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Nuclear-Coulomb Interference in Inelastic Scattering of α Particles from ^{168}Er, ^{184}W, and ^{186}W†

I. Y. Lee, J. X. Saladin, C. Baktash, J. E. Holden, and J. O'Brien
Scaife Nuclear Physics Laboratory, University of Pittsburgh, Pittsburgh, Pennsylvania 15260
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Excitation functions for the 0^+, 2^+, and 4^+ states of ^{168}Er, ^{184}W, and ^{186}W were measured at 140° and 173.5° with incident α-particle energies between 12.5 and 19 MeV. Strong destructive nuclear-Coulomb interference effects were observed. A coupled-channels code was developed and the deformation parameters β_2^N and β_4^N of the optical potential were determined and are compared with those of the charge distribution, β_2^c and β_4^c.

In recent years systematic experimental information has been accumulated concerning quadrupole and hexadecapole deformations in the rare-earth region using two different methods, i.e., Coulomb excitation by means of α particles[1-4] and inelastic scattering of α particles well above the Coulomb barrier.[5,6] From Coulomb excitation experiments it is possible, after making some reasonable model assumptions, to extract deformation parameters β_2^c and β_4^c for the *charge* distribution. Experiments far above the barrier, on the other hand, yield the deformation parameters of the optical potential. Early comparisons between the two types of experiments seemed to indicate that the deformation parameters obtained from high-energy (α, α') scattering were substantially smaller than those obtained by electromagnetic methods. This difference may, in principle, be attributed to one or both of two reasons:

(1) There is no *a priori* reason why the electric charge distribution and the optical potential need to have *exactly* the same deformation parameters, though one would certainly expect them not to be too different.

(2) It is well known that the quantities determined by inelastic scattering are not the β_λ's rather something like a deformation length $\beta_\lambda R$,[7] where R is the nuclear radius. The extraction of β_λ's from electromagnetic moments, on the other hand, depends critically (approximately like $R^{-\lambda}$) on the radius parameter chosen for the charge distribution. Thus some care must be exercised in comparing β_λ's obtained by these two different classes of experiments.

The question of the connection between the deformation of the charge distribution and that of the optical potential is an intriguing one. We expected experiments in the interference region between Coulomb and nuclear excitation to be most sensitive to possible differences or equalities in the two types of deformation parameters. Experiments were carried out at laboratory scattering angles of 140° and 173.5° in the energy range from 12 to 19 MeV on the isotopes ^{168}Er, ^{184}W, and ^{186}W. The experimental setup was similar to that used in earlier work of our group.[2] Its salient features are an annular surface-barrier detector at 173.5°, two surface-barrier detectors at ±140°, and another pair of detectors at ±30° which served as monitors. All detectors were cooled to

~ – 20°C and overbiased. Spot targets of 3 mm diam, about 20 $\mu g/cm^2$ thick, on 20-$\mu g/cm^2$ carbon backing were produced by vacuum evaporation of the enriched isotope oxides. The energy resolution for all detectors was ~25 keV full width at half-maximum. The yields of the elastic and inelastic peaks were evaluated by means of an iterative computer program which is based on the assumption that all peaks have the same shape. The overall uncertainty in the 2^+ and 4^+ yields is about 2% and 3–5%, respectively. In the sub-Coulomb region the uncertainties are smaller, about 1% and 3%. The values of the reduced matrix elements $\langle 0^+ \| \mathfrak{M}(E2) \| 2^+ \rangle$ and $\langle 0^+ \| \mathfrak{M}(E4) \| 4^+ \rangle$ were extracted from the low-energy cross-section ratios with the help of the quantum-mechanical coupled-channels code AROSA for Coulomb excitation.[8] From the moments, charge deformation parameters $\beta_\lambda{}^c$ were derived within the framework of the axially symmetric rigid-rotor model, using a deformed Fermi-charge distribution

$$\rho(r,\theta)=\rho_0(1+\exp\{[r-R(\theta)]/a\})^{-1},$$

with

$$R(\theta)=R^c[1+\beta_2{}^cY_{20}(\theta)+\beta_4{}^cY_{40}(\theta)].$$

The procedure for choosing the parameters R_0, ρ_0, and a was identical to that used in earlier work of our group.[2] The experimental points in Fig. 1(a) represent the ratios $d\sigma_{\text{exp}}/d\sigma_{cE}$ of the experimental elastic cross section to the elastic cross section calculated under the assumption of pure Coulomb excitation. The experimental points in Fig. 1(b) are the double ratios $(d\sigma_{2^+}/d\sigma_{0^+})_{\text{expt}}/d\sigma_{2^+}/d\sigma_{0^+})_{cE}$ plotted as a function of energy. Figure 1(c) is an analogous plot for the 4° double ratios. The most conspicuous feature of the data is the sharp increase in destructive interference of the excitation of the 4^+ state between ^{168}Er and the W isotopes. This is in agreement with recent results of Bemis *et al.*[9] on ^{154}Sm, ^{166}Er, and ^{182}W. In order to interpret the data in the interference region we expanded the coupled-channels Coulomb excitation code AROSA to include excitation via a deformed optical potential. We found it necessary to integrate the radial equations out to arguments $\rho = kr = 450$. In the region of nonvanishing optical potential a Numerov integration routine is used. Further out a Taylor expansion is employed which is particularly fast in regions where the interaction is purely electromagnetic.[8] The convergence in the sum of partial amplitudes from different channel spins is considerably improved by using a procedure which is related to the Padé approximation.[8,10] Thus in a typical case only about 45 partial waves need to be cal-

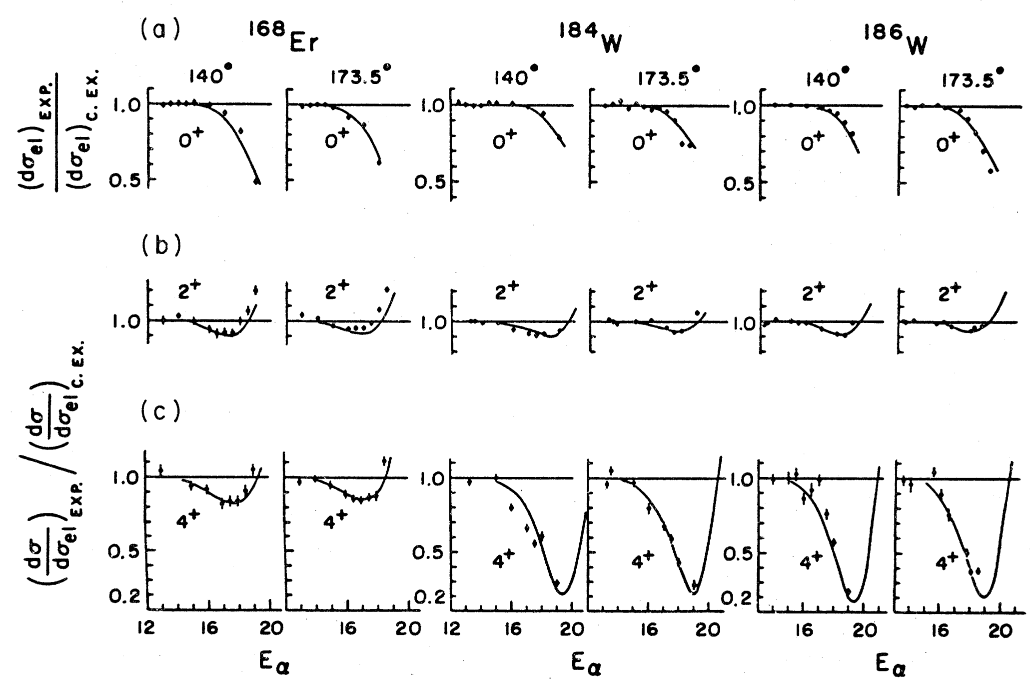

FIG. 1: Coulomb-nuclear interference in the elastic and inelastic scattering of α particles. The solid curves represent the results of coupled-channel calculations. For details, see text.

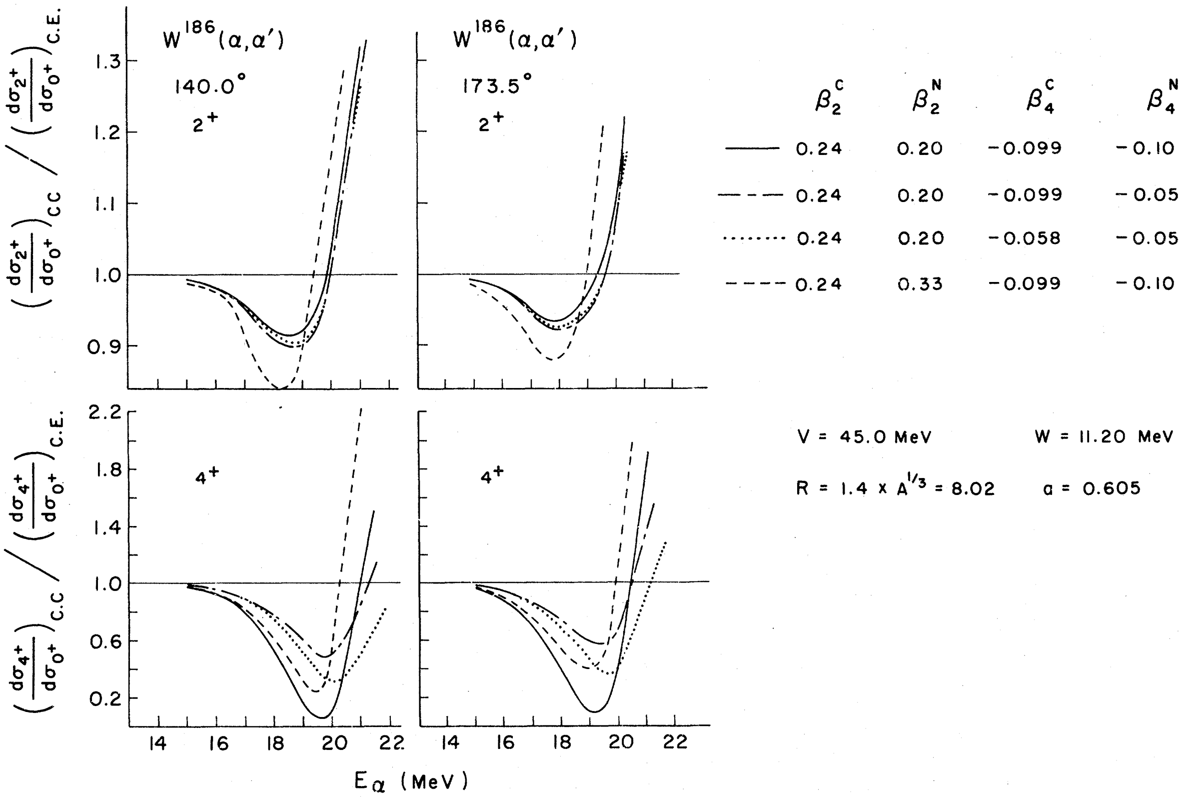

FIG. 2: Sensitivity of coupled-channels calculations to the relative size of charge and optical potential deformation.

culated explicitly where about 300 partial waves would be needed without the use of the Padé approximation.

In fitting the data, the following procedure was adopted. We started out using essentially the same optical potential used by Aponick *et al.* for the interpretation of (α, α') scattering far above the Coulomb barrier.[6] This potential was originally derived from elastic scattering on nuclei just before the onset of permanent deformation. The interaction radius and potential depth V were then slightly adjusted such as to fit the elastic scattering. The final parameters were $V=47$ MeV, $W=11.2$ MeV, and $a=0.605$ fm. The other ingredients of the calculations are the reduced matrix elements $\langle 0^+ \| \mathfrak{M}(E2) \| 2^+ \rangle$ and $\langle 0^+ \| \mathfrak{M}(E4) \| 4^+ \rangle$ and the deformation parameters β_2^N and β_4^N of the optical potential. The Coulomb-excitation contribution from the interior of the nucleus was calculated by assuming a uniform charge distribution $\rho=\rho_0$ for $r<R^u(\theta)$ with $R^u(\theta)=R^u(1+\beta_2^u Y_{20}+\beta_4^u Y_{40})$, where R^u and the β_λ^u are adjusted such as to give the correct charge and reduced matrix elements. The position, depth, and shape of the interference minimum for the 4^+ state is very sensitive to both the nuclear deformation parameter β_4^N and the reduced matrix element $\langle 0^+ \| \mathfrak{M}(E4) \| 4^+ \rangle$. Thus the interference data permit a more precise (though somewhat more model-dependent) determination of the $E4$ moments than do sub-Coulomb measurements alone, particularly in the region of negative β_4 values where the sub-Coulomb cross section is rather insensitive to the $E4$ moment. Figure 1 shows the best fits to the data. The sensitivity of the calculations to the parameters is shown in Fig. 2. The sensitivity of the experiments is ~20% in the β_4^N and the β_4^c deformations. It has been pointed out earlier by Brückner *et al.*[11] and is confirmed by the present investigation that measurements in the interference region permit an unambiguous determination of the sign of the β_4 deformation which is not possible from sub-Coulomb data alone. Table I summarizes the results. The deformation parameters β_λ^c are those derived from the E_λ moments, whereas $\beta_2^{N,\text{int}}$ are the optical potential deformation parameters derived from tne interference data. R^N is the radius of the optical potential.

TABLE I. Reduced $E2$ and $E4$ transition moments and charge and optical-potential deformation parameters. For details, see text.

	$\langle 0^+ \Vert \mathfrak{M}(E2) \Vert 2^+ \rangle$ (e b)	$\beta_2{}^c$	$\beta_2{}^{N'}$	$\beta_2{}^{N,\mathrm{int}}$	$\langle 0^+ \Vert \mathfrak{M}(E4) \Vert 4^+ \rangle$ (e b^2)	$\beta_4{}^c$	$\beta_4{}^{N'}$	$\beta_4{}^{N,\mathrm{int}}$	R^c (Fm)	$R^{N'}$	$R^{N,\mathrm{int}}$
^{168}Er	2.42 (0.03)	0.336	0.260	0.263	0.179 (0.054)	−0.019	−0.025	−0.028	6.07	7.92	7.91
^{184}W	1.94 (0.03)	0.254	0.192	0.192	−0.243 (0.090)	−0.089	−0.075	−0.076	6.257	8.09	8.17
^{186}W	1.83 (0.03)	0.239	0.181	0.182	−0.266 (0.095)	−0.090	−0.075	−0.077	6.28	8.11	8.18

Our β_4 deformation parameters for $W^{184,186}$ are considerably smaller than those obtained by Bemis *et al.*[9] from sub-Coulomb data on W^{182}. However if one includes in the analysis of the Oak Ridge National Laboratory data the interference results, one obtains a $\beta_4{}^c$ value of ~0.11.

Hendrie[12] has recently proposed a method of scaling between the two types of deformation parameters which takes into account the geometry of the situation. The parameters $\beta_c{}^{N'}$ and $R^{N'}$ are those calculated from the $\beta_\lambda{}^c$ and R^c using Hendrie's prescription. The agreement between the $\beta_\lambda{}^{N'}$ and the $\beta_\lambda{}^{N,\mathrm{int}}$ is extremely good, showing that the two types of deformation parameters are in the sense of Hendrie's model very consistent with each other. It should be pointed out that the expressions of Ref. 12 are derived under the assumption that projectile and target have sharp surfaces. They are therefore not strictly correct but they are probably a pretty good approximation for the geometrical aspects of the situation. A more rigorous approach would involve the use of folded potentials which has been successfully employed by West, Cotanch, and Robson.[13]

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