entropy but preserve "small" mass fluctuations of galactic size. Such a solution would necessarily imply a growth of the specific entropy in time or a corresponding increase of $\bar{\mu}$ extrapolated back in time, with $\bar{\mu} \gg 1$ allowed in principle.

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[1]D. A. Kirzhnits and A. D. Linde, Phys. Lett. 42B, 471 (1972); D. A. Kirzhnits, Pis'ma Zh. Eksp. Teor. Fiz. 15, 745 (1972) [JETP Lett. 15, 529 (1972)].
[2]S. Weinberg, "Gauge and Global Symmetries at High Temperature" (to be published).
[3]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Physics*, edited by N. Svartholm (Almquist and Wiksells, Stockholm, Sweden, 1968), p. 367.
[4]S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).
[5]A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971), Chap. 7.
[6]S. Weinberg, Phys. Rev. D 7, 1887 (1973).
[7]C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).
[8]E. R. Harrison, Annu. Rev. Astron. Astrophys. 2, 155 (1973).
[9]S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (Wiley, New York, 1972).
[10]In the composite-particle model [see R. Hagedorn, Nuovo Cimento, Suppl. 3, 147 (1965); S. Fubini and G. Veneziano, Nuovo Cimento 64A, 811 (1969)], the chemical potential grows without bound while there is a maximum temperature, so that the condition $\bar{\mu} \gg 1$ is attained. However, because of the richness of fermion states, the typical thermodynamic quantities (e.g., pressure, density, and number density) grow exponentially with the chemical potential [see K. Huang and S. Weinberg, Phys. Rev. Lett. 25, 895 (1970)], so that the validity of a one-loop approximation would be questionable.
[11]C. W. Misner, Astrophys. J. 151, 431 (1968).
[12]S. Weinberg, Astrophys. J. 168, 175 (1971).
[13]R. Matzner and C. W. Misner, Astrophys. J. 171, 415 (1972).

Inclusive π^0 Production in pp Collisions at 50–400 GeV/c*

D. C. Carey, J. R. Johnson, R. Kammerud, M. Peters,† D. J. Ritchie, A. Roberts, J. R. Sauer, R. Shafer, D. Theriot, and J. K. Walker
National Accelerator Laboratory, Batavia, Illinois 60510

and

F. E. Taylor
Physics Department, Northern Illinois University, DeKalb, Illinois 60115
(Received 15 May 1974)

We have measured the single-photon cross section in the reaction $p+p\to\gamma+$anything for incident proton momenta from 50 to 400 GeV/c and lab angles of 80, 100, and 120 mrad. It is shown that in the range $p_\perp=0.3$ to 4.3 GeV/c, the derived π^0 invariant cross section can be factorized into a product of two functions, one in $p_\perp$ and the other in a new scaling variable $x_R=p^*/p_{\max}^*$, where p^* is the total c.m. momentum of the π^0.

The study of the production of pions with large transverse momentum $p_\perp$ in proton-proton collisions is expected to give insight into the short-distance structure of the proton.[1-3] Great interest in this field has been stimulated by the experimental results obtained at the CERN intersecting storage rings[4-6] and at the National Accelerator Laboratory (NAL).[7,8] Büsser *et al.*[4] at the intersecting storage rings were the first to show that near 90° in the pp c.m. system, the large-$p_\perp$ data were consistent with a scaling behavior with respect to the variable $x_\perp=2p_\perp/\sqrt{s}$. On the other hand, it has been known for several years that the small-$p_\perp$ single-pion inclusive data exhibit scaling with respect to the variable $x_\parallel=2p_\parallel^*/\sqrt{s}$ at all c.m. angles. In this experiment, data have been obtained on single-π^0 inclusive spectra from 40 to 110° in the pp c.m. system and for $0.3\leq p_\perp \leq 4.3$ GeV/c. It has been found that the cross sections scale when the "radial" variable $x_R=p^*/$

p_{max}^* is used. This new radial scaling variable, x_R, may be regarded as a generalization of the previous variables $x_\perp$ and $x_\parallel$. Clearly, in the appropriate limits of small $p_\perp$, x_R tends to $x_\parallel$, while at small $p_\parallel^*$, x_R tends to $x_\perp$.

We have measured the photon energy spectrum in the process $p + p \to \gamma +$ anything for incident proton energies from 50 to 400 GeV. Data were accumulated at three angles, 80, 100, and 120 mrad, over a range in photon transverse momentum from 0.3 to about 4.3 GeV/c. On the assumption that all the detected photons came from π^0 decay,[4,9] the observed photon energy spectra were then converted into invariant π^0 cross sections using Sternheimer's[10] prescription.

The experiment was performed at the internal target area at NAL, using both a hydrogen gas jet[11] and a rotating carbon-fiber target which intercepted the accelerator's internal proton beam. The detection system[7] consisted of a lead-glass shower counter which was used to measure the γ-ray energy, and scintillation counters which were used to separate incident γ rays from hadrons and muons. Because the γ-ray opening angle in the decay $\pi^0 \to 2\gamma$ was larger than the angular acceptance of the spectrometer, the detector was sensitive to only one of the two π^0-decay photons.

At each angle, the normalization of the γ-ray cross sections, and hence the π^0 invariant cross sections, was determined from a short run using the hydrogen-jet target, where a solid-state detector was used to monitor the rate of recoil protons elastically scattered at small momentum transfer. By using the known[12,13] pp elastic cross section to determine the luminosity, the absolute γ-ray cross section was then obtained. The major portion of the data at each angle was taken while using the rotating carbon-fiber target. To establish that the experiment was not sensitive to complex target effects, γ-ray data were collected for both carbon and hydrogen at 100 mrad for incident energies between 50 and 400 GeV and $p_\perp$ up to 3 GeV/c. There was no observable difference in the shapes of the γ-ray spectra from the two types of targets.[7]

Figure 1 displays the $p_\perp$ dependence of the π^0 invariant cross section for the data obtained at 80 mrad. At a fixed $p_\perp$, the cross section rises with increasing incident momentum; at $p_\perp = 2$ GeV/c, there is roughly a factor of 25 between the cross sections at incident momenta of 50 and 250 GeV/c. Furthermore, the slope of the cross section at large $p_\perp$ decreases with increasing incident momentum.

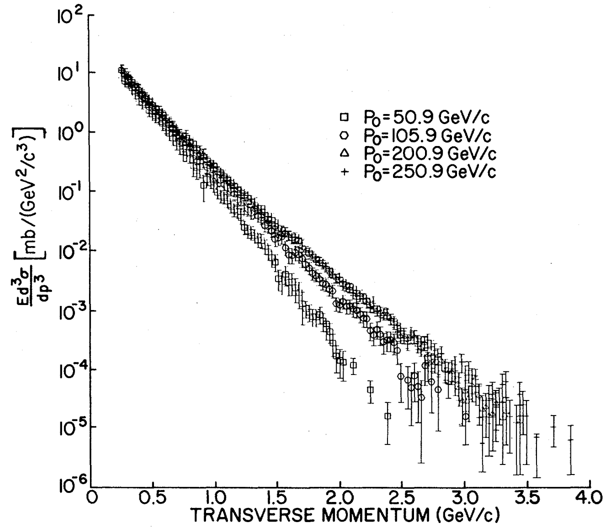

FIG. 1. The invariant π^0 cross sections at $\theta_{lab} = 80$ mrad plotted versus $p_\perp$ for four incident proton momenta, 50.9, 105.9, 200.9, and 250.9 GeV/c.

The π^0 invariant cross section reveals a strikingly simple behavior when it is expressed in terms of certain variables. For the three lab angles 80, 100, and 120 mrad, we find that the invariant π^0 cross section is well represented by the product of two functions, $f(x_R)$ and $g(p_\perp)$; that is,

$$E\,d^3\sigma/dp^3 = f(x_R)g(p_\perp).$$

The variable $p_\perp$ is the familiar transverse momentum of the π^0 while the radial scaling variable x_R is the ratio of the π^0 momentum to the maximum momentum,

$$x_R \equiv p^*/p_{max}^* \cong 2p^*/\sqrt{s}.$$

For a particular lab angle, the data from this experiment cover a large range of x_R for fixed $p_\perp$ and thereby allow an accurate determination of the scaling function $f(x_R)$. By an iterative procedure, self-consistent solutions for the functions $f(x_R)$ and $g(p_\perp)$ were obtained from the invariant π^0 cross sections. The resulting functions $f(x_R)$ and $g(p_\perp)$ for 80 mrad are shown in Figs. 2 and 3.[14]

The degree to which the radial scaling function $f(x_R)$ is independent of the transverse momentum $p_\perp$ for a particular lab angle can be seen in Fig. 2. This figure shows that $f(x_R)$ is, to a good approximation, a unique function of x_R, which demonstrates the validity of the factorization of the π^0 invariant cross section. The solid line of Fig.

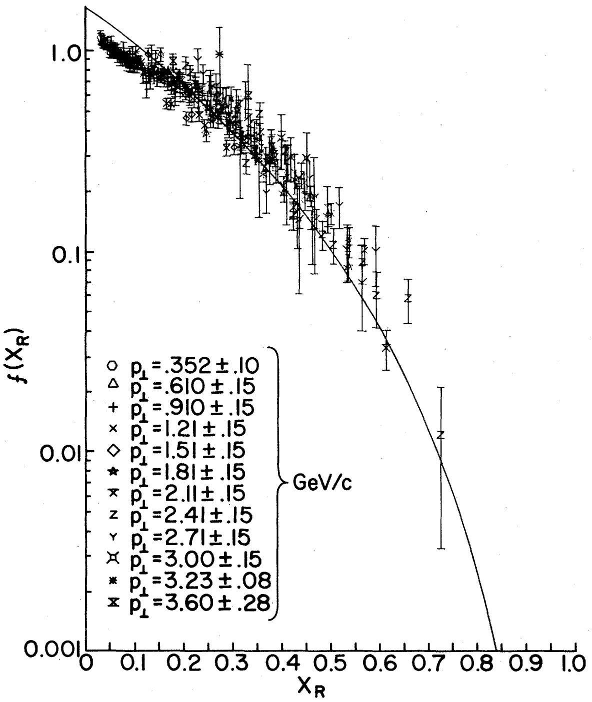

FIG. 2. The functions $f(x_R)$ calculated for twelve fixed $p_\perp$ values for the 80-mrad data are compared with each other and with the functional form $(1-x_R)^4$.

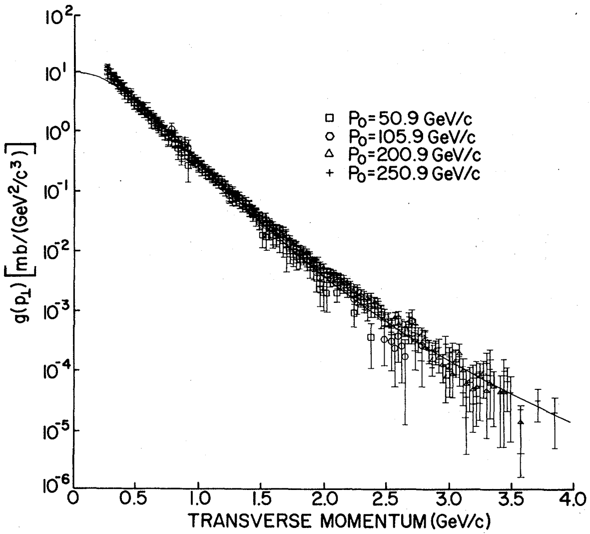

FIG. 3. The functions $g(p_\perp)$ calculated for four incident momenta at 80 mrad are compared with each other and with the functional form $(p_\perp^2+0.86)^{-4.5}$.

2 is a constant times $(1-x_R)^4$ and gives a convenient although rough description of the scaling function $f(x_R)$. The representation of $f(x_R)$ used in subsequent analysis was a minimum-χ^2 fit by a fourth-order polynomial in x_R.

Having determined a unique $f(x_R)$, we compute the transverse momentum function $g(p_\perp)$ for each incident momentum as the invariant cross section at that momentum divided by $f(x_R)$. In Fig. 3, this result has been plotted for four different incident proton momenta from 50 to 250 GeV/c for the same lab angle, 80 mrad. It can be seen that $g(p_\perp)$ appears to be a universal function over six decades in magnitude, even though, as noted previously, the π^0 inclusive cross sections vary widely in shape and magnitude over this same range in $p_\perp$ and incident momentum. This implies that $g(p_\perp)$ has reached its asymptotic limit even at an incident momentum as low as 50 GeV/c. The solid line in Fig. 3 has the functional dependence $(p_\perp^2+0.86)^{-4.5}$ and fits the data well for $p_\perp > 0.5$ GeV/c.

The functions $f(x_R)$ and $g(p_\perp)$ were determined separately at each angle of 80, 100, and 120 mrad. The radial scaling functions $f(x_R)$ agreed to within ±10% for $0.1 \le x_R \le 0.7$. Similarly the three intrinsic transverse momentum functions $g(p_\perp)$ were consistent with each other within the errors of the data. A three-parameter minimum-χ^2 fit was made to the 80-mrad $g(p_\perp)$ function for each incident momentum bin from 50 to 300 GeV/c, and an average fit was determined. Comparing this average fit to the 80-, 100-, and 120-mrad $g(p_\perp)$ functions over their respective incident momentum ranges yielded an average χ^2 per degree of freedom of 0.7, 0.4, and 1.6, respectively.

In conclusion, from 40 to 110° in the c.m. system and from 50 to 400 GeV/c incident momenta, we have shown the following: (a) The π^0 inclusive production cross section in pp collisions factorizes into a product of two functions, $f(x_R)$ and $g(p_\perp)$. (b) Both the radial scaling function $f(x_R)$, where x_R is a new scaling variable, and the transverse-momentum function $g(p_\perp)$ are universal functions over these kinematic ranges. (c) Approximate analytic expressions for these functions are

$$f(x_R) \propto (1-x_R)^4$$

and

$$g(p_\perp) \propto (p_\perp^2+0.86)^{-4.5}.$$

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†Permanent address: Physics Department, University of Hawaii, Honolulu, Haw. 96822.

[1]R. Blankenbecler, S. J. Brodsky, and J. F. Gunion, Phys. Lett. 42B, 461 (1972).
[2]P. V. Landshoff and J. C. Polkinghorne, Phys. Lett. 45B, 361 (1973).
[3]J. D. Bjorken and J. Kogut, Phys. Rev. D 8, 1341 (1973).
[4]F. W. Büsser et al., Phys. Lett. 46B, 471 (1973).
[5]M. Banner et al., Phys. Lett. 41B, 547 (1972).
[6]B. Alper et al., Phys. Lett. 44B, 521, 527 (1973).
[7]D. C. Carey et al., Phys. Rev. Lett. 32, 24 (1974).
[8]J. W. Cronin et al., Phys. Rev. Lett. 31, 1426 (1973).
[9]D. Swanson et al., ANL Report No. ANL/HEP 7357, 1973 (unpublished); G. R. Charlton and G. H. Thomas, Phys. Lett. 40B, 378 (1972); S. N. Ganguli and P. K. Malhotra, Phys. Lett. 39B, 632 (1972).
[10]R. M. Sternheimer, Phys. Rev. 99, 277 (1955).
[11]V. D. Bartenev et al., in *Advances in Cryogenic Engineering*, edited by K. D. Timmerhaus (Plenum, New York, 1973), Vol. 18, p. 460.
[12]U. Amaldi et al., Phys. Lett. 43B, 231 (1973).
[13]V. Bartenev et al., Phys. Rev. Lett. 31, 1367 (1973).
[14]The shape of the radial function $f(x_R)$ is sensitive to errors in the energy dependence of our normalization to the pp elastic cross section and in the energy calibration of our lead glass counter, especially near the phase-space boundary $x_R=1$. The absolute normalization of the data is reflected in the magnitude of the $p_\perp$ function.

Unified Description of Single-Particle Production in *pp* Collisions*

D. C. Carey, J. R. Johnson, R. Kammerud, M. Peters,† D. J. Ritchie, A. Roberts, J. R. Sauer, R. Shafer, D. Theriot, and J. K. Walker
National Accelerator Laboratory, Batavia, Illinois 60510

and

F. E. Taylor
Physics Department, Northern Illinois University, DeKalb, Illinois 60115
(Received 15 May 1974)

We show that over a very wide range of c.m. angles, energies, and transverse momenta, the invariant cross sections for inclusive production of hadrons in pp collisions show a very simple behavior when expressed in terms of a new radial scaling variable $x_R \equiv p^*/p_{\max}^*$, where p^* is the total c.m. momentum of the hadron.

In a separate Letter,[1] we have presented data on inclusive π^0 production from pp collisions at the National Accelerator Laboratory (NAL), showing that for c.m. angles from 40 to 110°, transverse momenta from 0.3 to 4.3 GeV/c, and incident energies from 50 to 400 GeV, the invariant cross sections factorize very simply into the product of two universal functions:

$$E\, d^3\sigma/dp^3 = f(x_R) g(p_\perp), \tag{1}$$

where x_R is an angle-independent scaling variable,

$$x_R \equiv p^*/p_{\max}^* \approx 2p^*/\sqrt{s}. \tag{2}$$

The observed dependence on x_R can be approximated by

$$f(x_R) \propto (1-x_R)^4, \tag{3}$$

and the intrinsic transverse-momentum dependence by

$$g(p_\perp) \propto (p_\perp^2 + m^2)^{-4.5}, \quad m^2 = 0.86 \text{ GeV}^2. \tag{4}$$

The simplicity of this factorization and the wide kinematic range over which we have found it applicable make it desirable to examine other pp inclusive data. Although the available data are limited, we feel it is important to provide an initial overview of an emerging pattern for inclusive measurements in strong interactions.

We have compared our results with other inclu-