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Multinucleon Removal in the Absorption of π^- at Rest and at 60 MeV*

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The intensities of γ rays from π^- absorption at rest on O, F, Mg, P, Ca, and Nb have been measured. The results are compared with those at 220 MeV, previously published. The α removal processes which have been reported to dominate at higher energies are not as pronounced in the present work.

Recent studies on the interaction of 220-[1] and 380-MeV[2] negative pions with nuclei have shown unexpectedly large cross sections for integral "α-particle" (i.e., multiples of two neutrons + two protons) removal. This is surprising, either as an intrinsic property of pion interaction, or from the point of view of nuclear structure. The measurements have been made via the observation of de-excitation γ rays in the final nucleus, so that it is not possible to distinguish the course of the preceding sequence of particle emissions, or even their identities (single nucleons or heavier particles). From the design of these experiments there has also been no distinction made between absorption and nonabsorption processes.

The purpose of this Letter is to present results of a similar study with π^- at rest for a range of nuclei and a case of π^- interaction at 60 MeV kinetic energy. In this study the pion interaction is clearly an absorption process for π^- at rest. Because of the experimental conditions, the results for π^- at 60 MeV are also due predominantly to absorption.

The measurements were done with a negative pion beam from the CERN synchrocyclotron. Targets with 5 g/cm^2 nominal thickness of natural H_2O, LiF, Mg, P, Ca, and Nb were used in a standard stopped-pion experimental arrangement as described earlier.[3] γ rays were detected by a 40-cm^3 Ge(Li) detector (at 90° to the beam direction) in coincidence with stopped-pion logic $(1,2,3,\bar{4})$ from the beam telescope. The absolute yields per stopped pion were determined by comparison with muonic x rays.[3] In Fig. 1 (upper curve), as an example, the prompt γ-ray spectrum from pions absorbed at rest on ^{31}P is shown, with an energy resolution of 3 keV (full width at half-maximum) at 1 MeV. Background lines determined in a separate measurement are indicated by arrows below the spectrum.

For the measurement with pions in flight the same arrangement has been used. Since counter 4 was close behind the target, and both were tilted by 45° with respect to the beam direction, a $(1,2,3,+4)$ trigger condition covers all processes where the pion is scattered in the forward direction, and to some extent also pions with larger scattering angles. In a phosphorus target measurement made under this trigger condition, none of the γ-ray lines from residual nuclei present in the upper curve of Fig. 1 were observed. The γ spectrum taken under the complementary condition $(1,2,3,\bar{4})$, however, shows a striking similarity to the spectrum obtained with π^- at rest from the same target (Fig. 1). The drastic reduction of the pionic x-ray yield from phosphorus in the lower curve proves that the contribution from pions at rest is negligible in this spectrum. In Table I the ratios of intensities of corresponding lines in the two spectra are listed, and are seen to be similar within about a factor of 2. This indicates that absorption at rest and at 60 MeV are similar processes. Also, since the order of magnitude of the 60-MeV absorption cross sections is 10 mb, considerations[4] which neglect absorption at nonzero pion energies are questionable.

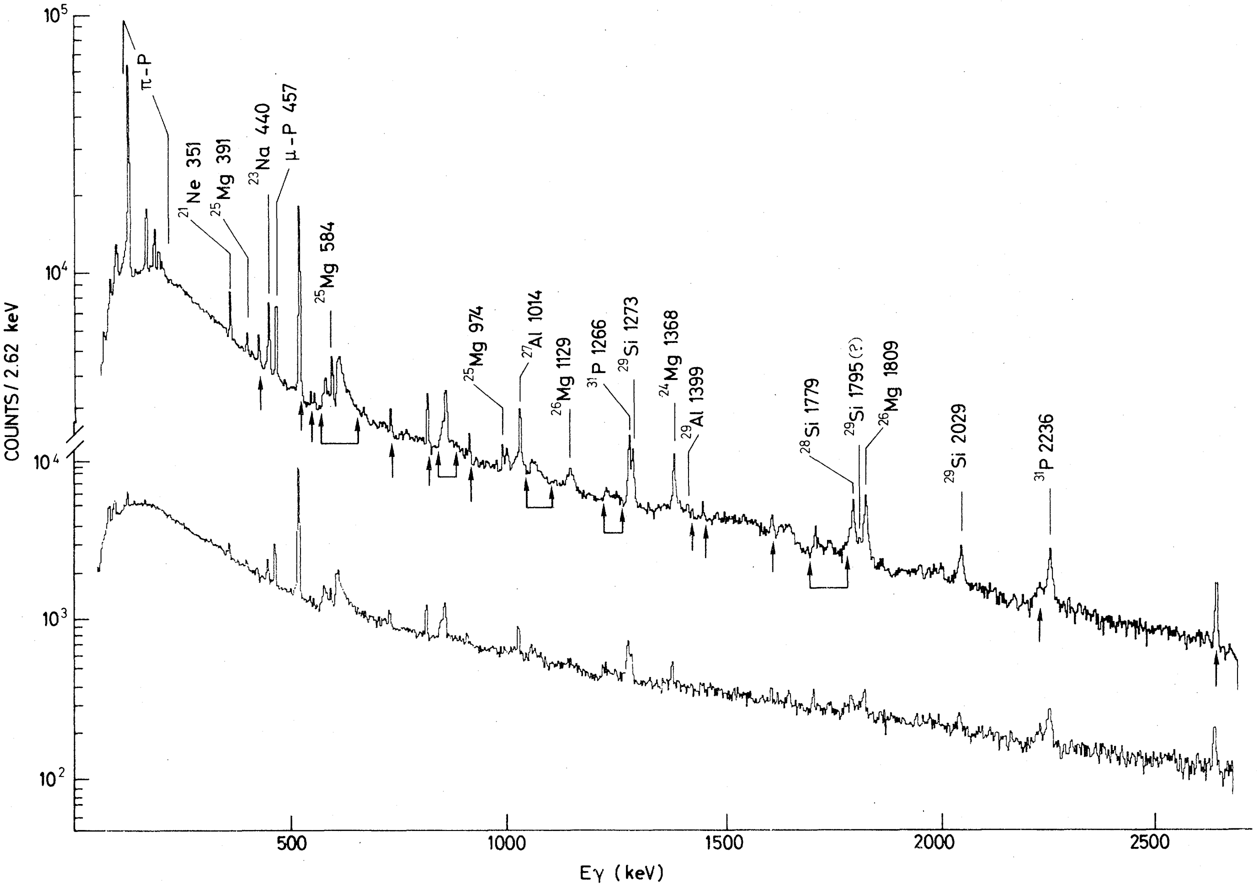

FIG. 1. γ-ray spectra from π^- interaction on ^{31}P with pions at rest (upper curve) and at 60 MeV (lower curve) under trigger condition $(1,2,3,\bar{4})$.

Table I also compares all cross sections given in Ref. 1 for multinucleon removal processes with 220-MeV pions with the respective yields per stopped pion from our measurements for the three targets common to the two studies. Systematic uncertainties in all the yield values may be as large as 30%, but relative errors should be less than 10%. For meaningful comparison the conventions in the two sets of measurements are the same: (a) Only the lowest observed γ-ray transition (usually first to ground) in each residual nucleus is listed, and (b) the yield and cross section for each γ ray listed is without regard to whether it occurs by direct excitation or by γ-ray cascading. The comparison shows no simple proportionality between cross sections and yields for corresponding transitions. We must conclude that pion interaction at 220 MeV is very different from absorption at rest.

It should also be pointed out that Table I is not complete as far as our own results are concerned. This can be seen in Fig. 2, where the observed γ-ray yields from π^- absorption at rest are shown for six nuclei. Again, the same conventions as before are used. Lines with ambiguous assignments as well as lines from the target nuclei have been omitted. The complete results of our measurements will be presented elsewhere.

In Fig. 2 it is apparent that π^- absorption at rest leads very frequently to the emission of more than two nucleons. It can also be seen that most of the observed processes are found along the diagonal line, with the number of protons and neutrons being about equal. The processes connected with the removal of one or two α particles are, in fact, observed with relatively high abundance. For the three lighter nuclei they give rise to the strongest transition. For the heavier nuclei, however, they are less important. In the case of niobium, no removal of α particles has been observed.

Multinucleon removal caused by pion absorption

TABLE I. Comparison of γ-ray production by π^- at rest and in flight. The ratio of γ-ray intensities at 0 and 60 MeV from Fig. 1 is given by R. The absolute value of R has no significance. The 220-MeV cross sections are from Ref. 1.

Observed transition (keV)	Nucleons removed	0 MeV yield (%)	R or σ(mb)
^{31}P Target			R
^{29}Si(1273-0)	1p 1n	3.4	4.9
^{28}Si(1779-0)	1p 2n	5.0	7.3
^{27}Al(1014-0)	2p 2n	2.0	3.8
^{26}Mg(1809-0)	3p 2n	4.0	4.9
^{25}Mg(585-0)	3p 3n	2.0	7.4
^{24}Mg(1369-0)	3p 4n	2.9	4.7
^{23}Na(440-0)	4p 4n	3.0	4.9
^{21}Ne(351-0)	5p 5n	1.1	3.4
^{16}O Target			σ(220 MeV)
^{13}C (3684-0)	2p 1n	1.3	<1.4
^{12}C (4440-0)	2p 2n	4.0	16.8
^{24}Mg Target			σ(220 MeV)
^{21}Ne(2789-351)	2p 1n	0.4	6.3
^{20}Ne(1634-0)	2p 2n	2.5	18.4
^{16}O (6131-0)	4p 4n	2.9	0.5
^{40}Ca Target			σ(220 MeV)
^{37}Ar(1410-0)	2p 1n	0.7	21.7
^{36}Ar(1970-0)	2p 2n	1.5	137.9
^{32}S (2230-0)	4p 4n	0.7^{a}	114.8
^{28}Si(1779-0)	6p 6n	<0.1	66.1
^{24}Mg(1369-0)	8p 8n	<0.2	36.2
^{20}Ne(1634-0)	10p10n	<0.1	27.4

[a] Large uncertainty from line shape.

may in principle be explained by the following reaction mechanisms:

(i) The initial pion interaction involves more than two nucleons. Indications for such processes have already been reported,[3,5] especially for the case of α-cluster absorption.

(ii) The pion is absorbed by two nucleons with high separation energies, thus producing highly excited two-hole states. These states decay via nuclear Auger effects with the emission of, preferentially, two more nucleons. Since in the first step the removal of a neutron-proton pair seems to be dominant, the subsequential emission of a second neutron-proton pair is likely.

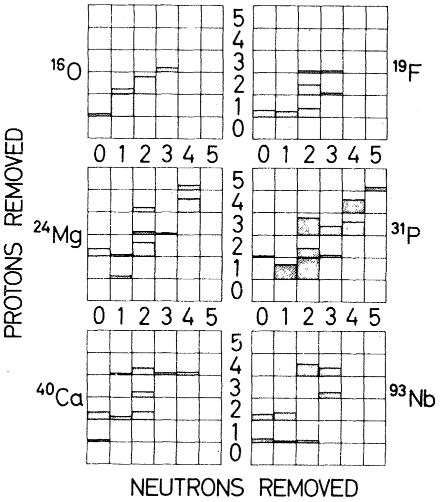

FIG. 2. γ-ray yields in residual nuclei from π^- absorption at rest, plotted versus the corresponding number of missing nucleons. Yields of the lowest observed γ-ray transition only (usually first to ground) are shown. An entire square corresponds to 5% per stopped pion.

(iii) The initial pion interaction leads to the emission of two loosely bound nucleons with high kinetic energy giving rise to an intranuclear cascade. Numerical calculations based on such a model show narrow Z distributions for residual nuclei of the same mass.[6]

All three mechanisms can finally lead to evaporation processes, where eventually α-particle emission is favored because of low thresholds. From our data, in fact, none of the three principal possibilities can be excluded.

The present results distinguish pion absorption from general interaction in flight for the first time in this class of experiments. The main qualitative result is the small variation between "zero energy" and 60 MeV, and the large change between 60 and 220 MeV. Recent results from a radiochemical study[7] of π^- interaction at 65 and 215–373 MeV on Cu also support the latter observation. Finally, according to our results at low pion energies, cluster removal is comparable to other reaction channels, while at higher energies (Refs. 1 and 2) the removal of α clusters seems to be dominant.

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Size of Barely Bound Many-Body Systems

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It is pointed out that, in contrast to the case of S-wave two-body bound states, the size of a many-body bound state remains finite (irrespective of its angular momentum) even if its binding energy vanishes. Physical implications for barely bound many-body systems are outlined.

The size of an S-wave two-body bound state diverges as the binding energy of the bound state vanishes.[1] This has an important physical implication, recognized long ago in connection with the first investigations of the deuteron[2]: The size of an S-wave barely bound state is approximately $1/(2\mu|E|)^{1/2} = 1/q$, where $\mu = m_1m_2/(m_1+m_2)$ is the reduced mass of the two-body system and E is its binding energy ($\hbar = c = 1$). Thus any (reasonable) model of the neutron-proton interaction that fits the deuteron binding energy yields a deuteron wave function having essentially the same extension, $\sim 1/q$ (much larger than the range r_0 of the interaction[3]). Therefore any additional information on the neutron-proton interaction, besides that implied by the value of the binding energy, can result only from measurements of the deuteron size that are sufficiently accurate to display corrections of order qr_0 relative to the dominant term.[2,4]

This phenomenon is peculiar to S-wave two-body bound states. In fact, the wave function of a two-body l-wave bound state is proportional, for $r \gg r_0$ (where r is the interparticle separation, and r_0 is the range of the forces[5]), to the (asymptotically vanishing) free solution of the l-wave Schrödinger equation, $(qr)^{-1/2}K_{l+1/2}(qr)$,[6] and therefore it becomes $c(2l-1)!!(qr)^{-(l+1)}$ for $r_0 \ll r \ll 1/q$ and $c(qr)^{-1}\exp(-qr)$ for $r \gg 1/q$, c being a constant whose value is determined by the behavior of the wave function for $r \lesssim r_0$, and by the normalization condition. It follows that, in the zero-energy case, the wave function is asymptotically proportional to $r^{-(l+1)}$, being therefore, for $l > \frac{1}{2}$, still normalizable, and implying that the expectation value of $r^{|p|}$ is finite for $|p| < p_0 = 2l-1$. Moreover, simple power counting shows that, for an l-wave two-body barely bound state (i.e., such that $qr_0 \ll 1$), the expectation value of $r^{|p|}$ is of order $r_0^{|p|}$ for $|p| < p_0$, $-r_0^{|p|}\ln(qr_0)$ for $|p| = p_0$, and $r_0^{|p|}(qr_0)^{p_0-|p|}$ for $|p| > p_0$. These estimates refer to the case $l > \frac{1}{2}$, i.e., when the normalization integral remains finite for $q = 0$; note that in all cases the result depends on r_0. For $l < \frac{1}{2}$ one finds instead, as a result of the divergence of the normalization integral for $q = 0$, that the expectation value of $r^{|p|}$, is of order $q^{-|p|}$, i.e., independent of the value of r_0 (provided $qr_0 \ll 1$).

These results display the exceptional nature of the two-body S-wave case, and imply that even rough measurements on two-body higher-wave barely bound states would yield more information on the forces than that conveyed by the binding-energy value—in contrast to the S-wave case.

This property of higher-wave barely bound states originates from the normalizability of the bound-state wave function in the zero-energy case, or, equivalently, from the finite size of the zero-energy bound state. The purpose of this Letter is to point out that, in analogy to the two-body higher-wave case, and in contrast to the