Observation of the Coupled Mode of a Collisional Drift Wave and an Alfvén Wave

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We observe a low-frequency instability with $\omega \ll \omega_{ci}$ in a steady high-density plasma ($n_0 \simeq 10^{13-14}$ cm^{-3}). This instability has components of magnetic field fluctuations as well as density fluctuations. By introducing the dispersion relation, the instability is interpreted as the coupled mode of the collisional drift wave and the Alfvén wave.

There have been many works on the collisional drift wave in rather low-density, low-β plasmas ($\beta \ll m/M$), in which only an electrostatic mode is precisely considered.[1] In rather high-density plasmas with $1 \gg \beta \gg m/M$, however, the hydromagnetic mode must be taken into account as the electrostatic approximation breaks down. β is the ratio of the plasma pressure to the magnetic pressure, and m and M are the mass of an electron and an ion, respectively. Kadomtsev[2] has shown in a collisionless plasma the existence of the normal mode of the coupled Aflvén wave and collisionless drift wave, but no experimental verification has been reported. Woods[3] extended a theory on the current-convective instability presented by Kadomtsev[4] to that of the hydromagnetic wave, and has shown that the hydromagnetic mode is unstable if a radial resistivity gradient and/or a steady current along the magnetic field exists. But this mode is different from the drift-wave branch characterized in Refs. 1 or 2.

In the present Letter, we show that a low-frequency oscillation ($\omega \ll \omega_{ci}$) observed in a rather high-density, large-β ($1 \gg \beta \gg m/M$) plasma without a current along the magnetic field may be interpreted as the coupled mode of the collisional drift wave and the Alfvén wave. The same kinds of experiments have been reported,[5,6] but all of them have not necessarily satisfied the large-β condition mentioned here, and no observation of the collisional drift wave in such a high-density plasma has been reported. But in fusion devices in which high-density high-temperature plasma is produced and in which β is high, the coupled collisional and Alfvén waves may be expected to be unstable, resulting in reduced confinement of the plasma by the magnetic field.

Experiments are performed on the "TPD-1" machine of Nagoya University.[7] A helium plasma, which is produced by an arc discharge of a maximum current of 100 A in a high–neutral-pressure region ($p \simeq 0.5$–1 Torr) under a strong magnetic field, diffuses into a low-pressure experimental region with weak magnetic field through an anode orifice (diameter 8 mm), and ends on the floating target electrode. So, the fully ionized, current-free plasma exists in the experimental region. The magnetic field B_0 in the experimental region is varied up to 4.0 kG. The plasma density n_0 measured by a HCN laser[8] ranges from 10^{12} to 2×10^{14} cm^{-3} in the column center, with a background neutral pressure of $(6-7) \times 10^{-4}$ Torr. The plasma diameter is about 15 mm. Langmuir probes are used for measuring plasma parameters. The electron temperature T_e is about 6–10 eV, and the temperature ratio $R = T_i/T_e$ is about 0.5–1 depending on the discharge conditions, where T_i is an ion temperature.

Plasma profiles of the steady state and of fluctuations are measured by optical probes which are constructed with an optical guide and a photomultiplier. The optical guide has a collimating slit with a hole 2.0 mm and a length 42.3 mm. The optical signal detected is the He II (4686 Å) line. The fluctuating signals are analyzed by a real-time autocorrelator and cross correlator and a spectrum analyzer. The velocity of the plasma rotation around the axis is measured by the Doppler shift of the He II line.[9] Fluctuations of the magnetic field are detected by a water-cooled magnetic probe which, unfortunately, cannot be inserted into the center of the plasma column because the probe would melt, although the wave dominated region can be measured.

In the region where the radial density gradient

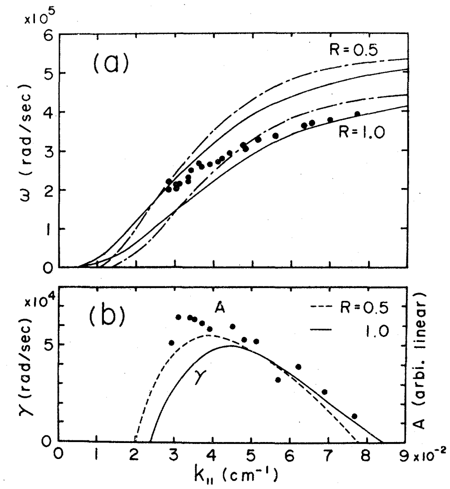

FIG. 1. (a) Dispersion relation of the collisional drift mode coupled with the Alfvén mode. Dot-dashed lines show the results for $D_0=0$ (see text). $k_\perp=2.5$ cm^{-1}, $k_x=1.5$ cm^{-1}, $k_y=2.0$ cm^{-1}, $\lambda=1.2$ cm^{-1}, $B_0=1.5$ kG, $n_0=3.0\times10^{13}$ cm^{-3}, and $T_e=10$ eV. Closed circles show experimental results. (b) Growth rate γ as a function of parallel wave number for the collisional drift mode obtained from Eq. (1). Parameters are the same as in (a). Closed circles are the instability amplitude $A=n_1/n_0$ obtained experimentally.

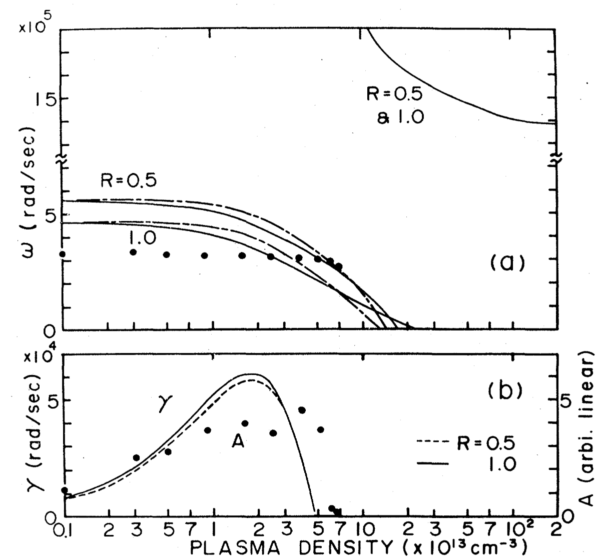

FIG. 2. (a) Frequency ω versus the plasma density. Solid lines are obtained from Eq. (1), and dot-dashed lines from $D_0=0$. Upper branch is the Alfvén branch and the lower ones are for the collisional drift mode. The parameters are the same as in Fig. 1 except with $k_\parallel=0.05$ cm^{-1}. Closed circles are the experimental results. (b) Growth rate γ versus the plasma density calculated from Eq. (1) for lower-branch mode with the same parameters as in (a). Closed circles are the instability amplitude.

is maximum, 4–6 mm from the column center, there exists typically an ~50-kHz oscillation (f_1 mode) at the magnetic field intensity of 1.5 kG, while in the column center there exists an ~10-kHz oscillation (f_2 mode). Here, we are interested in the f_1 mode. The maximum relative density fluctuation of the f_1 mode is 10–15%. The variation of the frequency is measured as a function of the steady magnetic field intensity. The frequency of the instability increases almost linearly with the magnetic field strength till about 3 kG, but it becomes almost constant under stronger field. The amplitude becomes larger until $B_0 \simeq 1.8$ kG, and decreases gradually with increasing magnetic field strength, although the spectrum becomes broader.

The mode number in the azimuthal direction is found to be $m=1$, from the cross-correlation measurement, and the wave travels in the direction of the electron diamagnetic velocity. A higher mode ($m=2$) is also observed, but its amplitude is much smaller than the fundamental mode. The instability amplitude is also measured along the plasma column (z direction), and is found to be minimum near the anode and target electrodes, and maximum near the middle of the column. The phase is constant over the whole of the plasma column length at fixed radial and azimuthal position. Thus, the half wavelength along the plasma column should be determined by the length between the anode and target electrodes. When the column length is made shorter, the frequency increases, while the instability amplitude decreases. An example of the dispersion relation is shown in Fig. 1, which is obtained by changing the plasma column length, after correcting for plasma rotation which will be mentioned later. In this example, the instability disappears at $k_\parallel \simeq 8.0\times10^{-2}$ cm^{-1} as seen in Fig. 1(b).

Steady-state plasma rotation has been observed,[9] and it has been found that the rotation angular velocity in the electron diamagnetic direction is constant at about $(4-5)\times10^5$ cm/sec within 2.5–3 mm from the column axis at $B_0=4.0$ kG. Outside of this region where the wave amplitude is maximum, a small obscure rotation of the plasma column is observed [$(1-2)\times10^5$ cm/sec].

When the discharge current I_d is changed from 10 to 100 A, the plasma density can change from 1×10^{12} to 2×10^{14} cm^{-3} in the column center. The density is not necessarily proportional to the discharge current. The present instability onsets at about $n_0 \gtrsim 3\times10^{12}$ cm^{-3} at $B_0 = 1.5$ kG, and the instability amplitude n_1/n_0 increases until $n_0 \simeq 4\times10^{13}$ cm^{-3}, as shown in Fig. 2(b). After that the amplitude decreases strongly, being extinguished at $n_0 \simeq 7\times10^{13}$ cm^{-3}. The instability frequency decreases monotonically from 53 to 42 kHz as the discharge current increases.

Fluctuations of the magnetic field are measured on the azimuthal B_θ and axial B_z components, as there is the possibility of hydromagnetic instability or of plasma rotation. An example of the results is shown in Fig. 3. We cannot measure the values in the plasma column center, because the water-cooled magnetic probe would melt and, further, it would disturb the plasma strongly. Figure 3 shows that both B_θ and B_r are larger than B_z, although B_θ is a little larger than B_r. These facts are consistent with the theoretical results, and indicate the existence of a nonelectrostatic instability.

The interpretation of this instability is based on the coupled mode of the collisional drift mode and the Alfvén mode. By starting with the ion and electron fluid equations with ion viscosities across and electron collisions with ions along the magnetic field, and without current, we obtain the dispersion relation[10]

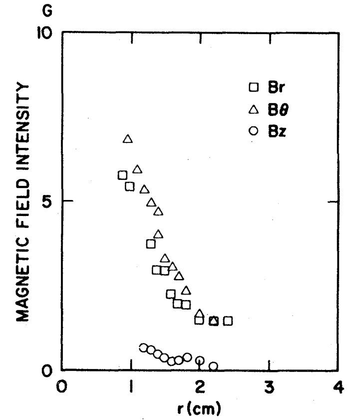

FIG. 3. Magnetic field fluctuations measured by the magnetic probe at $n_0 \simeq 4.5\times10^{13}$ cm^{-3} and $B_0 = 1.5$ kG.

$$D_0(k,\omega) + D_A(k,\omega) = 0, \qquad (1)$$

where

$$D_0(k,\omega) = i\frac{\Omega}{R}\omega^2 - \frac{\Omega}{bR}\{\omega_\perp + [R + b(1+R)]\omega_\parallel + i\Omega\omega_*\}\omega + \frac{\Omega}{b}(\omega_\parallel - \omega_\perp)\omega_* - i\frac{\Omega}{b}\frac{1+R}{R}\omega_\parallel\omega_\perp,$$

$$D_A(k,\omega) = \omega^3 + \left[(R-1)\omega_* + i\left(\frac{\omega_\perp}{b} - \kappa\frac{1+R}{R}b\omega_{ci}\right)\right]\omega^2$$

$$+\left[-R\omega_*^2 + \kappa\frac{1+R}{R}\omega_{ci}\omega_\perp + i\left(\frac{R-1}{b}\omega_*\omega_\perp + \kappa\frac{1+R}{R}(b+R)\omega_{ci}\omega_*\right)\right]\omega$$

$$-\kappa\frac{1+R}{R}\omega_{ci}\omega_\perp\omega_* - i\left(\frac{R}{b}\omega_\perp\omega_*^2 + \kappa(1+R)\omega_{ci}\omega_*^2\right),$$

$\kappa = k_\parallel(k_x - k_y)/k_\perp^2$, and $k_\perp^2 = k_x^2 + k_y^2$.

Here, we assume that the fluctuating electric field is nonelectrostatic, i.e., $\vec{E} = -\nabla\varphi - (1/c)\times(\partial A_z/\partial t)\cdot\hat{z}$, and $1 \gg \beta \gg m/M$ and $\nabla n_0/n_0 \gg \nabla T/T$ are the experimental conditions. Further we assume $k_y, k_x \gg \lambda$, although in the experiments $k_x \gtrsim \lambda$. φ is the scalar potential, $\vec{A}$ is the vector potential, $b = \rho_i^2 k_\perp^2/2$, $\omega_\parallel = T_e k_\parallel^2/m\nu_{ei}$, $\omega_\perp = b^2\nu_{ii}/4$, $\omega_A = V_A k_\parallel$, V_A is the Alfvén speed, $\lambda = -(\partial/\partial x)\ln n_0$, $\Omega = \omega_A^2 b/\omega_\parallel$, and ω_* is the drift frequency caused by the density gradient. Other notations are standard. From the experimental results the effect of parallel motion of the ions and $\nabla T/T$ can be neglected, but strictly these should be taken into account.[11] If we use the electrostatic approximation, Eq. (1) reduces to the dispersion relation of the collisional drift wave obtained by Hendel *et al.*,[12] i.e., $D_0 = 0$.

Examples of numerical values of Eq. (1) are shown in Fig. 1 by the solid lines, which are obtained by the use of experimental parameters. In the same figure the results obtained from $D_0 = 0$ are also shown by the dot-dashed lines.

We further mention that the growth rate of this mode increases with density, to a maximum at about 2×10^{13} cm^{-3}, and decreases rapidly to become negative for $n_0 \geq 5\times10^{13}$ cm^{-3}. The calculated frequency is almost constant for $n_0 \leq 1\times10^{13}$ cm^{-3} and decreases for densities above this value. These characteristics agree with those observed experimentally as also shown in Fig. 2. Little differences between the theory and experiments may depend on the fact that the plasma density in the experiments is taken for the values in the column center. The lower branch of Reω in Fig. 2(a) seems not to be a pure collisional drift mode but a mode coupled with the Alfvén mode, and the upper one is the Alfvén mode modified by the collisional drift mode. The Alfvén mode always damps in the present conditions.

In conclusion, the unstable mode of the collisional drift wave coupled with the Alfvén mode would be observed in a steady high-density plasma with $1 \gg \beta \gg m/M$, although the Alfvén mode is not unstable in under these conditions. In a large-β plasma with an axial current both the Alfvén and the collisional drift modes may be unstable,[3] resulting in reduced confinement of the plasma by the magnetic field. More precise experiments on coexcitation and control of these modes are now in progress.

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Magnetic Compression of Intense Ion Rings*

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It is shown how Lawson criterion for controlled fusion might be achieved by magnetic compression of intense ion rings to field reversal.

It is well known that Astron E layers[1] consisting of relativistic electrons are not practical as a fusion-reactor concept because at the required electron energy (> 100 MeV) and magnetic fields the synchrotron radiation is prohibitive. This restriction does not apply to the use of protons or heavier ions as pointed out by Christofilos. Given a sufficiently powerful ion source intense ion rings could be created by exploiting the injection techniques successfully used for electron rings.[2] The possibility of producing intense ion beams by modern high-power electrical pulse technology has been suggested.[3] Indeed Humphries, Lee, and Sudan[4] have recently reported 50-nsec proton pulses of 500 A at 100 kV and 5000 A at 300 kV at current density ~10 A/cm^2 from a triode configuration in which the net electron current was suppressed by a factor of 20–30. With the advent of these powerful ion sources the possibility of creating ion rings intense enough to cause the