Remarks on Dirac's New Theory

J. L. Pietenpol, R. Incoul, and D. Speiser
Institut de Physique Théorique, Université Catholique de Louvain, B-1348 Louvain-la-Neuve, Belgium
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We show that Dirac's proposed scalar-tensor theory of gravitation and electromagnetism can be rewritten in such a way that the scalar field decouples from the rest of the world. The symmetry breaking implicit in the formalism is shown to be unobservable.

Recently, Dirac has proposed a foundation for a new theory of gravitation and electromagnetism.[1] The fundamental objects of the theory are a metric tensor $g_{\mu\nu}=g_{\nu\mu}$, a vector field κ_μ (to be identified with the electromagnetic potential), and a scalar field β (assumed to be everywhere positive). The equations of the theory are obtained from an action functional which is required to be invariant under gauge transformations in the sense of Weyl:

$$g_{\mu\nu}'(x)=\lambda^2(x)g_{\mu\nu}(x),$$
$$\kappa_\mu'(x)=\kappa_\mu(x)+\partial_\mu\ln\lambda(x),$$
$$\beta'(x)=\lambda^{-1}(x)\beta(x),$$

where λ is an arbitrary positive function. The metric and the vector field form the basis of Weyl's geometry, while the β field is introduced in order to avoid fourth derivatives of the metric in the field equations.

Our purpose is to point out that the theory can be recast in a form such that the scalar field is completely decoupled from the other dynamical variables.[2] We define a new set of variables as follows:

$$h_{\mu\nu}(x)\equiv\beta^2(x)g_{\mu\nu}(x),$$
$$A_\mu(x)\equiv\kappa_\mu(x)+\partial_\mu\ln\beta(x).$$

These relations are invertible (because $\beta\neq0$), so that instead of $g_{\mu\nu}$, κ_μ, and β we can just as well take $h_{\mu\nu}$, A_μ, and β as independent field variables. Under a Weyl gauge transformation, we have

$$h_{\mu\nu}'(x)=h_{\mu\nu}(x),$$
$$A_\mu'(x)=A_\mu(x),$$
$$\beta'(x)=\lambda^{-1}(x)\beta(x).$$

Thus, in terms of the new variables, an action functional I is Weyl gauge invariant if and only if it is independent of β. The field equation $\delta I/\delta\beta(x)=0$ then reduces to the identity $0=0$, while the other field equations are independent of β. The Weyl gauge invariance of the theory thus seems to be devoid of physical content.

In general, there will be additional dynamical variables to describe the source. If these are Weyl gauge invariant, the foregoing argument still holds. If they are "cotensors", say,

$$q'(x)=\lambda^p(x)q(x),$$

we can define new variables

$$Q(x)\equiv\beta^p(x)q(x)$$

and then apply the preceding argument to the set of variables $h_{\mu\nu}$, A_μ, β, and Q. (Even if the transformation law of q should be more complicated then that of a cotensor, it suffices to define Q as the value taken by q in the gauge where $\beta\equiv1$.)

Dirac's action functional for the vacuum (his equation 5.3),

$$I=\int(\tfrac{1}{4}g^{\mu\lambda}g^{\nu\sigma}F_{\mu\nu}F_{\lambda\sigma}-\beta^2R+6g^{\mu\nu}\beta_{,\mu}\beta_{,\nu}+c\beta^4)\sqrt{-g}\,d^4x,\quad F_{\mu\nu}\equiv\kappa_{\mu,\nu}-\kappa_{\nu,\mu},$$

when re-expressed in terms of the new variables, becomes

$$I=\int(\tfrac{1}{4}h^{\mu\lambda}h^{\nu\sigma}F_{\mu\nu}F_{\lambda\sigma}-R_h+c)\sqrt{-h}\,d^4x,\quad F_{\mu\nu}\equiv A_{\mu,\nu}-A_{\nu,\mu},$$

where R_h is the scalar curvature associated with $h_{\mu\nu}$, and a divergence has been dropped from the action density. Apart from numerical coefficients, this is just the usual action for general relativity with cosmological constant and electromagnetic field. Likewise, the proposed action for a charged particle (Dirac's equations 8.1 and 8.3),

$$I_1+I_2=-m\int\beta\,ds+e\int\kappa_\mu dx^\mu,$$

becomes

$$I_1+I_2=-m\int ds_h+e\int A_\mu dx^\mu,$$

with neglect of a total differential, and again we have the same expression as in the standard theory.[3]

Dirac suggests that an observable violation of charge-conjugation and time-reversal invariance will result if the particle action contains a term of the form (his equation 10.1)

$$I_3 = a\int g^{\mu\nu}(\beta^{-3}\beta_{,\mu}\beta_{,\nu} + 2\beta^{-2}\beta_{,\mu}\kappa_\nu + \beta^{-1}\kappa_\mu\kappa_\nu)\,ds.$$

In terms of the new variables this becomes

$$I_3 = a\int h^{\mu\nu}A_\mu A_\nu\,ds_h$$

and, while such a term does violate ordinary gauge invariance (which is equivalent to Dirac's "transformations of the type 3"), it does not violate invariance under C or T. Dirac's contention that it does rests on the assertion that, while one may always work in the "Einstein gauge" $\beta \equiv 1$, one must first obtain the field equation from the variation of β before putting $\beta = 1$, and "this field equation will still show symmetry breaking."[4] But we have shown that $\delta I/\delta\beta = 0$ is an identity for one set of independent variables, and it is clear that, for *any* choice of variables, this equation cannot be independent of the other dynamical equations. We conclude that the symmetry breaking, while undeniably present in Dirac's *formalism*,[5] is unobservable.

Despite the pessimistic nature of our remarks, it is tempting to try to salvage something from Dirac's theory. In section 2 of his paper, he proposes two special gauges: an "atomic" gauge corresponding to the space-time interval defined by an atomic standard; and an Einstein gauge, in which the Schwarzschild solution is valid *and* in which a test particle follows geodesic motion. In this way it would be possible to accommodate a variable gravitational constant (expressed in atomic units), as suggested by numerical coincidences among certain large dimensionless quantities,[6] without spoiling the prediction of general relativity for the perihelion precession. Unfortunately, the atomic metric is never introduced into the formalism; moreover, to do so would obviously break the Weyl gauge invariance by singling out a particular gauge, which is just what Dirac wishes to avoid.[7]

Nevertheless, it is interesting to try to incorporate the two-metric idea into a theory (without insisting on Weyl gauge invariance), and an attempt in this direction will be published elsewhere.[8] The theory the one is led to gives the same predictions as general relativity for the classic tests, but suffers from at least one serious disease: It seems impossible to incorporate ordinary electromagnetic gauge invariance in a satisfactory way.

[1]P. A. M. Dirac, Proc. Roy. Soc., Ser. A 333, 403 (1973).

[2]A similar observation, in the context of the Brans-Dicke theory, has been made by J. L. Anderson, Phys. Rev. D 3, 1689 (1971).

[3]Except that the sign of the mass term seems to be inconsistent with the overall sign of the vacuum action.

[4]Ref. 1, p. 417.

[5]For example, in Dirac's formalism the charge e is a dimensionless number, so that the two signs of charge are formally inequivalent.

[6]P. A. M. Dirac, Proc. Roy. Soc., Ser. A 165, 199 (1938).

[7]Ref. 1, p. 409.

[8]J. L. Pietenpol, "A New Scalar-Tensor Theory of Gravitation" (to be published).