Upper Bounds on Massive-Lepton-Pair Production, $pN \to l^+ l^- X$, in the Quark-Parton Model

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We present rigorous bounds on the process $pN \to l^+ l^- X$ derived in the context of quark-parton models. As constraints we use the positivity of the parton probability functions and data on deep inelastic leptoproduction. Generally, our upper bounds fall below the Brookhaven National Laboratory data on muon-pair production. This result forces us to conclude that the Brookhaven data do not represent the scaling limit for this process, and/or that the usual ideas of the (colored) quark-parton model are not all correct.

The intriguing successes of parton models for processes involving large spacelike momentum transfers, especially deep inelastic lepton scattering, have been well demonstrated over the past several years. In stark contrast are the results on e^+e^- annihilation from the Cambridge Electron Accelerator, and more recently from SPEAR. These results are in gross disagreement with parton-model expectations. Another process which has been the subject of much theoretical and experimental study is the production of massive lepton pairs in hadronic collisions. This process, first discussed in the parton model by Drell and Yan,[1] is supposed to occur when a parton from one incident hadron annihilates with an antiparton from the other incident hadron, producing a heavy photon which finally decays into a lepton pair. The cross section for this process in proton-proton collisions is given by

$$Q^4 d\sigma/dQ^2 = \tfrac{4}{3}\pi\alpha^2\tau \int_\tau^1 (dx/x) \sum_i e_i^2 [q_i(x)\bar{q}_i(\tau/x) + \bar{q}_i(x) q_i(\tau/x)], \quad (1)$$

where $\tau = Q^2/s$, e_i is the charge of parton of type i, and $q_i(x)$ $[\bar{q}_i(x)]$ is the probability of finding a parton (antiparton) of type i with a fraction x of the parent proton's longitudinal momentum.

The rates for this process as measured at Brookhaven National Laboratory (BNL) by Christensen *et al.*[2] have generally been larger than theoretical estimates, but no firm conclusions about whether the parton model could describe these results have been drawn. In this Letter we present some rigorous bounds on this cross section and compare them with the BNL data. The bounds are based on the positivity of the parton probability distributions and the constraints on these distributions imposed first by the Stanford Linear Accelerator Center (SLAC) electroproduction data[3] and, later, by the neutrino experiments at Gargamelle.[4] The probability distributions are regarded as generalized coordinates in a variational problem, and constraints are imposed by using the method of Lagrange multipliers generalized to incorporate inequality constraints.[5] The positivity requirements are

$$q_i(x), \bar{q}_i(x) \geq 0, \quad (2)$$

for all x and i.

In the first problem that we consider, we use only the data from SLAC on νW_2 for protons and neutrons.[3] These data are quite good, and so our bound will provide a very clean test of the parton model. In the parton model these structure functions are given by certain linear combinations of the parton probability distributions. Specifically, if the partons are quarks and carry, in addition, SU(3) color, we have

$$\nu W_2^{\gamma p}(x)/x = \tfrac{4}{9}U(x) + \tfrac{1}{9}D(x) + \tfrac{1}{9}S(x),$$

$$\nu W_2^{\gamma n}(x)/x = \tfrac{4}{9}D(x) + \tfrac{1}{9}U(x) + \tfrac{1}{9}S(x),$$

where

$$U(x) = u_r(x) + \bar{u}_r(x) + u_w(x) + \bar{u}_w(x) + u_b(x) + \bar{u}_b(x),$$

and similarly for $S(x)$ and $D(x)$. The subscripts refer to the color degrees of freedom.

When combined with the inequalities (2), these constraints provide rather severe restrictions on the cross section (1). The full derivation of this bound is too long to describe here, and will be presented in detail (along with a number of additional results) elsewhere.[6] The result is shown and compared with the BNL data in Fig. 1. Curve A is the bound computed with the constraints described for the process $pU \to \mu^+\mu^- X$, since the target used was uranium. That is, it is a weighted average of the upper bounds for $pp \to \mu^+\mu^- X$ and $pn \to \mu^+\mu^- X$. Curve B is the same as curve A, corrected to incorporate the detection efficiency of the BNL experiment, and curve C is the 29.5-GeV/c data from BNL.[7] For $0.2 \lesssim Q^2/s \lesssim 0.5$ the

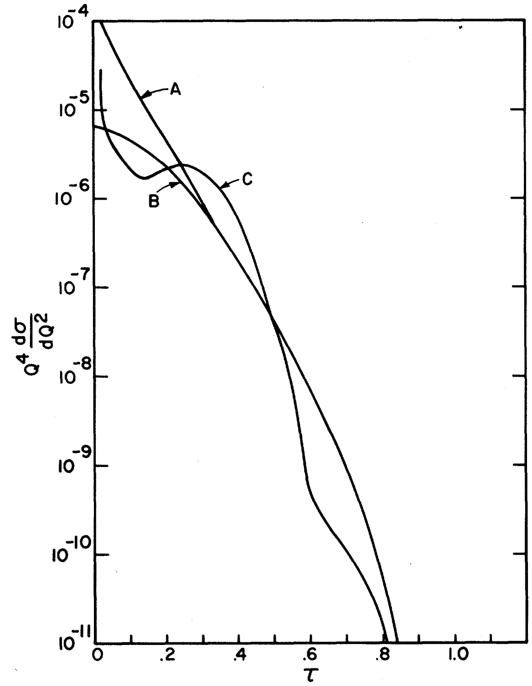

FIG. 1. Curve A, upper bounds for the process $pU \to \mu^+\mu^- X$ calculated as a weighted average of bounds on $pp \to \mu^+\mu^- X$ and $pn \to \mu^+\mu^- X$. Curve B, same bound corrected to include detection efficiency at BNL. Curve C, 29.5-GeV/c data from BNL.

upper bound falls significantly below the data proving that the colored-quark-parton model cannot possibly describe the result.

Corresponding bounds for the quark-parton model without color are obtained by multiplying curves A and B by a factor of 3. Even in this theory, the data come dangerously close to violating the bounds.

In the problem considered above, the q_i and $\bar{q}_i$ distributions enter symmetrically. However, for nonzero x, the usual notions of the parton model suggest that there is a significant difference between the quark and antiquark distributions. To implement this difference in our variational problem, we turn to a consideration of neutrino scattering. Using the preliminary data on neutrino-nucleon scattering from Gargamelle,[4] and the SLAC data on νW_2 for protons and neutrons, one can deduce the four different linear combinations of quark probability distributions shown in Fig. 2(a). In the colored-quark-parton model, the labels of the curves in this figure should be understood as sums over color, e.g., $u(x)=u_r(x)+u_w(x)+u_b(x)$, etc. There are many reasons to be skeptical about the preliminary data from Gargamelle, and their interpretation in terms of parton distributions (for instance, most of the data are not at large enough Q^2 to expect scaling), but at the very least, they are probably indicative of what cleaner data will show. Interpolating these points and modifying slightly the curves at small Q^2 (i.e., x),[8] we take as our "data" the curves of Fig. 2(b). A detailed discussion of the Gargamelle data and further elaboration of Fig. 2(b) may be found in Ref. 6.

We can now use these data, which distinguish between quarks and antiquarks, as constraints in a new variational problem. Again we impose the positivity requirements (2), and maximize the functional. The result of this tedious, but straightforward, calculation[6] is shown in Fig. 3,

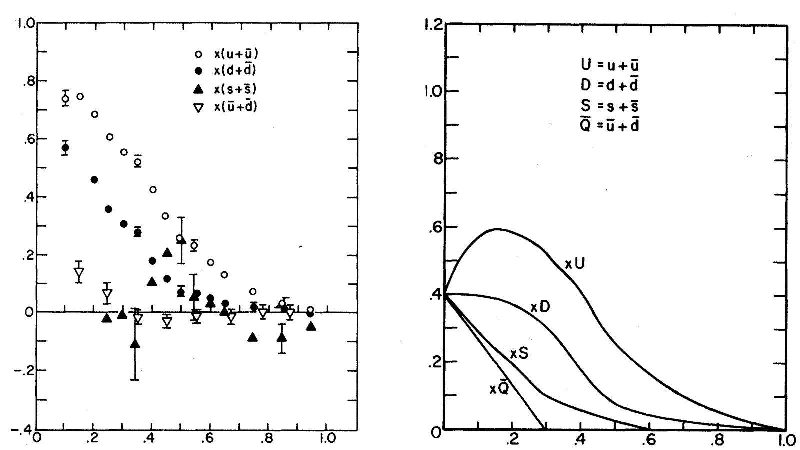

FIG. 2. (a) Parton distribution functions gleaned from SLAC and Gargamelle data. (b) Modified version of (a) (see text).

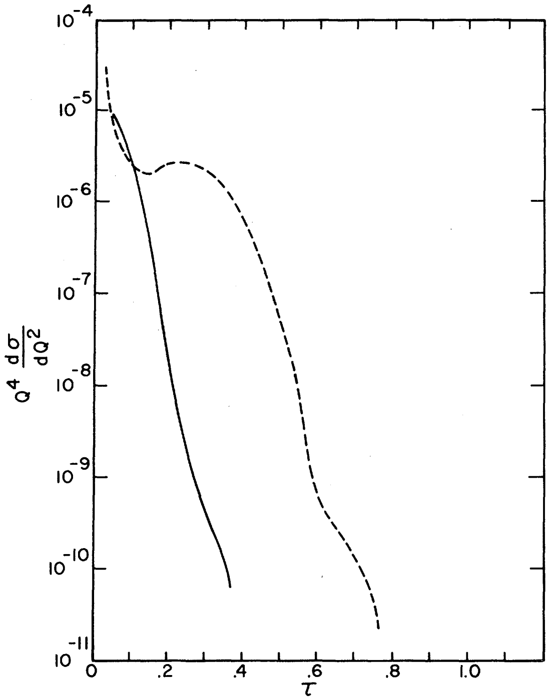

FIG. 3. Upper bound derived using data of Fig. 2 (solid line) compared with BNL data (dashed line).

where it is compared with the BNL data.[9] Again, the upper bound falls far below the data leading to the conclusion that the parton model and the BNL data are incompatible. As before, the same bound derived in the uncolored-quark model is precisely 3 times as large as the bound of Fig. 3.

One might worry that, since the Gargamelle data have such large error bars, and since the process in question is so sensitive to the size of the antiquark distributions at large x, the bound of Fig. 3 could change drastically within the error bars of the data. However, model studies[6] indicate that while the upper bound does change significantly when one changes the data, it is still at least 1 or 2 orders of magnitude smaller than the data for $0.2 \lesssim \tau \lesssim 0.6$. In any event, no matter what the neutrino data eventually turn out to be, the bound can never rise above that of Fig. 1 which depends only on electron scattering and makes no assumptions about the weak interactions. We also want to stress that the method of solution for this problem, which is given in Ref. 6, does not depend on the form of the data in Fig. 2(b) and can easily be applied to other data when they become available.

These bounds, which are perfectly rigorous in the context of the usual parton models, force us to conclude that at least one of the following statements is correct. (i) The BNL data for the experimentally observed cross section per nucleon are not the scaling limit for the process $pN \to \mu^+\mu^- X$. (ii) The Drell-Yan formula for $pp \to \mu^+\mu^- X$ must be modified. (iii) The colored-quark-parton model is wrong.

Let us briefly comment on each possibility:

(i) Nuclear effects which were thought to be well understood may, in fact, significantly alter the observed cross section per nucleon. Inasmuch as experiments on hydrogen will not be available for quite some time, an experiment utilizing different nuclear targets is urgently needed. Another effect which may be important at large τ is the interactions of secondary mesons produced in the target. Since mesons are supposed to have many more antiquarks at large x than baryons, this effect may be important in a thick target such as that at BNL. It must be remarked, however, that model-dependent estimates suggest that this contribution is still at least an order of magnitude below the observed signal.[10] Finally, it is possible that the Brookhaven experiment is not the scaling limit for this reaction. Similar experiments at other energies such as those being planned and carried out at the National Accelerator Laboratory are obviously of great interest.

(ii) Theoretically, the most unsatisfactory aspect of the Drell-Yan formula is its apparent neglect of strong interactions. Such interactions are generally required in deep inelastic phenomena, especially in a quark-parton model, to dispose of the isolated quark quantum numbers. However, these effects may leave the parton-model results for deep inelastic lepton scattering unchanged, but alter the predicted cross sections for processes, such as the one discussed here, which involve more than one hadron in both the initial and final states.[11] Our results make this fundamental and unresolved question that much more salient.

(iii) The colored-quark-parton model may be incorrect in one or both of two ways. On the one hand, the correct parton model may have constituents other than colored quarks. For such theories, bounds similar to those presented here can be derived. In general, we can anticipate that the greater the mean squared charge, the smaller the bound will be, so the situation will be worse in most such models. On the other hand, the ideas of the naive parton model may not be applicable in their present form to processes involving timelike photons. [This possibility is related to option (ii), discussed above.] Indeed, it is intriguing that in the two large-Q^2 processes involving timelike photons which have been measured—e^+e^- annihilation and massive-muon-pair production—the observed cross sections are larger than the predictions of the parton model.

The ideas of the parton model, which have worked so well in describing processes with large spacelike momentum transfers, seem to fail, in their present form, in the timelike region. We have shown in this paper, by presenting rigorous bounds in the context of parton models, that the most popular versions cannot describe the BNL data on $pN \to \mu^+\mu^- X$. The Cambridge Electron Accelerator and SPEAR results on e^+e^- annihilation, and the present bounds on massive-lepton-pair production, make it extremely important to carry out further experiments in the deep timelike region, and to focus our theoretical attention on the exciting paradoxes presented by the parton model.

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[2]J. H. Christensen *et al.*, Phys. Rev. D 8, 2016 (1973).

[3]A. Bodek, Ph.D. thesis, Massachusetts Institute of Technology, MIT Laboratory for Nuclear Studies Report No. COO-3069-116, 1972 (unpublished); E. M. Riordan, Ph.D. thesis Massachusetts Institute of Technology, MIT Laboratory for Nuclear Studies Report No. COO-3-69-176, 1973 (unpublished).

[4]D. H. Perkins, Oxford University Report No. Ref. 67-73 (to be published).

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[6]M. B. Einhorn and R. Savit, NAL Report No. NAL-Pub-74/35-THY (to be published).

[7]In presenting the data, we have drawn a smooth curve through the data points and have not indicated statistical errors, which range between 5 and 25% for $0.02 \le \tau \le 0.45$ and are larger for larger τ. However, the dominant errors are systematic, ranging between 25 and 65% for $0.02 \le \tau \le 0.61$, and are correlated from point to point. Consideration of these errors does not vitiate our conclusions. We also point out that the difference between calculating the average of the upper bounds on p and n targets and the upper bounds on the average nucleon in uranium is negligible for our purposes. For further details, see Ref. 6.

[8]The modification is desirable for several reasons: for example, to make the probability distributions nonpositive.

[9]Actually, the curve in Fig. 3 is a bound for the process $pp \to \mu^+\mu^- X$, not $p\mathrm{U} \to \mu^+\mu^- X$ as in Fig. 1. However, the bounds for proton and neutron targets typically differ by a factor of 2 or so, and for the purpose of Fig. 3 this difference is insignificant. See Ref. 6 for more details.

[10]G. Farrar and G. C. Fox, private communication.

[11]Different points of view on this question are suggested by R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, New York, 1972), p. 152; and P. V. Landshoff and J. C. Polkinghorne, Nucl. Phys. B33, 221 (1971). See also Ref. 6 above.