tering rate and the impurity scattering rate. The leading umklapp process will decrease exponentially with temperature and, depending on sample conditions, may be the dominant contribution.

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Diffraction, Refraction, and Interference Phenomena in Heavy-Ion Transfer Reactions

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The forward-angle oscillations recently observed in heavy-ion transfer reactions are explained as the Young interference pattern of a refractive two-slit "optical system" in l space, and a new phenomenon is predicted.

The forward-angle oscillations observed[1-3] in heavy-ion transfer reactions undergo a characteristic modification as the bombarding energy is increased into the range where the nuclear force significantly modifies the Coulomb trajectory. We offer here an analysis of distorted-wave Born-approximation (DWBA) calculations which fit these oscillations, to show that their energy dependence can be understood in simple optical terms (diffraction, refraction, and interference), and to predict an interesting new feature of the oscillatory pattern which should appear at bombarding energies somewhat higher than those employed up to now.

An example of these angular distributions is shown in Fig. 1, which displays LOLA[4] calculations of the (one-step) reaction $^{48}Ca(^{16}O, {}^{14}C)^{50}Ti$ to the ground state. The familiar "grazing-angle peak" seen in the 40-MeV curve moves to more forward angles and becomes "inundated" by the oscillatory pattern at 56 MeV. This is further accentuated in the 85-MeV distribution, which also exhibits the qualitatively new feature, a "modulation" of the envelope of the oscillations, giving them exceptionally large peak-to-valley ratios at 0, 35, and 70°, with small peak-to-valley ratios at 15 and 50°.

Several interesting papers[1,3,5-7] have recently provided partial explanations of certain of these features. Our purpose here is to indicate how these discussions can be unified, and extended to explain the amplitude modulation seen in the 85-MeV curve of Fig. 1. The oscillations, as pointed out by Chasman, Kahana, and Schneider,[1] (CKS) are an interference phenomenon, arising basically from the highly peripheral nature of these transfer reactions. This "surface peaking" forces the trajectories which contribute substantially to the reaction to pass through two narrow "l windows," centered at $l=l_0$, on either side of the target nucleus. The narrowness of these windows (they have a full width $\Delta l \approx 4$ to 6 in the cases studied here) causes the passing beams to fan out diffractively into two "grazing-angle peaks" of full width $\Delta\theta \geqslant 2/\Delta l$, which at low energy are separated by an angular distance $2\theta_0$. This is the equivalent of a two-slit optical system with oppositely slanted prisms behind the slits to separate the two beams; the forward oscillations seen at 40 MeV in Fig. 1 are the Young interference pattern resulting from the overlap of the tails of the grazing-angle peaks centered at $\pm\theta_0$.

To see more clearly how this comes about, we consider first the simplest case of zero l transfer (the conclusions, however, are general) and, for the semiclassical case $l_0 \gg 1$, employ the large-l approximation to $P_l(\cos\theta)$ for $\theta > 1/l$. The partial-wave decomposition of the transfer amplitude, $f(\theta)=\sum f(l-l_0)P_l(\cos\theta)$, then takes the form

$$f(\theta)\approx(\tfrac{1}{2}\pi l_0\sin\theta)^{1/2}\{\exp(i[(l_0+\tfrac{1}{2})\theta-\tfrac{1}{4}\pi])\sum f(l-l_0)\exp[i(l-l_0)\theta] + \exp(-i[(l_0+\tfrac{1}{2})\theta-\tfrac{1}{4}\pi])\sum f(l-l_0)\exp[-i(l-l_0)\theta]\}, \qquad (1)$$

in which each of the two sums represents a beam passing through one of the l windows. In order for them to interfere destructively to produce deep minima, their amplitudes must be equal at the same θ. To see where this occurs, we define the single-slit amplitude

$$g(\theta)=\sum f(l-l_0)\exp[-i(l-l_0)\theta], \qquad (2)$$

which is the Fourier transform of $f(l-l_0)$. The single-slit pattern (or grazing-angle peak) has the shape $|g(\theta)|^2/\sin\theta$, and Fig. 2 displays $|g(\theta)|^2$ and $|g(-\theta)|^2$ for the three cases of Fig. 1. It shows the merging of the two peaks with increasing energy, and in addition makes clear the way in which the amplitude modulations of Fig. 1 arise from the curve crossings of the two peaks of Fig. 2. These modulations are due entirely to

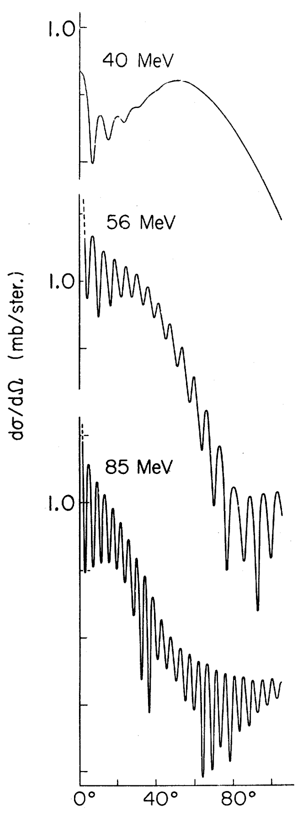

FIG. 1. DWBA calculations (log scale) for $^{48}Ca(^{16}O, ^{14}C)^{50}Ti$, with the parameters of Ref. 2. The optical potential has a transparent surface, $R_V - R_W = 0.5$ fm.

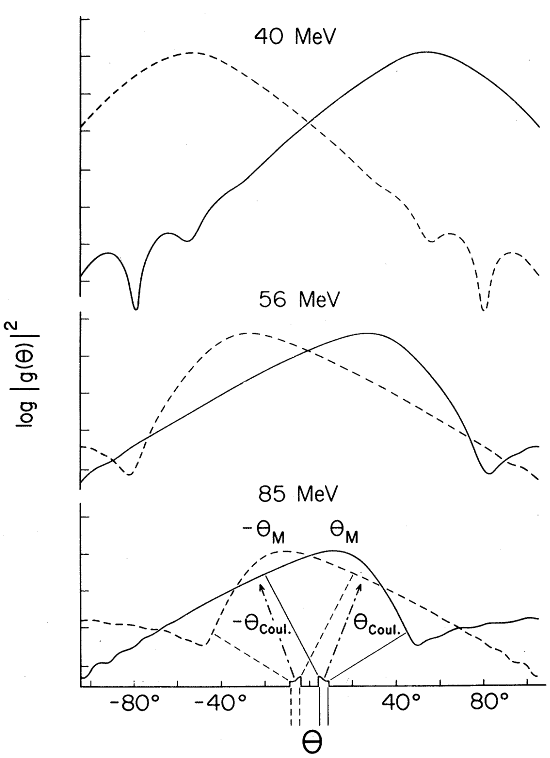

FIG. 2. Single-slit diffraction-refraction patterns from the DWBA calculations of Fig. 1. The refractive two-slit system is suggested at the bottom of the figure.

(a) the *asymmetries* and (b) the *minima* shown by the single-slit peaks. We turn next to a consideration of their origin.

Writing $f(l-l_0)=\rho(l-l_0)\exp[2i\delta(l-l_0)]$, we can associate $\rho(l-l_0)$, the "shape" of the l window, with the *diffractive* spreading of the beam, and $\delta(l-l_0)$ with its *refractive* focusing and deflection, as though lenses were placed behind the windows to produce different "optical paths" for different impact parameters. As an example, CKS assume a phase linear in l, $\delta(l-l_0)=(l-l_0)\theta_0$, which will clearly just shift the Fourier transform of $\rho(l-l_0)$ by the angle θ_0: A linear phase implies prismatic refraction.

In order to study the refractive and diffractive effects of $\delta(l-l_0)$ and $\rho(l-l_0)$, we must examine the single-slit patterns $g(\theta)$ which are the Fourier transforms of the reaction amplitudes $f(l-l_0)$. Insight into their properties may be obtained by converting the l sum to an integral and evaluating the latter by the method of stationary phase, a technique fruitfully exploited for elastic scattering by Ford and Wheeler. For this purpose it is convenient to consider $f(l-l_0)$ in the form $\exp[i\Delta(l-l_0)]$, where $\Delta(l-l_0)=2\delta(l-l_0)+i\ln\rho(l-l_0)$. With $\ln\rho(l-l_0)=0$, one can carry out the conventional stationary-phase treatment of *refraction,* and determine the trajectories for which l and the "classical" reaction angle θ are related by $2d\delta(l-l_0)/dl=\theta(l)$. At the other extreme, i.e., for $\delta(l-l_0)=0$, the stationary-phase evaluation leads to the familiar *diffraction* results. For example, a $\Delta(l-l_0)$ of $i(l-l_0)^2/\Gamma^2$ provides $g(\theta)\sim\exp[-(\Gamma^2\theta^2)/4]$ which comes from a "stationary phase" associated with a complex l, $l=l_0+i\frac{1}{2}\Gamma^2\theta$. Clearly a stationary-phase treatment of a fully complex Δ_l is capable of treating simultaneously both refractive and diffractive effects.

In Fig. 3 we have plotted $\rho(l-l_0)$, and $2\delta(l-l_0)$ obtained from the 85-MeV LOLA calculation. We have also plotted $\theta=2d\delta(l-l_0)/dl$, which is seen to have a minimum as a function of l. Such a minimum is analogous to the minimum scattering angle conventionally referred to as a "rainbow" angle θ_R, in semiclassical terms. This type of refractive dip was seen at each of nine energies ranging from 40 to 120 MeV, and in each case was positioned with striking accuracy at the center of the l window, l_0. For an optical potential whose imaginary part is of shorter range than its real part, this seems to have a simple explanation in the Sopkovich model,[8] in which $2\delta(l-l_0)$ is the sum of the channel phases. The channel Cou-

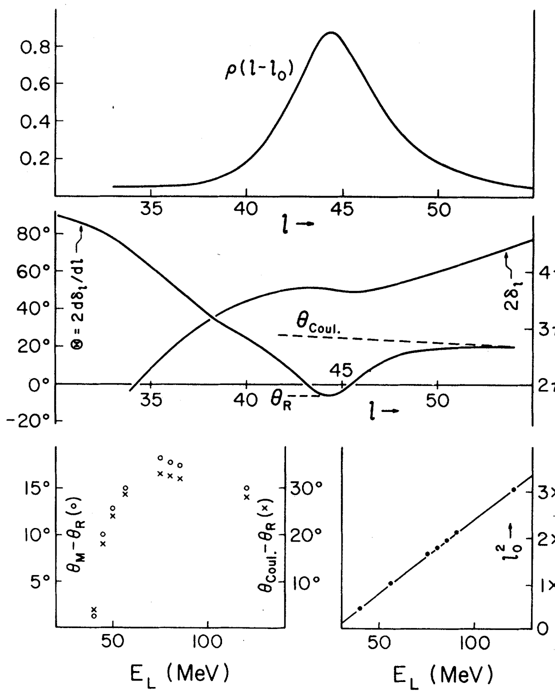

FIG. 3. Top and center: magnitude, phase, and phase derivative for the 85-MeV DWBA amplitudes. Lower figures are explained in text.

lomb phases together give $2d\sigma_c/dl\approx2\tan^{-1}\eta_c/l_c=\theta_{\text{Coul}}$ which provides the monotonic background from which the dip "hangs." The dip, however, is determined by the behavior of the nuclear phase shift δ^N. For large l, δ_l^N is zero; approaching the window from above, δ_l^N begins to rise as the nuclear force pulls the flux forward from θ_{Coul}, but with the onset of absorption the rise is stalled and a point of inflection is introduced at which $d^2\delta_l^N/dl^2=0$. Note, this inflection point (and consequently the minimum θ) comes at the onset of the absorptive effects and thus always occurs at approximately l_0, or the center of the window. The value of θ_R, on the other hand, is determined by both Coulomb and nuclear forces, the Columb force giving the background, and the nuclear force determining the extent of the dip. This dip, at the center of the l window, guarantees both asymmetry and broadening in the single-slit patterns $|g(\theta)|^2$.

To obtain qualitative features arising from the interplay of refractive and diffractive effects we have used stationary-phase techniques to examine $g(\theta)$. A model which can reproduce the general features of the one-slit patterns of Fig. 2 is $f(l-l_0)=\exp[-(l-l_0)^2/\Gamma^2]\exp[2i\delta(l-l_0)]$, with $\delta(l-l_0)=(l-l_0)\theta_R+\frac{1}{3}\beta(l-l_0)^3$; this approximates the "nuclear-rainbow" dip in $\theta(l)$ by a parabola. It is convenient to define a "refraction parameter" Φ, as that area of the dip which is contained within the l window. For the present model $\Phi = \frac{4}{3}\beta\Gamma^3$. In nuclear terms, it is a measure of the amount by which the deflection angle deviates from $\theta_{\rm Coul}$, for trajectories which pass through the l window.

The resulting $|g(\theta)|^2$, which is very similar to those of Fig. 2, is best understood by noting that the curves in Fig. 2 bear a striking resenblance to the Airy function which describes a classical rainbow. The right-hand (solid) curves are oriented with the bright side of the rainbow (steeper falloff from the principal maximum) toward larger angles and the dark side (gentler falloff from the principal maximum) toward smaller angles. The minimum, on the bright side (at 80° at 56 MeV and at 50° at 85 MeV), is to be associated with the first rainbow zero; the rainbow angle θ_R is on the opposite side of the main peak (at $-5°$ in the 85-MeV case, as Fig. 3 indicates).

The $g(\theta)$ for the present model has two simple limits for large and small Φ. For $\Phi \to 0$, $|g(\theta)|^2 \to \exp[-\frac{1}{2}\Gamma^2(\theta-\theta_R)^2]$, the symmetric CKS peak, centered at $\theta=\theta_R$. However, if $\Phi = \frac{4}{3}\beta\Gamma^3$ is increased by opening the window, the asymmetric refraction due to β is introduced. This shifts the peak at θ_M away from θ_R toward the bright (right-hand) side of the rainbow and simultaneously brings a zero down toward the peak from above, producing the asymmetric principal peak of the Airy function. Note that this is exactly the type of change observed in the curves of Fig. 2 as the energy rises above the Coulomb barrier and the nuclear influence on the trajectory, measured by β, causes an increase in Φ. This interpretation is reinforced by the lower left-hand inset in Fig. 3, which shows the two differences, $\theta_M - \theta_R$, and $\theta_{\rm Coul} - \theta_R$, as a function of laboratory energy E_L.

In summary, it is our conclusion that the minimum seen in the 56- and 85-MeV curves is primarily a *refractive* or rainbow-type minimum, due principally to the nuclear-force influence on $\delta(l-l_0)$, rather than a diffraction zero arising from $\rho(l-l_0)$. In fact, the Fourier transform of the LOLA $\rho(l-l_0)$ alone is found to exhibit no minima at any of these energies.[9] Thus the amplitude modulation of Fig. 1, which is in part a consequence of this minimum, appears to be extremely sensitive to the tail of the optical potential, and may prove to be a means for determining this tail with great accuracy. In this regard, the lower right-hand inset in Fig. 3 shows that $l_0 \approx kR$ with $R = 8.7$ fm; for comparison, $R_V = 8.3$ fm and $R_W = 7.8$ fm.

Ascuitto and Glendenning[5] have noted that decreasing R_W by 5% can increase the $\theta=0$ cross section by a factor of 10. Careful inspection of their curve suggests that this is also best understood as a similar asymmetric-refraction effect: Decreasing R_W increases Φ by opening the l window and increasing $\theta_R - \theta_{\rm Coul}$. This tips the $|g(\theta)|^2$ so as to increase the cross section at angles below the grazing-angle peak, and decreases it at angles above the peak. An even more interesting phase effect can be seen by comparing one- and two-nucleon transfers. The smaller binding energy and mass transfer of the former increases its form-factor range by 2 relative to that for the two-nucleon transfer, thus opening the right-hand side of the l window substantially. This exposes much more of the slowly-varying Coulomb phase for $l > l_0$ and so builds up a large grazing-angle peak at $\theta = \theta_{\rm Coul}$ which is missing from the two-nucleon cross section. This is illustrated clearly in the $^{16}O + {}^{64}Ni$ data of Ref. 3, and might even suffice to distinguish sequential transfer of two nucleons from a one-step process.

We are very grateful to W. Henning, D. G. Kovar, B. Zeidman, and J. R. Erskine of the Argonne National Laboratory for making their data available prior to publication and providing the LOLA calculations analyzed here.[10]

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tive.

[10]We thank H. L. Harney, P. Braun-Munzinger, and C. K. Gelbke for a preprint that arrived while this manuscript was in preparation, which similarly emphasizes some of the two-slit features of these cross sections.

Search for a Δ-Δ Component in the Deuteron Wave Function Using $\bar{p}d$ Interactions at 5.55 GeV/c

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From a bubble-chamber experiment, we studied the $\bar{p}d \to p_s\bar{p}p\pi^-$, $p_s\bar{p}p\pi^-\pi^0$, $p_s\bar{n}p\pi^-\pi^-$ reactions at 5.55 GeV/c in order to see whether a Δ-Δ component of the deuteron wave function can be observed in our data. This was achieved by searching for spectator $\Delta \to p_s\pi$ systems. We observed significant production of $\Delta \to p_s\pi$, a fraction of which is emitted in the backward laboratory hemisphere. The meaning of this high $\Delta \to p_s\pi$ production rate is discussed.

Recently the possibility that virtual Δ's may be present in the nucleus has been considered.[1,2] It has been suggested that a way to test such a picture is to study inelastic reactions on the deuteron.[3,4] Because of isospin conservation, the deuteron can transform into only two Δ's. Then following the impulse approximation, a fraction of the interactions induced by fast beam particles may occur on a virtual Δ, the other appearing as a Δ spectator (Δ_s). In this approach the observation of Δ_s in inelastic channels may give some information about the Δ-Δ component of the deuteron wave function. Furthermore the presence of Δ in the deuteron may also offer a way to study interactions on virtual Δ targets.

The percentage of the Δ-Δ component in the deuteron wave function has been estimated from theoretical calculations to be a few percent.[1,2] Although this percentage is small, we attempted to see if Δ_s appear among the outgoing particles in $\bar{p}d$ interactions at 5.55 GeV/c by collecting data from the following channels

$$\bar{p}d \to p_s\bar{p}p\pi^- \quad (1348 \text{ events}), \qquad (1)$$

$$\to p_s\bar{p}p\pi^-\pi^0 \quad (651 \text{ events}), \qquad (2)$$

$$\to p_s\bar{n}p\pi^-\pi^- \quad (221 \text{ events}). \qquad (3)$$

The present data which have already been partially studied[5] were obtained from a bubble chamber experiment. All the considered events contain an outgoing proton (p_s) stopping in the chamber. One has thus a sample in which the p_s presents roughly the behavior of a spectator proton. In order to search for Δ_s in the final state, we will consider the various outgoing $p_s\pi$ systems, and discuss later the biases introduced by taking only events having a proton stopping in the chamber. To decrease complications due to resonance reflection, we studied $p_s\pi$ systems in which the π does not contribute to resonance production with the remaining outgoing particles. In Reaction (1) we thus excluded the $\bar{p}d \to p_s\bar{\Delta}^{--}p$ subsample obtaining 1348 events. For channel (2) we consider all of the $p_s\pi^0$ systems since the π^0 contribute in a negligible amount to resonance production. Within the present statistics this is likewise true for the π^- in channel (3), allowing us to consider the two $p_s\pi^-$ combinations.

Using the $p_s\pi$ systems thus selected, we present in Fig. 1(a) the scatter plot of $\cos\theta$ versus the $p_s\pi$ effective mass ($M_{p_s\pi}$). Here θ is the laboratory emission angle of the $p_s\pi$ system defined with respect to the incoming beam direction. One sees from this plot that the events having their $M_{p_s\pi}$ in the Δ band ($1.15 < M_{p_s\pi} < 1.32$ GeV/c^2) are distributed over all the $\cos\theta$ range. A similar structure observed[3] in the reaction $\pi^+d \to p_sp\pi^+\pi^-\pi^0$ at 15 GeV/c[3] was accounted for by the existence of virtual Δ in the deuteron since the $p_s\pi^+$ systems in the Δ band seem to be emitted isotropically in the laboratory system as predicted by the impulse-approximation approach.[3] Even in this model such a picture is oversimplified because one neglects, among other things, the following effects: the cross-section variation of the studied process for the c.m. energy spread