channeled H_2^+ ions through gold foils show that a significant fraction emerges from the foils as molecules. H^+H^+ pairs locked in spatial correlation may contribute significantly to this transmission probability. It would be interesting to ascertain the importance of wake-riding states of protons behind heavier ions, for example, in transmission experiments with incident fast molecules such as LiH, CH, OH, or hydrogen halides.

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Fluctuation Resistivity in One-Dimensional Metals

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The resistivity of one-dimensional metals is calculated for two models in which impurity scattering is the only mechanism for dissipating momentum. In the first model, impurity scattering is added to the Fröhlich Hamiltonian. In the fluctuation regime above an incommensurate Peierls transition it is found that the resistivity is enhanced. For a general two-body interaction Hamiltonian it is found that charge-density–wave fluctuations predominate over Cooper-pair fluctuations leading to enhanced resistivity.

There has been much speculation recently that enhanced conductivity can be obtained from fluctuation contributions in one-dimensional metals.[1-6] A general feature of one-dimensional metals is their inherent instability towards the formation of a charge-density wave with a period of twice the Fermi wave vector ($2k_F$).[7-10] In this Letter we examine two models for which impurity scattering is the only dissipative mechanism for momentum and find that the density fluctuations lead to an enhanced resistivity, not conductivity. One is the Peierls instability arising from the electron-phonon coupling. The second problem is that of a one-dimensional metal with a general two-body Hamiltonian. In this case even though attractive interactions cause a divergence in the Cooper-pair response function, the scattering time is dominated by the divergence in the electronic polarizability at $2k_F$, and the fluctuation contribution leads to a divergence in the resistivity.

We consider first the Peierls instability with a

Hamiltonian of the form

$$H = H_0 + H_1 + H_2, \tag{1}$$

$$H_0 = \sum_{k\sigma} \epsilon(k) a_{k,\sigma}^{\dagger} a_{k,\sigma}, \tag{2}$$

$$H_1 = L^{-1/2} \sum_{kk'\sigma j} U \exp[i(k-k')R_j] a_{k',\sigma}^{\dagger} a_{k,\sigma}, \tag{3}$$

$$H_2 = L^{-1/2} \sum_{k\sigma q} g[a_{k+q,\sigma}^{\dagger} a_{k,\sigma} (b_{-q}^{\dagger} + b_q) + \text{H.c.}] + \sum_q \omega_q b_q^{\dagger} b_q. \tag{4}$$

The first term is the kinetic energy of the electrons and $a_{k,\sigma}^{\dagger}$ is an electron creation operator. The third term is the Fröhlich electron-phonon coupling term and $b_q^{\dagger}$ is a phonon creation operator. We have included in the Hamiltonian a term representing the impurity scattering with an impurity potential U.

A calculation of the conductivity is meaningful only in the presence of impurity or umklapp phonon scattering. Ohterwise, as Peierls has pointed out,[7] there is perfect phonon drag and no relaxation of the current. We will consider an incommensurate model with the main relaxation arising from impurity scattering. In a strongly commensurate case, such as a half-filled band, Patton and Sham[11] have shown that the fluctuation contributions enhance the resistivity. In discussing the conductivity in one-dimensional metals it is more convenient to use the momentum-relaxation-rate formulation rather than the current-current response-function formulation. A convenient formulation of the former has been given recently by Mori[12] and by Götze and Wölfle[13] and used by Luther and Peschel[14] in the context of impurity scattering in the Tomonaga model. However, it requires some care to apply this formula to the electron-phonon system where the above-mentioned phonon drag is essential. First we note that within the Born approximation for impurity scattering the relaxation rate τ^{-1} of the total momentum P, defined by

$$P = \sum_{k\sigma} k a_{k,\sigma}^{\dagger} a_{k,\sigma} + \sum_q q b_q^{\dagger} b_q, \tag{5}$$

can be expanded in powers of c as

$$\tau^{-1} = \frac{c}{\omega} U^2 \sum_{kk'} \left(\frac{\partial \epsilon}{\partial k} - \frac{\partial \epsilon}{\partial k'} \right)^2 \mathrm{Im} N(k-k', \omega), \tag{6}$$

where c is the concentration of impurities. $N(q, \omega)$ is the density-density response function evaluated for $H_0 + H_2$, Eqs. (3) and (4). In the steady state ($\omega = 0$) under a weak static electric field, which is the usual circumstance of measuring conductivity, the relaxation time of the electronic current is the same as that of the total momentum P, since within the Hamiltonian (1) phonons cannot dissipate their momentum without the scattering of electrons.

The calculation of τ^{-1} reduces then to the calculation of $\mathrm{Im} N(2k_F, \omega)$. If we assume a finite value for the transition temperature T_P, then an expansion of $\mathrm{Im} N(2k_F, \omega)$ in terms of ϵ [$= (T - T_P)/T_P$] leads to a consideration of the diagram in Fig. 1(b) as the leading correction to the lowest-order diagram in Fig. 1(a), where the wavy line represents a fluctuation propagator of the form $D^{-1}(q, \omega) = -\omega_0^2(\epsilon + c_{\parallel} v_F^2 q_{\parallel}^2/T^2 + c_{\perp} v_F^2 q_{\perp}^2/T^2 - i\pi\omega/8T)$. Here $\vec{q}$ is the deviation from $2k_F$, v_F is the Fermi velocity, and $c_{\parallel}$ and $c_{\perp}$ are numerical constants. We have used a more general form for D^{-1} than that of Ref. 11 in order to include possible phonon dispersion in the directions perpendicular to the chains. If we denote the zone-boundary value of $q_{\perp}$ by q_m, the condition to be met in the three-dimensional limit is $c_{\perp} v_F^2 q_m^2/T^2 > \epsilon$, and in this limit the leading correction to the rate is

$$\tau^{-1} = \tau_0^{-1} \left(1 + \frac{\pi \ln^2(E_F/T)}{32 c_{\parallel}^{1/2} c_{\perp} \epsilon^{1/2} v_F^2 q_m^2/T^2} \right), \tag{7}$$

where $\tau_0^{-1} = 2cU^2 v_F^{-1}$ is the normal momentum relaxation rate. In the opposite limit the behavior is one-dimensional and

$$\tau^{-1} = \tau_0^{-1} \left(1 + \frac{\pi \ln^2(E_F/T)}{16 c_{\parallel}^{1/2} \epsilon^{3/2}} \right). \tag{8}$$

In these formulas E_F is the electronic energy cutoff. In both cases an enhancement of the momentum relaxation rate is obtained. In the one-dimensional case, the expression is only the leading order in an uncontrolled expansion, e.g., Fig. 1(c) contributes a term $\sim \epsilon^{-5/2}$ to the expansion.

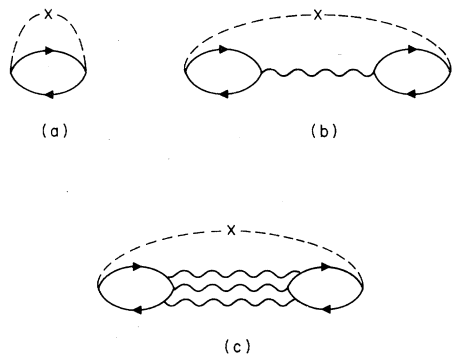

FIG. 1. Leading fluctuation contributions to the momentum relaxation rate according to Eq. (6).

While the coefficient of $\epsilon^{-3/2}$ in (8) is larger by the factor $\ln^2(E_F/T)$ it is clear that there is no choice of parameters for which this expansion is convergent. This implies that the critical region in this problem is of the order of the transition temperature and nonlinear fluctuation effects are important. However, the main consequence of the higher-order fluctuations is to renormalize the critical temperature to zero temperature, but they are not expected to change the overall qualitative behavior of N. It is noteworthy that the term shown in Fig. 1(b) is the high-temperature counterpart of the pinning term which was found[5] to destroy Fröhlich superconductivity in an ordered state. In such a state it is the collective-mode scattering with momentum transfer of $2k_F$ to the impurity which is responsible for the pinning of the charge-density wave and destruction of Fröhlich superconductivity. In the fluctuation regime, this process leads to an enhancement of the momentum relaxation rate and therefore of the resistivity.

In passing from τ^{-1} to the resistivity ρ a factor of m^*/ne^2 enters, where n is the electron density and m^* is an effective mass. The value of m^* may be obtained by examining the transport equations in the phonon-drag regime in the absence of impurities, or the expression for ρ^{-1} obtained in the ordered phase.[5] In both cases m^* is enhanced by a factor containing the number of phonons at $2k_F$ and this factor will give rise to an additional enhancement of the resistivity in the fluctuation regime.

Let us now consider a general two-body interaction Hamiltonian of the form $H_0+H_1+H_3$, where

$$H_3=(g_1/L)\sum a_{k_1,\alpha}^{\dagger}a_{k_2-2k_F,\beta}^{\dagger}a_{k_3,\beta}a_{k_4-2k_F,\alpha}$$
$$+(g_2/L)\sum a_{k_1,\alpha}^{\dagger}a_{k_2-2k_F,\beta}^{\dagger}a_{k_3-2k_F,\beta}a_{k_4,\alpha}. \quad (9)$$

All the k_i are near $+k_F$. The coupling constants g_1 and g_2 represent scattering with momentum transfer near $2k_F$ and 0, respectively. Again one can relate the momentum relaxation rate τ^{-1} to $\mathrm{Im}N(2k_F,\omega)$ within the Born approximation. The response functions for a momentum transfer near $2k_F$ have been calculated using the renormalization-group method by Sólyom.[10] For an attractive interaction at $2k_F$ ($g_1<0$) the first-order renormalization method leads to a singularity at finite T and the coupling scales to the strong-coupling limit. Second-order renormalization-group methods remove the singularity to zero temperature as must be the case for a one-dimensional system[15] and Sólyom[10] finds the result

$$\mathrm{Re}N(2k_F+q,\omega)\sim[\max(v_F|q|,\omega,T)/E_F]^{-3/2}. \quad (10)$$

Using the Kramers-Kronig relation one obtains corresponding results for $\mathrm{Im}N(2k_F+q,\omega)$, and one obtains the result, using (10), that

$$\tau^{-1}\sim\tau_0^{-1}(T/E_F)^{-3/2}. \quad (11)$$

Again the fluctuations enhance the momentum relaxation rate and thus lead to an *enhanced resistivity*. In this system the Cooper-pair response function Δ is also diverging and the effect of this divergence can be traced as a high-order set of terms in the perturbation expansion for N, which are opposite in sign to the dominant terms arising from the Peierls instability. Formula (6) is applicable only in the weak-scattering limit ($\tau T>1$). As the scattering rate grows this condition is violated and one must include impurity scattering to higher order, especially in the calculation of N itself.

The Hubbard model represents a special case of the Hamiltonian (9), in which $g_1\equiv g_2=g$. Thus for $g<0$, resistive fluctuations are predicted. An interesting limiting case occurs if we set $g_1\equiv 0$. The model is then the Tomonaga model. The form of $N(2k_F+q,\omega)$ within the renormalization method is drastically changed and one finds

$$N(2k_F+q,\omega)\sim[\max(v_F|q|,\omega,T)/E_F]^{-g_2/\pi v_F}. \quad (12)$$

Thus for attractive interactions ($g_2<0$), $N\to 0$ as $T\to 0$. A similar result holds for the imaginary part and one finds $\tau^{-1}\sim\tau_0^{-1}(T/E_F)^{-g_2/\pi v_F}$. Within this model the fluctuations *enhance the conductivity*. These results are in agreement with the exact results of Luther and Peschel[14] and of Mattis[16] for the Tomonaga model. For the more general model with $g_1\neq 0$, the behavior will be governed by g_1 primarily and the conditions to obtain enhanced conductivity, i.e., $N\to 0$ rather than $N\to\infty$, are $g_1>0$, and $2g_2-g_1<0$. In practice these conditions will be hard to realize since a study[17] of the combined electron-phonon and electron-electron Hamiltonian shows a strengthening of the singularity in $N(2k_F,\omega)$ and a diminishing of that in $\Delta(2k_F,\omega)$.

In conclusion, we find that for most circumstances, a one-dimensional metal will have a divergence in the electronic polarizability near $2k_F$ which will lead to enhanced momentum scattering rate from impurities in the fluctuation regime. However the overall behavior of the resistivity as the temperature is decreased will be determined by the combined umklapp phonon scat-

tering rate and the impurity scattering rate. The leading umklapp process will decrease exponentially with temperature and, depending on sample conditions, may be the dominant contribution.

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Diffraction, Refraction, and Interference Phenomena in Heavy-Ion Transfer Reactions

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The forward-angle oscillations recently observed in heavy-ion transfer reactions are explained as the Young interference pattern of a refractive two-slit "optical system" in l space, and a new phenomenon is predicted.

The forward-angle oscillations observed[1-3] in heavy-ion transfer reactions undergo a characteristic modification as the bombarding energy is increased into the range where the nuclear force significantly modifies the Coulomb trajectory. We offer here an analysis of distorted-wave Born-approximation (DWBA) calculations which fit these oscillations, to show that their energy dependence can be understood in simple optical terms (diffraction, refraction, and interference), and to predict an interesting new feature of the oscillatory pattern which should appear at bombarding energies somewhat higher than those employed up to now.

An example of these angular distributions is shown in Fig. 1, which displays LOLA[4] calculations of the (one-step) reaction ^{48}Ca(^{16}O, ^{14}C)^{50}Ti to the ground state. The familiar "grazing-angle peak" seen in the 40-MeV curve moves to more forward angles and becomes "inundated" by the oscillatory pattern at 56 MeV. This is further accentuated in the 85-MeV distribution, which also exhibits the qualitatively new feature, a "modulation" of the envelope of the oscillations, giving them exceptionally large peak-to-valley ratios at 0, 35, and 70°, with small peak-to-valley ratios at 15 and 50°.

Several interesting papers[1,3,5-7] have recently provided partial explanations of certain of these features. Our purpose here is to indicate how these discussions can be unified, and extended to explain the amplitude modulation seen in the 85-MeV curve of Fig. 1. The oscillations, as pointed out by Chasman, Kahana, and Schneider,[1] (CKS) are an interference phenomenon, arising basically from the highly peripheral nature of