Systematics of Nuclear Single-Particle States

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Semiempirical model-independent formulas for single-particle energies of neutrons and protons in bound nuclei have been obtained as functions of nuclear parameters A and Z for given states specified by nlj. These formulas are almost as convenient as the harmonic oscillator energy formulas to use. The single-particle energies computed from these formulas have been compared with the experimental data and found in good agreement for occupied states.

In several recent works, various models have been used to obtain the energy levels of nuclei along the line of β stability and have yielded extensive information on the energies of single-particle states.[1-5] These energies seem to follow some regular patterns from nucleus to nucleus. Recently, Millener and Hodgson[1] have shown that the energies of 2s single-particle states vary in a regular and systematic way from nucleus to nucleus in the range $35 \leqslant A \leqslant 65$.[1] One would naturally think as a next step that a simple formula for the single-particle energies would be more convenient to use, for example, as an initial energy guess for an energy-dependent potential. The purpose of this study is to develop model-independent, semiempirical formulas for the single-particle energies of states for the entire range of nuclei.

The semiempirical formulas for these single-particle energies are in terms of the nuclear parameters $(A,Z;n,l,j)$. Various combinations of these parameters have been tested for their correlations with the single-particle energies. After a reasonably complete set of variables has been generated, the problem is to find the coefficients of the significant variables. This is accomplished by using a stepwise multiple linear regression analysis.[6] Only linear combinations of the variables in the given set are used in the analysis. Coefficients for the most significant terms are computed first, followed by the next most significant terms in the descending order of significance. The constants are reevaluated on the addition of each term until there are no more variables left or until the significance of the remaining variables is below some specified value. Single-particle neutron and proton energies for nuclei along the β-stable line have been used as the input data[5] in the regression analysis. The single-particle energy states up to the Fermi level have been used in this analysis including 352 states for neutrons and 250 states for protons. The resulting model-independent, semiempirical energy formulas for neutron and proton states, given by

$$E_n(\mathrm{MeV}) = -94.902 + 70.838l/A^{1/3} + 17.951n + 90.000n/A^{1/3} - 18.038j(j+1)/A^{2/3} - 2.572n^2 + 43.285l(l+1)/A\,, \tag{1}$$

and

$$E_p(\mathrm{MeV}) = -95.020 + 82.382l/A^{1/3} + 24.020n + 76.284n/A^{1/3} - 17.540j(j+1)/A^{2/3} - 2.954n^2, \tag{2}$$

have multiple correlation coefficients 0.9938 and 0.9956, respectively. The first six terms of E_n and E_p are the same, the neutron-proton differences being reflected in the values of the coefficients. The first two variables of Eqs. (1) and (2) involving the terms $l/A^{1/3}$ and n as the predominant terms are about equally significant and they account for the major effects, with their multiple correlation coefficient $\simeq 0.92$.

The constant terms of Eqs. (1) and (2) place the energies at a very negative value. The next three terms ($+l/A^{1/3}$, $+n$, and $+n/A^{1/3}$) can be attributed to harmonic oscillator effects and give the required increase (decrease in magnitude) in energy as n and/or l increases. The term $-j(j+1)/A^{2/3}$ gives the spin-orbit splitting proportional to $2l+1$, exhibiting the proper doublet sequence, and also shows the reduced spin-orbit effect as A increases. The n^2 term illustrates a slight nonlinearity in the principal quantum number. The $l(l+1)/A$ term in Eq. (1) accounts for a residual

TABLE I. Single-particle energies (MeV) of protons. Question marks indicate level assignment is uncertain.

State	Cal.	Exp.
^{16}O		a
$1s_{1/2}$	45.75	44±7
$1p_{3/2}$	21.35	19.1±1.4
$1p_{1/2}$	13.06	12.7±1.4
^{32}S		a
$1s_{1/2}$	51.23	---
$1p_{3/2}$	30.50	(p) 29.7±6.1?
$1p_{1/2}$	25.28	
$1d_{5/2}$	13.26	16.2±1.6
$1d_{3/2}$	4.55	13.3±1.6
$2s_{1/2}$	12.05	9.5±1.4
^{40}Ca		b
$1s_{1/2}$	52.77	49.1±12 77±14
$1p_{3/2}$	33.18	33.3±6.5
$1p_{1/2}$	28.68	32±4^{c}
$1d_{5/2}$	16.59	14.9±2.5 13.8±7.5
$1d_{3/2}$	9.09	8.4±0.5 7.7±2.6^{c}
$2s_{1/2}$	15.31	10.6±1.1^{a} 12.1±5.4
^{58}Ni		b
$1s_{1/2}$	55.12	57.3±7.5?
$1p_{3/2}$	37.35	(p) 37.6±7.9
$1p_{1/2}$	33.84	---
$1d_{5/2}$	21.92	(d) 20.2±9.8
$1d_{3/2}$	16.07	11.72^{3,f}
$2s_{1/2}$	20.26	12.6$^{+6.3}_{-2.7}$ 11.19^{e,f}
$1f_{7/2}$	8.84	10.1±5.6
^{120}Sn		b
$1s_{1/2}$	59.03	54.0±8.1?
$1p_{3/2}$	44.49	(1p)40.2±7.2?
$1p_{1/2}$	42.33	---
$1d_{5/2}$	31.39	(d) 30.0±6.9
$1d_{2/2}$	27.79	---
$2s_{1/2}$	28.41	29.0±6.8
$2p_{3/2}$	13.87	16±6.3
$2p_{1/2}$	11.70	---
$1f_{7/2}$	19.74	---
$1f_{5/2}$	14.69	(1f+1g)16±6.8
$1g_{7/2}$	9.52	---
^{208}Pb		g
$1g_{7/2}$	13.33	11.45
$2d_{5/2}$	9.61	9.65
$1h_{11/2}$	9.42	9.30
$2d_{3/2}$	7.11	8.30
$3s_{1/2}$	11.30	7.95 9.0^{b}
$1h_{9/2}$	3.92	3.8
$2f_{7/2}$	+0.80	2.9
$1i_{13/2}$	2.01	2.2
$2f_{5/2}$	+4.29	0.8^{h}
$3p_{3/2}$	+1.11	0.5^{h}

[a] Quoted in Ref. 7.
[b] Ref. 8.
[c] Ref. 9.
[d] Quoted in Ref. 10.
[e] Quoted in Ref. 11.
[f] Ref. 11.
[g] Quoted in Ref. 12.
[h] Quoted in Ref. 13.

TABLE II. Single-particle energies (MeV) of neutrons. The experimental values are taken from Ref. 12.

State	Calc	Expt
	^{16}O	
$1s_{1/2}$	45.94	47.00
$1p_{3/2}$	20.94	22.00
$1p_{1/2}$	12.41	15.70
	^{40}Ca	
$1d_{5/2}$	18.78	21.30
$2s_{1/2}$	17.81	18.20
$1d_{3/2}$	11.07	15.80
	^{208}Pb	
$1h_{9/2}$	11.03	10.70
$2f_{7/2}$	8.64	9.50
$1i_{13/2}$	8.91	8.90
$3p_{3/2}$	8.18	8.20
$2f_{5/2}$	5.04	7.80
$3p_{1/2}$	6.64	7.30
$4s_{1/2}$	3.88	1.91

effect in the orbital angular momentum not taken care of by the $l/A^{1/3}$ term.

The nuclear systematics obtained from Eqs. (1) and (2) show the distinctiveness of the single-particle states, the increase in binding energies as A increases, and the tendency to level off for deeply bound states in heavy nuclei, thus exhibiting the saturation property of nuclear forces. Both the neutron and proton systematics show gaps in the shell structure at all the observed magic numbers. Gaps are also exhibited for all the semimagic numbers in the appropriate region.

Calculated and experimental single-particle energies are listed in Tables I and II. There is generally a reasonable agreement with the experimental data.[7-12] Nevertheless, it seems appropriate to make a few remarks in general on the discrepancies that one may observe between the results of this work and the experimental data compared with them. They are partly due to the errors associated with the experimental data themselves. The calculated levels are generally too low for unoccupied or partially occupied levels. This effect may be attributed to the input data which were used for the occupied state primarily. Nevertheless, the semiempirical formulas for single-particle energies given by Eqs. (1) and (2) give an overall good description of the bound-state spectrum for occupied states in nuclei and these formulas are almost as convenient to use as ones belonging to the harmonic oscillator potential.

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Multinucleon Removal in the Absorption of π^- at Rest and at 60 MeV*

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The intensities of γ rays from π^- absorption at rest on O, F, Mg, P, Ca, and Nb have been measured. The results are compared with those at 220 MeV, previously published. The α removal processes which have been reported to dominate at higher energies are not as pronounced in the present work.

Recent studies on the interaction of 220-[1] and 380-MeV[2] negative pions with nuclei have shown unexpectedly large cross sections for integral "α-particle" (i.e., multiples of two neutrons + two protons) removal. This is surprising, either as an intrinsic property of pion interaction, or from the point of view of nuclear structure. The measurements have been made via the observation of de-excitation γ rays in the final nucleus, so that it is not possible to distinguish the course of the preceding sequence of particle emissions, or even their identities (single nucleons or heavier particles). From the design of these experiments there has also been no distinction made between absorption and nonabsorption processes.

The purpose of this Letter is to present results of a similar study with π^- at rest for a range of nuclei and a case of π^- interaction at 60 MeV kinetic energy. In this study the pion interaction is clearly an absorption process for π^- at rest. Because of the experimental conditions, the results for π^- at 60 MeV are also due predominantly to absorption.

The measurements were done with a negative pion beam from the CERN synchrocyclotron. Targets with 5 g/cm^2 nominal thickness of natural H_2O, LiF, Mg, P, Ca, and Nb were used in a standard stopped-pion experimental arrangement as described earlier.[3] γ rays were detected by a 40-cm^3 Ge(Li) detector (at 90° to the beam direction) in coincidence with stopped-pion logic $(1,2,3,\bar{4})$ from the beam telescope. The absolute yields per stopped pion were determined by comparison with muonic x rays.[3] In Fig. 1 (upper curve), as an example, the prompt γ-ray spectrum from pions absorbed at rest on ^{31}P is shown, with an energy resolution of 3 keV (full width at half-maximum) at 1 MeV. Background lines determined in a separate measurement are indicated by arrows below the spectrum.

For the measurement with pions in flight the same arrangement has been used. Since counter 4 was close behind the target, and both were tilted by 45° with respect to the beam direction, a $(1,2,3,+4)$ trigger condition covers all processes where the pion is scattered in the forward direction, and to some extent also pions with larger scattering angles. In a phosphorus target measurement made under this trigger condition, none of the γ-ray lines from residual nuclei present in the upper curve of Fig. 1 were observed. The γ spectrum taken under the complementary condition $(1,2,3,\bar{4})$, however, shows a striking similarity to the spectrum obtained with π^- at rest from the same target (Fig. 1). The drastic reduction of the pionic x-ray yield from phosphorus in the lower curve proves that the contribution from pions at rest is negligible in this spectrum. In Table I the ratios of intensities of corresponding lines in the two spectra are listed, and are seen to be similar within about a factor of 2. This indicates that absorption at rest and at 60 MeV are similar processes. Also, since the order of magnitude of the 60-MeV absorption cross sections is 10 mb, considerations[4] which neglect absorption at nonzero pion energies are questionable.