×10^{14}	0.207
0.05	2×10^{11}	0.248

It is intriguing that we are led to contemplate elementary particle masses as high as 2×10^{17} GeV, of about the same order of magnitude as the Planck mass, $G^{-1/2} = 1.2206 \times 10^{19}$ GeV. Perhaps gravitation has something to do with the superstrong spontaneous symmetry breaking, or perhaps the spontaneous breakdown of the simple gauge group has something to do with setting the scale of the gravitational interaction.

Equation (15) predicts lower values for $\sin\theta$ than does Eq. (4). While the available data favor the higher value, they are rather preliminary, and the strong constraints on $\sin^2\theta$ follow only if the Z mass is assumed to satisfy the relation $M_W^2 = M_Z^2 \cos^2\theta_W$, which depends on the Higgs structure of the model.¹¹ If this relation is abandoned, $\sin^2\theta_W$ must be inferred from the ratio of

the ratio of neutral- to charged-current events seen in neutrino scattering to that seen in anti-neutrino scattering.¹¹

In the SU(5) example, it is not necessarily true that all effects of order m/M are negligible, because some of the superheavy vector bosons mediate proton decay into lepton plus pions. Since the proton is otherwise stable, such very small effects may be observable. Calculation of the proton lifetime involves details of the strong interactions at small momenta, but we can give an order-of-magnitude estimate on dimensional grounds. The lifetime must be proportional to M^4 and so it must approximately equal M^4/m_p^5 . Taking $M = 5 \times 10^{15}$ GeV, for example, gives a proton lifetime of about 6×10^{31} yr. The present experimental lower limit is 10^{30} yr.¹² The observation of proton decay with a lifetime of this order of magnitude would be a startling confirmation of the ideas discussed here.

Before concluding, we emphasize again the approximation which went into the derivation of Eqs. (14) and (15). We have idealized the two transition regions: the region in momentum scale around M where the three coupling constants are merging into one, and the region from m into the timelike domain where we actually measure e^2 . The corrections to (14) and (15) due to changes of the coupling constants in these regions can be calculated using perturbation theory in the relevant coupling constants. The corrections for the second region are electromagnetic and therefore small (g_{3_0} can in principle be measured directly in the spacelike region through the observation of logarithmic violations of scaling in electroproduction). The corrections from the first region will be small if $g_G(M)$ is small. To calculate $g_G(M)$ we need to know the fermion and scalar-meson content of the theory. For the SU(5) model with $M = 5 \times 10^{15}$ GeV (see the table), $g_G^2(M)/4\pi = (48 \pm 1)^{-1}$.

We have also assumed that the lowest-order form for β_i , Eq. (8), is valid down to $\mu = m$. Next-order corrections to β have been calculated by Belavin and Migdal.¹³ We use their results and find the ratio of the correction to the lowest-order value in the SU(5) theory to be about $0.6g^2/4\pi$. Such corrections are obviously only relevant for $g_3(\mu)$, and even for $g_{3_0}^2/4\pi = 0.5$, the largest value used in the table, the correction is only 30%.

Finally we want to emphasize what seems to us to be the most disturbing feature of the class of models discussed here, that is, the existence of

two stages of spontaneous symmetry breaking characterized by radically different mass scales. In the context of the conventional Higgs mechanism, we can find no natural explanation of the enormous ratio of superheavy mass to ordinary mass. We have nothing quantitative to say about this mystery, but the following speculation seems attractive to us. Suppose that only superstrong breaking takes place via the Higgs mechanism. There is only one mass scale in the theory and it is superheavy. All of the scalar mesons are either superheavy or Goldstone bosons (note that this obviates the difficulties associated with superlarge trilinear couplings among ordinary-mass scalars). Well below the superheavy mass scale the theory is an effective $SU(3) \otimes SU(2) \otimes U(1)$ theory containing only gauge fields and fermions. The next stage of symmetry breaking is dynamical and hence nonperturbative. The mass scale associated with this stage is the mass at which $g_3(\mu)$ gets large enough that nonperturbative effects become important.

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