1/f Noise from Thermal Fluctuations in Metal Films*

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The observed dependence of the $1/f$ noise in metal films on the temperature coefficient of resistance and on sample volume, and the observation of frequency-dependent spatial correlations of the noise, suggest the noise is due to a thermal-diffusion mechanism. The assumption of a spectrum for the intrinsic temperature fluctuations with an explicit $1/f$ region, and the condition $\langle(\Delta T)^2\rangle = kT^2/C_V$, yield quantitative predictions of the noise power in excellent agreement with experiment.

Several authors[1,2] have studied diffusion of heat or particles as a possible mechanism for $1/f$ noise. The spectra calculated for simple uniform systems, although "$1/f$-like," do not exhibit a convincing $1/f$ behavior over many decades of frequency. As a consequence, diffusion has not been accepted as a source of $1/f$ noise.[2] In this Letter, we present experimental evidence that strongly supports a thermal-diffusion mechanism as the source of $1/f$ noise in thin metal films.

As an example of a thermal-diffusion system, consider an infinite uniform medium with thermal diffusivity D. If $\theta(\vec{x}, t)$ is the temperature at $\vec{x}$ at time t, θ obeys the diffusion equation $D\nabla^2\theta = \partial\theta/\partial t$. The average temperature of a bar-shaped volume Ω of length l_1 and rectangular cross section $l_2 l_3$ $(l_1 \gg l_2 \gg l_3)$, $T(t) = \Omega^{-1}\int_\Omega\theta(\vec{x}, t)d^3x$, is a fluctuating quantity.[3] In equilibrium at temperature T, $\langle(\Delta T)^2\rangle = k_B T^2/C_V$, and is independent of D, where C_V is the heat capacity of the bar. The spectrum of the fluctuations, $S_T(\omega) \equiv \langle T^*(\omega)T(\omega)\rangle$, does, however, depend on D, and on the manner in which a given fluctuation decays in space and time. The characteristic dimensions of the bar, $l_3 \ll l_2 \ll l_1$, define characteristic frequencies, $\omega_3 \gg \omega_2 \gg \omega_1$, through the relation $\omega_i = D/l_i^2$. The spectrum[1,4] is approximated by $S(\omega) \propto$ const $(\omega \ll \omega_1)$, $S(\omega) \propto \ln(1/\omega)$ $(\omega_1 \ll \omega \ll \omega_2)$, $S(\omega) \propto \omega^{-1/2}$ $(\omega_2 \ll \omega \ll \omega_3)$, and $S(\omega) \propto \omega^{-3/2}$ $(\omega \gg \omega_3)$. For two dimensions, $S(\omega) \propto \ln(1/\omega)$ $(\omega \ll \omega_1)$, $S(\omega) \propto \omega^{-1/2}$ $(\omega_1 \ll \omega \ll \omega_2)$, and $S(\omega) \propto \omega^{-3/2}$ $(\omega \gg \omega_2)$. For one dimension, $S(\omega) \propto \omega^{-1/2}$ $(\omega \ll \omega_1)$, and $S(\omega) \propto \omega^{-3/2}$ $(\omega \gg \omega_1)$. Normalization is achieved by $\int_0^\infty S_T(\omega)d\omega = \langle(\Delta T)^2\rangle$.

The spectra are best understood in terms of the frequency-dependent correlation length, $\lambda(\omega) \equiv (2D/\omega)^{1/2}$. The quantity $c(\vec{r}, \omega) \equiv \langle\theta^*(\vec{x}, \omega)\theta(\vec{x} + \vec{r}, \omega)\rangle$ is a measure of how fluctuations at $\vec{x}$ are related to those $\vec{r}$ away at frequency ω. In one dimension

$$c_1(\vec{r}, \omega) \propto \exp(-|\vec{r}|/\lambda)\cos(\pi/4 + |\vec{r}|/\lambda);$$

in three dimensions

$$c_3(\vec{r}, \omega) \propto |\vec{r}|^{-1}\exp(-|\vec{r}|/\lambda)\cos(|\vec{r}|/\lambda).$$

λ is thus a measure of the average spatial extent of a fluctuation at ω. Low-ω fluctuations on the average take place over larger volumes and have greater amplitudes than those at high ω. Changes in the spectrum occur when $\lambda \approx l_i$.

Although the heat flow is uniform throughout the system, we assume that electrical conduction is confined to the bar, of resistance R and temperature coefficient of resistance $\beta \equiv (1/R) \times \partial R/\partial T$. A constant current in the bar produces an average voltage V and a voltage noise spectrum $S_V(\omega) = V^2\beta^2 S_T(\omega)$. The spectrum is $1/f$-like, increasing as f is lowered, but there is no substantial $1/f$ region.

Hooge and Hoppenbrouwers[5] have studied $1/f$ noise in gold films, and found that the voltage spectrum $S_V(f)$ could be described by

$$S_V(f)/V^2 \approx 2.4 \times 10^{-3}/N_C f, \tag{1}$$

where N_C is the number of carriers in the sample. Although both the numerical coefficient and the exact $1/f$ law represent a rather approximate fit for a large number of samples, Eq. (1) is a useful basis for comparison. We have repeated these experiments using films typically 150 μm $\times 10$ μm $\times 400$ to 2000 Å evaporated or sputtered onto glass substrates. A constant current was passed along each sample, and the voltage spectrum from 0.1 to 25 Hz was measured using tuned circuits modeled in a PDP-11 computer. The amplifier background spectrum $(V = 0)$, which was 1–4 orders of magnitude smaller, was measured and subtracted out for each sample. The samples were usually placed in a closed metal

box during the measurements. We also measured several samples under vacuum, and found no detectable change in the noise, indicating that atmospheric turbulence did not contribute to the noise. The observed spectra varied as $1/f^{\alpha}$ where $1.0 < \alpha < 1.4$. We also found $S_V(f) \propto V^2$, and by varying sample dimensions, that $S_V(f)$ was approximately inversely proportional to sample volume. Samples of identical geometry showed variations of up to a factor of 3 in noise power. The measured average values of $S_V(f)/V^2$ at 10 Hz for seven different materials are summarized in Table I, together with the measured temperature coefficients of resistance for the same films. The negative β for the bismuth was probably due to the somewhat amorphous nature of the evaporated films. The resistivities of all the materials, including bismuth, were close to the bulk values.

Table I contains important evidence for the diffusion mechanism. The measured β for the Manganin films was $< 10^{-4}$ K^{-1}, more than an order of magnitude below that of the other materials. There was no detectable $1/f$ noise in the Manganin films. This result strongly suggests that the noise in the other metal films arises from temperature modulation of the resistance.

As a second test of the diffusion mechanism, we measured the frequency-dependent spatial correlation. A bismuth film of thickness 1000 Å was cut into two strips each of length l and width 12 μm [Fig. 1(a)]. Separate batteries and large resistors R_0 were used to supply a constant current to each strip. The two noise voltages $V_1(t)$ and $V_2(t)$ were separately amplified and the spectrum of their sum or difference measured with the PDP-11. If $S_+(f)$ and $S_-(f)$ are the spectra of $[V_1(t) + V_2(t)]$ and $[V_1(t) - V_2(t)]$, the fractional
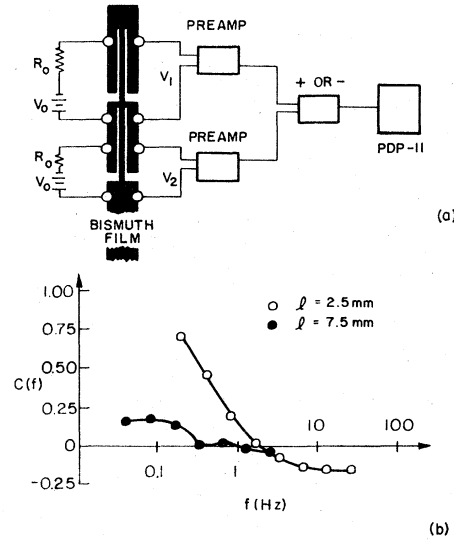

FIG. 1. (a) Experimental configuration for correlation measurement. (b) Fractional correlation for two samples.

correlation between the strips is

$$C(f) = [S_+(f) - S_-(f)]/[S_+(f) + S_-(f)]. \qquad (2)$$

When $V_1(t)$ and $V_2(t)$ are independent, $S_+(f) = S_-(f)$ and $C(f) = 0$. When the two strips are completely correlated $V_1(t) = V_2(t)$, $S_-(f) = 0$, and $C(f) = 1$. For temperature fluctuations at high f, $\lambda \ll l$ and $C(f) \rightarrow 0$, while at low f, $\lambda \gg l$ and $C(f) \rightarrow$ const. We expect the change from correlated to uncorrelated noise to occur when $\lambda \approx l$. Results for two different l are shown in Fig. 1(b). The condition $\lambda = l$ corresponds to frequencies of 0.13 and 1.2 Hz for $l = 7.5$ and $l = 2.5$ mm (with $D = 0.2$ cm^2 sec^{-1}). The measured $C(f)$ is in good agreement with our expectations. As l is increased,

TABLE I. The measured and calculated noise power at 10 Hz and the measured temperature coefficient of resistance for seven materials.

Material	$S_V(f)/V^2$ measured at 10 Hz $(10^{-16}$ Hz$^{-1})$	$S_V(f)/V^2$ calculated at 10 Hz $(10^{-16}$ Hz$^{-1})$	Measured β (K^{-1})
Cu	10	16	0.0038
Ag	10	2	0.0035
Au	1	0.76	0.0012
Sn	12	7.7	0.0036
Bi	20	9.3	-0.0029
Manganin	$< 10^{-2}$	$< 3.5 \times 10^{-3}$	$|\beta| < 10^{-4}$
InSb	3×10^4	20	-0.0054

the low-f limit of $C(f)$ decreases because (in two or three dimensions) a fluctuation in one strip has an increasing probability of decaying without influencing the other strip [cf. $c_3(\vec{r}, \omega)$]. For $l > 7.5$ mm, scatter in the data obscured any possible correlations.

Although the scaling of the noise as V^2/Ω, the absence of the noise in Manganin, and the frequency-dependent correlation all indicate a thermal-diffusion mechanism, the observed spectrum of a thin film differs from that calculated for the uniform system. There are several possible reasons for the discrepancy. The sample is not in thermal equilibrium. The power dissipated in the sample raises its temperature above ambient temperature. Moreover, a temperature fluctuation, by changing the resistivity, changes the power dissipated in its neighborhood resulting in a feedback effect. We expect this effect to change the dependence of the noise power on V^2 as well as to produce vastly different spectra for positive and negative feedback. The observed close dependence on V^2 and the similarity of the spectra for metals ($\beta > 0$) and bismuth ($\beta < 0$), however, lead us to believe that feedback is not generally a dominant mechanism. Only to lowest order is the sample resistance proportional to the average temperature. It is possible to have a temperature distribution which does not change the average temperature yet which does change the current paths and hence the resistance. We expect this effect to be negligible for λ greater than the strip width, and to be independent of β.

The major reason for the discrepancy, however, is probably the thermal inhomogeneity of the system. Although neither the very different diffusivities of ilm and substrate, nor the presence of boundary resistance can affect $\langle (\Delta T)^2 \rangle$, both can seriously modify $S_T(f)$, which depends on the decay of the temperature fluctuations. In an attempt to investigate the importance of the thermal environment of the film, we coated several samples with an electrically insulating layer of photoresist: No change in the noise was observed. However, a layer of copper about 5000 Å thick evaporated over the photoresist decreased the noise by roughly a factor of 2 below about 1 Hz, but did not affect the noise at higher frequencies. The spectrum, then, depends in a detailed way on the thermal environment of the film. At high frequency, we expect fluctuations to decay predominantly along the film (two-dimensional decay) because of the high thermal conductivity of the metal. Only at lower frequencies, where λ is

comparable with the sample length, does the substrate become important and the decay more three dimensional.

We therefore expect $S_T(f)$ to be constant for $f \ll f_1$, and to be proportional to $f^{-3/2}$ for $f \gg f_2$, where f_1 and f_2 are the characteristic frequencies for the length l_1 and the width l_2 of the strip. We postulate that the thermally inhomogeneous system has a $1/f$ spectrum in the intermediate frequency region. We model the spectrum by $S_T(f) = A/f_1$ ($f < f_1$), $S_T(f) = A/f$ ($f_1 < f < f_2$), and $S_T(f) = Af_2^{1/2}/f^{3/2}$ ($f > f_2$). Using the normalization condition, we find for the $1/f$ region

$$S_V(f) = V^2 \beta^2 k_B T^2 / C_V [3 + \ln(f_2/f_1)] f. \qquad (3)$$

At room temperature $C_V \approx 3N_A k_B$, where N_A is the number of atoms in the sample, and

$$S_V(f)/V^2 \approx \beta^2 T^2 / 3N_A [3 + 2\ln(l_1/l_2)] f. \qquad (4)$$

The last column of Table I contains the calculated values of $S_V(f)/V^2$ for the films at 10 Hz. Both metals ($N_C \approx N_A$) and bismuth ($N_A \approx 10^5 N_C$) show good agreement with Eq. (4). For the samples of Hooge and Hoppenbrouwers[5] (800 $\times$10 μm), using our measured β for gold, we find $S_V(f)/V^2 \approx 3.6 \times 10^{-3}/N_A f$ in remarkable agreement with Eq. (2), provided we set $N_C = N_A$.

We believe Eq. (3) should apply to any system. However, diffusion may not be the only or dominant source of $1/f$ noise. As an example, consider our results for evaporated InSb semiconducting films shown in Table I. The noise intensity was 3 orders of magnitude above that predicted by Eq. (3). In addition, we observed no spatial correlation of the noise. We conclude that the thermal-diffusion mechanism was *not* the dominant source of $1/f$ noise in these samples.

The frequency-dependent spatial correlation, the lack of $1/f$ noise in Manganin films, and the scaling of the noise as $V^2 \beta^2 / N_A$ constitute strong experimental evidence that the $1/f$ noise in metal films arises from the modulation of the resistance by intrinsic thermal fluctuations. We feel, therefore, that a diffusion mechanism can produce $1/f$ noise, and that the inability of theory to predict the observed $1/f$ spectrum is due to the approximation of the experimental system by a uniform diffusive medium. The assumption of an explicit $1/f$ region in $S_T(f)$, together with $\langle (\Delta T)^2 \rangle = k_B T^2 / C_V$, yields quantitative predictions in good agreement with experiment.

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tem fluctuates. Here, by temperature fluctuations, we
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Electron-Magnon Bound States

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Interactions between strongly correlated electrons (or holes) and spin waves are in-
vestigated with the aid of a simple physical model. The model is the large-interaction
limit of the Hubbard model with an additional ferromagnetic Heisenberg interaction be-
tween nearest-neighbor singly occupied sites. We find that even though there is no ob-
vious electron-magnon attraction in the model, electron-magnon bound states exist for
some crystal structures.

We have found electron-magnon bound states in
a simple model of a ferromagnetic semiconductor
or insulator. The bound states are characterized
by a large mass (about one hundred times the
electron mass), a binding energy considerably
less than the electron bandwidth, complicated
symmetries, and relatively large size. These
bound states are peculiar in the sense that they
can exist even though there is no *ad hoc* electron-
magnon interaction in this model. An effective
binding appears because for some crystal struc-
tures the kinetic energy of strongly correlated
electrons or holes is decreased by the presence
of a spin wave. We feel that spin waves can quite
generally influence electronic kinetic energies in
both ferromagnets and antiferromagnets to pro-
duce electron-magnon bound states. For simplic-
ity, the following discussion will be primarily re-
stricted to an exactly solvable, ferromagnetic
model.

Qualitatively, we envision the electrons to be
hopping from site to site, but because of the Cou-
lomb interaction two electrons cannot simulta-
neously occupy the same site. In addition, we as-
sume that there is an effective exchange interac-
tion between neighboring electron-spin moments.
Since we are interested in particle–spin-wave
bound states, only the case in which there is one
"hole" and one "spin flip" will be considered. We
are thus treating the case of hole-magnon bound
states. The corresponding case of electron-mag-
non bound states is identical to the above problem
with the sign of the hopping matrix element re-
versed.

For the ferromagnetic model with only nearest-
neighbor hopping, we find that the existence of
hole-magnon (or electron-magnon) bound states
depends crucially on the lattice structure. Elec-
tron kinetic energies can be lowered by spin
waves only in those lattices where "self-return-
ing paths" can have an odd number of hops. These
lattices include the face-centered cubic, hexa-
gonal close packed, and triangular structures.
Results presented here are primarily for the fcc