come negative for $n_0 \geq 5\times10^{13}$ cm^{-3}. The calculated frequency is almost constant for $n_0 \leq 1\times10^{13}$ cm^{-3} and decreases for densities above this value. These characteristics agree with those observed experimentally as also shown in Fig. 2. Little differences between the theory and experiments may depend on the fact that the plasma density in the experiments is taken for the values in the column center. The lower branch of Reω in Fig. 2(a) seems not to be a pure collisional drift mode but a mode coupled with the Alfvén mode, and the upper one is the Alfvén mode modified by the collisional drift mode. The Alfvén mode always damps in the present conditions.

In conclusion, the unstable mode of the collisional drift wave coupled with the Alfvén mode would be observed in a steady high-density plasma with $1 \gg \beta \gg m/M$, although the Alfvén mode is not unstable in under these conditions. In a large-β plasma with an axial current both the Alfvén and the collisional drift modes may be unstable,[3] resulting in reduced confinement of the plasma by the magnetic field. More precise experiments on coexcitation and control of these modes are now in progress.

The authors are grateful to Mr. A. Ogata for his measurement of cross-correlation and autocorrelation of the instability. They are also grateful to Mr. Y. Edano for his kind help in the numerical computations. One of the authors (Y.N.) is indebted to Professor S. Kawamata of Utsunomiya University for his continuous encouragements, and Professor F. F. Chen of the University of California, Los Angeles, for his discussions.

This work was performed under the Collaborating Research Program of the Institute of Plasma Physics, Nagoya University.

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Magnetic Compression of Intense Ion Rings*

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(Received 10 June 1974)

It is shown how Lawson criterion for controlled fusion might be achieved by magnetic compression of intense ion rings to field reversal.

It is well known that Astron E layers[1] consisting of relativistic electrons are not practical as a fusion-reactor concept because at the required electron energy (>100 MeV) and magnetic fields the synchrotron radiation is prohibitive. This restriction does not apply to the use of protons or heavier ions as pointed out by Christofilos. Given a sufficiently powerful ion source intense ion rings could be created by exploiting the injection techniques successfully used for electron rings.[2] The possibility of producing intense ion beams by modern high-power electrical pulse technology has been suggested.[3] Indeed Humphries, Lee, and Sudan[4] have recently reported 50-nsec proton pulses of 500 A at 100 kV and 5000 A at 300 kV at current density ~10 A/cm^2 from a triode configuration in which the net electron current was suppressed by a factor of 20–30. With the advent of these powerful ion sources the possibility of creating ion rings intense enough to cause the reversal of magnetic field in the ring and thus provide ideal magnetohydrodynamic stability for a confined thermonuclear plasma deserves serious attention. By increasing the axial magnetic field an ion ring can be compressed in both its major and minor radii, thus increasing the circulating current and ion energy. Furthermore since protons are practically nonrelativistic at energies up to a few hundred MeV the field reversal factor $\alpha \equiv B^s/B$ increases with compression; B is the axial Z component of the external magnetic field and B^s is the magnitude of the self-field at the surface of the ring. This is a significant point because an ion ring which at injection misses field reversal ($\alpha = 1$) by a large factor can by subsequent compression reach field reversal.[5] In what follows we discuss the dynamics of such an adiabatic compression of an ion ring and the subsequent plasma heating by the beam.

Ion beams are observed to emerge electrostatically neutralized[4] from the ion source by dragging along an equivalent number of electrons. We will therefore assume that an ion ring of major and minor radii R and r, respectively,[6] is electrostatically neutral, i.e., $Zn = n_e$ where n and n_e are beam and electron densities and Ze is the ion charge. The external field is pulsed to increase from B_0 to its final value B_1 in a time t_c and it is maintained at this level for a period t_d. For adiabatic compression the azimuthal momentum balance and radial force balance for a nonrelativistic ion ring are given by

$$\dot{u}_\varphi + \ddot{R}_\varphi = -\frac{Ze}{mcR}\frac{d}{dt}(RA_\varphi) = -\frac{Ze}{mc2\pi R}\frac{d}{dt}[\pi R^2 B + L(I + I_e)], \quad (1)$$

$$\frac{-u_\varphi^2}{R} = \frac{Ze}{m}\{E_r + \frac{1}{c}\langle u_\varphi(B + B_z^s)\rangle\}, \quad (2)$$

where

$$\langle u_\varphi B_z^s\rangle \equiv (\pi r^2)^{-1}\int d^2 s\, u_\varphi \frac{d}{\rho d\rho}(\rho A_\varphi) \approx u_\varphi L(I + Ie)/2\pi R^2 \quad (2a)$$

for $r \ll R$; the integral is over the cross section of the ring and u_φ is assumed approximately constant over this cross section, the ring inductance $L = (4\pi/c)R\{\ln(8R/r) - \frac{7}{4}\} \equiv Rl$, $I = ZNeu_\varphi/2\pi R$ is the beam current, N is the total number of beam ions, $I_e = -ZNev_\varphi/2\pi R$ is the electron current, and E_r is the radial polarization field. It is reasonable to assume that $L/\mathcal{R} \gg t_c$ (compression time) where the ring resistance $\mathcal{R} = 2R/\sigma r^2$, $\sigma = n_e e^2\tau_{eb}/m_e$, and τ_{eb} is the momentum exchange time between the electrons and the beam ions. In this case the electron equation is simply $\vec{E} + \langle\vec{v}\times(\vec{B} + \vec{B}^s)\rangle/c = 0$. The electrons drift radially inwards with the beam and the φ and r components of the electron equation furnish

$$E_r + \langle v_\varphi(B + B_z^s)\rangle/c = 0, \quad (3)$$

$$E_\varphi - \dot{R}(B + B_z^s)/c = 0. \quad (4)$$

Equation (4) states that the azimuthal electric field in the frame moving radially inwards with the beam vanishes which ensures that the flux linked by the ring [see Eq. (1)] remains constant, i.e.,

$$\pi R^2 B + L(I + I_e) = \Phi_0 \equiv \pi R_0^2 B_0 + L_0 I_0. \quad (5)$$

The electron current is taken to vanish in the initial state. From Eqs. (1) and (4) we obtain

$$Ru_\varphi = R_0 u_0, \quad (6)$$

$$R^2 I = R_0^2 I_0. \quad (7)$$

Substituting for E_r in Eq. (2) from (3) and taking the averages in the sense of (2a) we obtain[7]

$$R = -\frac{u_\varphi}{\Omega}\frac{(1 + W/K)}{(1 + I_e/I)} \quad (8)$$

with $\Omega = ZeB/mc$; $W = (1/2c)L(I + I_e)^2$ and $K = \frac{1}{2}Nmu_\varphi^2$ are the beam self-magnetic and kinetic energies, respectively. Equations (5), (7), and (8) completely define the problem of beam compression. It is possible to show that R/R_0 scales as $(B_0/B)^{1/2}$ for low compression and as $(B_0/B)^{2/5}$ for large compression. Combining Eqs. (6) and (7) we find that the external flux linked by the ring varies as

$$R^2 B = R_0^2 B_0(1 + W/K)[(1 + W_0/K_0)(1 + I_e/I)]^{-1}. \quad (9)$$

It can be readily checked by appealing to the principle of adiabatic compression that the minor radius also shrinks such that the aspect ratio scales as

$$(r/R) = (r_0/R_0)(1 + I_e/I)^{-1}. \quad (10)$$

The field reversal factor $\alpha = 2(I + I_e)/crB$ scales as

$$\alpha/\alpha_0 = (1 + I_e/I)^2 R_0^3 B_0/R^3 B. \quad (11)$$

Notice also that,

$$W/K = (1 + I_e/I)^2 NZ^2 e^2 l/(2\pi)^2 mRc \approx (R_0/R)(W_0/K_0)(1 + I_e/I)^2, \quad (12)$$

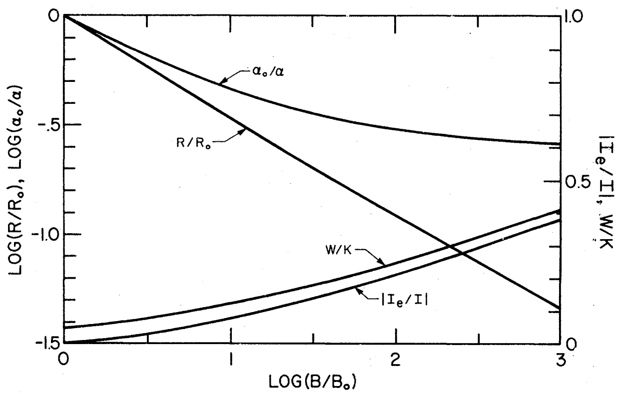

FIG. 1. Variation of R/R_0, $|I_e/I|$, W/K, and α_0/α with B/B_0, for $W_0/K_0=0.05$; note that I_e is actually negative, i.e., it opposes the beam current.

since $l \equiv L/R$ is relatively insensitive to changes in the aspect ratio. Figure 1 shows the variation of R/R_0, I_e/I, W/K, α/α_0 with B/B_0 from a numerical solution of Eqs. (5), (6), and (8).

Towards the end of the compression phase the plasma is created in the ring possibly by the introduction of a pellet which is rapidly ionized. The electron current decays quickly because of the high resistivity of the low-temperature plasma,[8] which results in a shrinkage of the minor radius and an increase in the field-reversal factor. The beam delivers energy to the plasma electrons and, since the beam ion velocity is much greater than the electron thermal speed, the heating time is given by the characteristic slowing down time of the ions,[9] viz., $\tau_s = 0.63 \times 10^{11}(m/m_{\rm pr})W^{3/2}/n_p$ sec, where W is the ion energy in MeV, $m_{\rm pr}$ is the proton mass, and n_p is the density of the plasma electrons in cm^{-3}. The beam ion deflection time $\tau_a \approx (m/m_e)\tau_s$ so that there is no significant scattering of the beam during this heating period. As the plasma heats up to keV temperatures its resistivity drops rapidly and $L/\mathfrak{R} \gg \tau_s$. In this stage the flux linking the ring is again conserved but the canonical azimuthal momentum is not. Introducing the slowing down time τ_s in Eq. (1) yields

$$Ru_\varphi = R_1 u_1(1 - 3t/\tau_{s1})^{1/3}, \qquad (13)$$

where the subscript 1 denotes quantities at the beginning of the heating stage. The mechanical momentum of the beam is picked up by the plasma ions since $(d/dt)(Nmu_\varphi + N_i m_i u_i) = 0$. For $N_i \gg N$ the bulk of the beam energy is transferred to the thermal energy of plasma electrons which eventually equilibrates with the ion thermal energy. As the beam slows down the major radius R does not shrink to zero because the flux will now be sustained by an induced plasma current. The asymptotic limit of this radius obtained from Eqs. (8) and (13) and flux conservation is

$$R_2 = R_1|1 - W_1/K_1|^{1/2}/(1 + W_1/K_1)^{1/2}. \qquad (14)$$

The number of beam ions needed for $\alpha = 1$, i.e., for creating closed lines of force in the ring is easily computed to be

$$NZ^2e^2/mc^2 \approx \pi r/[1 - 1.4r/R], \text{ for } r/R \sim \tfrac{1}{3}. \qquad (15)$$

To achieve Lawson condition the following additional requirements have to be satisfied:

$$n\tau = 10^{14}$$

(Lawson criterion),

$$2nT = (B^s)^2/8\pi$$

(plasma pressure balance),

$$NE > 2\pi^2 r^2 RnT$$

(heating energy requirement),

$$n\tau = 2.5 \times 10^{13} I^2 T^{1/2}$$

(pseudoclassical diffusion),[10] where I is in MA, E is the ion energy at the end of the compression stage, and n and T are plasma density and temperature, respectively. Assume that by the injection of a 10^{-7} sec, 150-kA pulse of 5-MeV protons containing $N \sim 10^{17}$ ions, a ring with initial parameters $I_0 = 125$ kA, $r_0 = 20$ cm, $R_0 = 60$ cm, $\alpha_0 = 0.226$ in a field $B_0 = 5.5$ kG and with $W/K = 0.08$ is created. A compression in radius by a factor of 10 results in the following final ring parameters (after decay of the electron current induced in the compression phase): $E \sim 500$ MeV, $I \sim 10^7$ A, $r = 2$ cm, $R = 6$ cm, $W/K \sim 0.80$, $B \sim 0.55$ MG, $\alpha = 2.26$. A plasma density $\sim 10^{18}$ cm^{-3} at 10 keV can be confined in such a ring, $\tau_s \lesssim 10^{-3}$ sec, and the confinement time obtained from the pseudoclassical diffusion formula far exceeds the Lawson time. The stability of such an ion ring to the equivalent of the Kruskal-Shafranov mode looks good because of the stabilization provided by the axial magnetic field B through the finite ion gyroradius effects. The optimization of such a scheme, the stability of the plasma loaded ion ring, and the role of microinstabilities if any will be discussed in a more detailed publication. However, it is noteworthy that since almost all of the beam energy is provided by magnetic compression the efficiency of the ion source is not a serious consideration. One technique available for compression to megagauss fields is to adopt the approach of magnetically imploding a cylindrical metallic liner, the so called LINUS concept.[11] Static, spatially varying, magnetic fields can also be considered for compression.[12]

Finally, we wish to state that the possibility of delivering the compressed ring energy to a DT pellet is available to achieve fusion by pellet compression as in electron-beam or laser fusion.[13] However, the following must be kept in mind: (a) The ring would have to be accelerated axially to impact the pellet in a time of a few nanoseconds (moving the pellet into a stationary ring would probably require unrealistically large pellet velocities). (b) Suitable high-Z, stripped, ring ions with appropriate stopping lengths with respect to the pellet size would be required.[14]

We wish to acknowledge the help of Dr. R. L. Ferch in computing the curves of Fig. 1 and interesting discussions with Dr. R. V. Lovelace, Dr. S. Humphreys, Jr., Dr. H. P. Furth, and Dr. M. N. Rosenbluth.

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*Work supported by the U. S. Office of Naval Research under Contract No. NOOO-4-67-A-0077-0025 and the U. S. Atomic Energy Commission under Contract No. AT(11-1)3170.

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