Evidence for a New Scaling Hypothesis in High-Energy Collisions*

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Single-particle distributions in transverse and longitudinal momenta have been studied in pp interactions at 300 GeV/c. By comparing these distributions for different multiplicities of produced particles and with published low-energy data, we put forth the hypothesis of a new type of scaling: The shape of distributions in p_T and p_L are independent of multiplicity and incident energy.

The scaling behavior of a distribution $f(x_1, x_2)$ with independent variables x_1 and x_2 can be defined in terms of a homogeneous equation,

$$f(\lambda_1 x_1, \lambda_2 x_2) = \lambda f(x_1, x_2). \quad (1)$$

The distribution f scales if there exist scaling parameters λ_1, λ_2, and λ for which Eq. (1) is satisfied. For the special choice $\lambda_1 = 1/\langle x_1\rangle$, $\lambda_2 = 1/\langle x_2\rangle$, and $\lambda = \langle x_1\rangle\langle x_2\rangle$, we obtain the scaling equation

$$f(x_1, x_2) = \frac{1}{\langle x_1\rangle\langle x_2\rangle} f\left(\frac{x_1}{\langle x_1\rangle}, \frac{x_2}{\langle x_2\rangle}\right). \quad (2)$$

We propose a new hypothesis of scaling for high-energy collisions: Distributions resulting in multiparticle production at high energies satisfy Eq. (2) when $x_1 = p_T$ and $x_2 = p_L$. In other words, the distributions maintain their shapes if they are plotted against the *mean* variable $x_i/\langle x_i\rangle$ and properly normalized. The energy, initial state, and multiplicity dependencies lie in the average value $\langle x_i\rangle$. It can be shown that Eq. (2) maintains its form when it is integrated over either one of the variables.

In this Letter we present an analysis of the inclusive single-particle distributions in the transverse (p_T) and the longitudinal (p_L) momentum variables in the c.m. system. These events were produced by 300-GeV/c protons in the 30-in. hydrogen bubble chamber at the Fermi National Accelerator Laboratory. A sample of 877 events (see Table I) with prong multiplicity as high as 26 prongs were track matched in three views by physicists and scanners on the high-magnification (×72) Fermi-Lab-designed "MOMM" tables using the bubble pattern of the charged track. These events were then measured and processed through the program TVGP. In this study we restricted ourselves to the negatively charged secondary tracks, to which we have assigned the pion mass. Based on our previous determination of the $K_S{}^0$ inclusive cross section[2] at this energy, we esti-

TABLE I. Events from 300-GeV/c pp interactions used in this analysis.

Prong multiplicity n_c	Number of events[a]	$\langle p_T\rangle$ (GeV/c)	$\langle p_L\rangle$ (GeV/c)
4	515	0.34 ± 0.02	0.93 ± 0.04
6	246	0.35 ± 0.02	0.72 ± 0.03
8	69	0.36 ± 0.02	0.64 ± 0.05
20	17	0.33 ± 0.02	0.41 ± 0.04
22	17	0.31 ± 0.02	0.43 ± 0.04
24	7	0.28 ± 0.02	0.44 ± 0.04
26	6	0.33 ± 0.03	0.41 ± 0.06

[a]To increase statistics, we have in our subsequent analysis combined these events into two groups, $4 \leq n_c \leq 8$ and $20 \leq n_c$, and properly weighted them according to the topological cross sections (see Ref. 1).

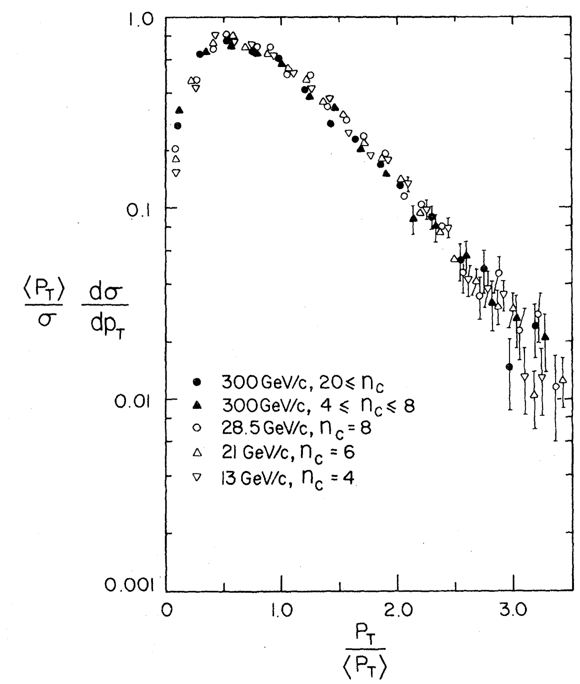

FIG. 1: Plot of $(\langle p_T\rangle/\sigma)d\sigma/dp_T$ versus $p_T/\langle p_T\rangle$ for the reactions $pp\rightarrow\pi^-$. Low-energy data at 13, 21, and 28.5 GeV/c are from Ref. 3.

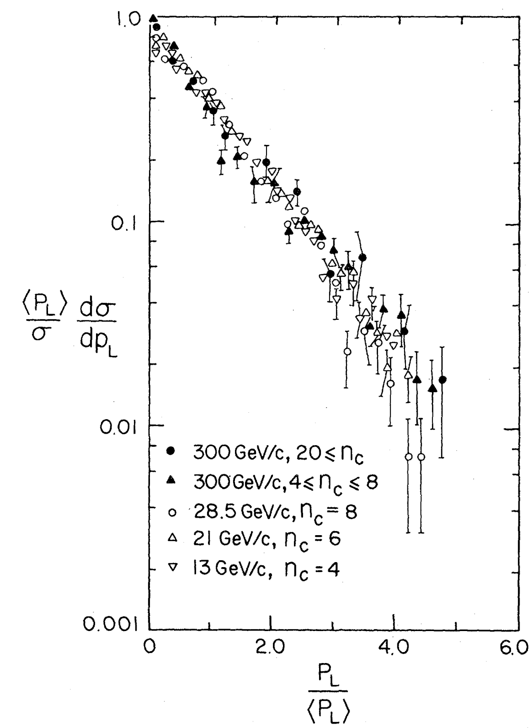

FIG. 2: Plot of $(\langle p_L\rangle/\sigma)d\sigma/dp_L$ versus $p_L/\langle p_L\rangle$ for $pp\rightarrow\pi^-$. Low-energy data at 13, 21, and 28.5 GeV/c are from Ref. 3.

mate our kaon contamination as about 10% of the negatively charged tracks.

According to the hypothesis mentioned above, we should test the following scaling equation:

$$\frac{1}{\sigma}\frac{d^2\sigma}{dp_T dp_L}=\frac{1}{\langle p_T\rangle\langle p_L\rangle}\,\varphi\left(\frac{p_T}{\langle p_T\rangle},\frac{p_L}{\langle p_L\rangle}\right), \qquad (3)$$

where φ is a function independent of incident energy, initial states, and prong multiplicity.

In Fig. 1 we examine the π^- spectra in transverse momentum from pp interactions at 13,[3] 21,[3] 28.5,[3] and 300 GeV/c by plotting $(d\sigma/dp_T)\times\langle p_T\rangle/\sigma$ versus $p_T/\langle p_T\rangle$. We observe that the data are remarkably similar and strongly hint at a universal function despite the fact that a wide range of incident momenta and prong multiplicities were selected.

In Fig. 2 we plot $(d\sigma/dp_L)\langle p_L\rangle/\sigma$ versus $p_L/\langle p_L\rangle$ (where the forward and backward hemispheres are folded together) and show again that the data are consistent with a universal function. We note that if these data are plotted in the Feynman variable x $(=2p_L/\sqrt{s}$, where $\sqrt{s}$ is the c.m. energy) instead of the mean variable $p_L/\langle p_L\rangle$, the distributions will not scale for such diverse prong multiplicities as we have presented in Fig. 2. The fact that the shapes of the distributions are similar leads us to speculate that (a) there is one dominant production mechanism in high-energy collisions, and (b) dependences in incident energy, initial state,[4] and prong multiplicity are manifested only in the average values.

To show that the data presented in Figs. 1 and 2 follow a universal distribution, we fitted the data to two functions suggested by the thermodynamic model,[3,5]

$$\varphi_T=a\left(\frac{p_T}{\langle p_T\rangle}\right)^c\exp\left[-b\left(\frac{p_T}{\langle p_T\rangle}\right)\right], \qquad (4)$$

$$\varphi_L=d\exp\left[-e\left(\frac{p_L}{\langle p_L\rangle}\right)-f\left(\frac{p_L}{\langle p_L\rangle}\right)^2\right], \qquad (5)$$

and obtained the following results: $a=6.23\pm0.52$, $b=2.37\pm0.04$, $c=1.37\pm0.03$, $d=0.91\pm0.15$, $e=0.83\pm0.04$, and $f=0.03\pm0.01$. The χ^2 per degree of freedom are 104/109 and 134/100, respectively.[6] If the average values are not divided out, the shapes of the distributions are different. To test this point, we fitted the same data without dividing them by the average value [namely, the distributions $(d\sigma/dp_T)/\sigma$ versus p_T and $(d\sigma/dp_L)/\sigma$ versus p_L] and by using the same functional form given by Eqs. (4) and (5). The results

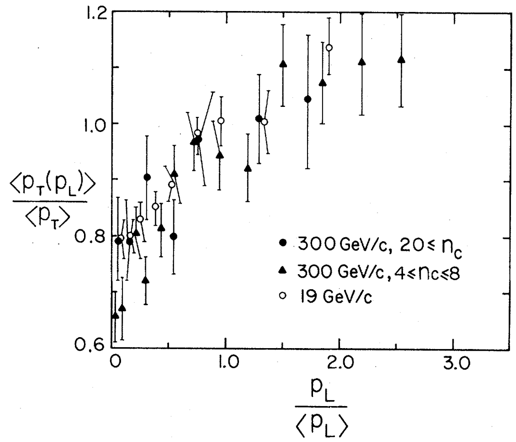

FIG. 3: Plot of $\langle p_T(p_L)\rangle/\langle p_T\rangle$ versus $p_L/\langle p_L\rangle$, where $\langle p_T(p_L)\rangle \equiv \int_0^\infty (d^2\sigma/dp_T\,dp_L)p_T\,dp_T/\int_0^\infty(d^2\sigma/dp_T\,dp_L)dp_T$, for $pp\to\pi^-$. Data at 19 GeV/c are from Ref. 6.

were poor and we obtained χ^2 of 757 and 1509, respectively.

The correlation between p_T and p_L can be studied in terms of the ratios of moments $\langle p_T{}^m p_L{}^n\rangle/\langle p_T\rangle^m\langle p_L\rangle^n$, which are constants according to Eq. (3). We examine the first moment of this correlation. From Eq. (3) one can derive

$$\langle p_T(p_L)\rangle \equiv \frac{\int_0^\infty p_T(d^2\sigma/dp_T\,dp_L)dp_T}{\int_0^\infty(d^2\sigma/dp_T\,dp_L)dp_T} = \langle p_T\rangle g(p_L/\langle p_L\rangle). \qquad (6)$$

In Fig. 3 we plot 300-GeV/c data and data at 19 GeV/c.[7] The figure shows that within the present statistics Eq. (6) is satisfied. Thus the shape of the well-known "seagull effect" (p_T and p_L correlation) as given by $g(p_L/\langle p_L\rangle)$ is also shown to be independent of energy and prong multiplicity.

Other variables can be used in Eq. (2). For example, if $x_1=n_c$, the number of charged particles, and $x_2=n_0$, the number of neutral particles, then we have the familar Koba-Nielsen-Olesen (KNO) scaling.[8] It may further be possible to generalize Eq. (2) to more than two variables.

In summary we note the following:

(i) The scaling hypothesis proposed in this Letter should be regarded as an attempt, without resorting to specific theoretical models, to summarize the bulk of the pion production data presently available from high-energy hadron collisions.

(ii) Insofar as these distributions are universal and independent of energy and initial state, we conclude that there is one dominant mechanism for pion production at high energies and the process can be described in terms of a few parameters. One such candidate is the thermodynamic model in which a very important concept (parameter) is the temperature of the hadron system.

(iii) The scaling hypothesis makes no mention of the energy dependence of the average value in the distribution. Consequently, one can study this dependence on energy and initial state as a separate problem (for example, to see whether $\langle p_T\rangle$ increases logarithmically or as a power).

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[1]A. Firestone *et al.*, Phys. Rev. D (to be published).

[2]F. T. Dao *et al.*, Phys. Rev. Lett. 30, 1151 (1973).

[3]D. B. Smith, Ph. D. thesis, Lawrence Berkeley Laboratory Report No. UCRL-20632, 1971 (unpublished). See also D. B. Smith *et al.*, Phys. Rev. Lett. 23, 1064 (1969).

[4]There are no published data on single-particle distributions from high-energy (>100 GeV/c) πp interactions. However we have studied the initial-state dependence of the functions φ_T and φ_L given in Eqs. (4) and (5), using the 25-GeV/c π^-p data of J. W. Elbert *et al.* [Phys. Rev. Lett. 20, 124 (1968)] and J. W. Elbert [Ph D. thesis, University of Wisconsin, 1971 (unpublished)]. We found that the π^+ (nonleading particle) spectra in this case have the same functional dependence as the π^- spectra from pp collisions. Since in πp collisions there is an asymmetry in p_L between forward and backward hemispheres, we scale p_L in each hemisphere by $\langle p_L\rangle$, the average value in that hemisphere.

[5]R. Hagedorn, Nuovo Cimento 25, 1017 (1962).

[6]The use of the ratios of moments may be a more sensitive test of this scaling hypothesis and possible systematic deviation from it. For example, the ratio of moments $\langle p_L\rangle/(\langle p_L{}^2\rangle-\langle p_L\rangle^2)^{1/2}$ has the following values: 0.98 ± 0.04 (300-GeV/c pp, $n_c\geq 20$), 1.10 ± 0.02 (300-GeV/c pp, $2\leq n_c\leq 10$), 1.16 ± 0.01 (13-GeV/c pp, $n_c=4$), 1.11 ± 0.01 (21-GeV/c pp, $n_c=6$), and 1.14 ± 0.02 (28-GeV/c pp, $n_c=8$).

[7]H. Bøggild *et al.*, Nucl. Phys. B57, 77 (1973).

[8]Z. Koba, H. B. Nielsen, and P. Olesen, Nucl. Phys. B40, 317 (1973). For an experimental review of KNO scaling, see P. Slattery, Phys. Rev. D 7, 2073 (1973); F. T. Dao and J. Whitmore, Phys. Lett. 46B, 252 (1973), and references therein. Of course KNO scaling is used here in the general sense where some modified scaling law of the same general type may be more applicable, for example, with Wroblewski's modification $x_1=n_c-1$ instead of $x_1=n_c$.