world of metastable states deserves a name of its own. We propose to name it the *cosmion* (from the Greek for jewel, beautiful, and cosmos). The cosmion gives the opportunity for the first time to search for antimatter in the Universe and thus settle the speculations regarding its existence at large.

We are grateful to our scanners for their conscientious and enthusiastic work. One of us (T.E.K.) expresses his gratitude to many people: Brookhaven National Laboratory for their support and encouragement; his many collaborators over the last twelve years; his colleagues at Syracuse University for their constructive criticism; his friends, secretary (B. Osborne), and wife for their understanding and patience; and last but certainly not least the National Science Foundation for its support. He would also like to thank Professor I. Shapiro of the Institute for Theoretical and Experimental Physics in Moscow for several private communications during the last three years on his work on $\bar{N}N$ bound and resonant states.[9]

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[1]T. E. Kalogeropoulos, in CERN Report No. 74-18, 1974 (unpublished), p. 44.

[2]T. E. Kalogeropoulos *et al.*, preceding Letter [Phys. Rev. Lett. 33, 1631 (1974)].

[3]Reviews by T. E. Kalogeropoulos, CERN Report No. 72-10, 1973 (unpublished), p. 319, and in Proceedings of the Seminar on Interactions of High Energy Particles with Nuclei and New Nuclear-Like Systems, Moscow, U. S. S. R., 1973 (to be published), and in Proceedings of the Fourth International Conference on Experimental Meson Spectroscopy, Boston, Massachusetts, 26–28 April 1974 (to be published).

[4]T. M. Knasel, DESY Reports No. 70/2 and No. 70/3, 1970 (unpublished).

[5]G. Ghesquiere, CERN Report No. 74-18, 1974 (unpublished), p. 436.

[6]H. J. Lipkin and M. Peshkin, Phys. Rev. Lett. 28, 862 (1972).

[7]Using data from Table I of Ref. 2.

[8]Inclusive π^+, π^-, K^0, $K^\pm$, Λ momentum distributions are essentially identical from $\bar{p}d$ annihilations at rest. See Ref. 4 of preceding Letter.

[9]I. S. Shapiro, Usp. Fiz. Nauk 109, 431 (1973) [Sov. Phys. Usp. 16, 173 (1973)].

Color-Symmetry Breaking and the Baryon Spectrum*

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In the colored-quark model, if the three vector gluons that correspond to an SU(2) subgroup of SU(3) are heavier than the other gluons, a quark-diquark structure for baryons results. Furthermore, the predicted baryon SU(6) representations are the 56 for even parity and the 70 for odd parity, in agreement with recent experimental indications.

Recent analyses of the baryon spectrum suggest that the even- and odd-parity baryons correspond exclusively to the SU(6) representations 56 and 70, respectively.[1] This contradicts the harmonic-oscillator quark model for all but the lowest two levels; for example, the model predicts even-parity resonances corresponding to the 56, 70, and 20 at the second excited level.[2]

Several years ago Lichtenberg, and later Ono, proposed that a baryon is a composite of a quark and a diquark.[3,4] The diquarks are assumed to correspond to the symmetric SU(6) representation 21, so that the unobserved twentyfold baryon representation is forbidden. There are three serious difficulties with this model. First, if there are just three fundamental quarks that do not satisfy Fermi statistics, it is hard to imagine a simple force that will bind two closely, and leave the third at larger distances. Second, quark-quark statistics are neglected, a proper procedure only if the diquark is pointlike. Third, 56 and 70 representations are predicted at every energy level. Lichtenberg showed that this last difficulty may be overcome by the introduction of a quark-exchange force, but this force is clearly of a different nature from that which binds the diquark.[3]

In this paper it is assumed that the quarks have color SU(3) indices as well as regular SU(3) and spin indices, and that the quark binding forces are transmitted by the exchange of an octet of vector gluons coupled to the color indices.[5] It is shown that a natural form of color-symmetry breaking leads simultaneously to the quark-diquark baryonic structure and to the correspondence of the $\underline{56}$ and $\underline{70}$ representations to even and odd parities. Harmonic oscillator wave functions are used in the calculations.

The colors are labeled A, B, and C, and the interactions are assumed invariant to the SU(2) subgroup of the A and B colors. The AB potential is taken to be very strong, but to have shorter range than the AC and BC potentials. A possible cause of this range difference is mass splitting of the vector gluon octet, if the three gluons coupled as the generators of the SU(2) of the A and B colors are heavier than the other gluons. This mechanism is similar to that used in some recent attempts to unify strong, electromagnetic, and weak interactions.

This mechanism mixes color singlets and octets. The effects of the lighter five gluons are not considered negligible, however, so the baryon states must contain some color-singlet component. This requires that each baryon contain one A quark, one B quark, and one C quark. If the difference in the ranges of the potentials is appreciable, the AB diquark will be relatively small. A convenient set of internal variables is

$$\vec{\lambda}_{AB} = (1/\sqrt{6})(\vec{r}_A + \vec{r}_B - 2\vec{r}_C), \tag{1a}$$

$$\vec{\rho}_{AB} = (1/\sqrt{2})(\vec{r}_A - \vec{r}_B). \tag{1b}$$

The gluon-exchange potential is a sum of two-body potentials $V = V_{\alpha\beta} + V_{\beta\gamma} + V_{\gamma\alpha}$, where α, β, and γ are the three quarks. We assume that $V_{\mu\nu}$ is the sum of a short-range part (V_s) and a long-range part (V_l), i.e.,

$$V_{\mu\nu} = \sum_{i=1}^{3} J_i^{\mu} J_i^{\nu} V_s(|\vec{r}_\mu - \vec{r}_\nu|) + \sum_{i=4}^{8} J_i^{\mu} J_i^{\nu} V_l(|\vec{r}_\mu - \vec{r}_\nu|), \tag{2}$$

where J_i^{μ} is the ith Hermitean generator of SU(3), operating in the color space of the quark μ, and J_1, J_2, and J_3 are the generators of the AB SU(2) subgroup of color SU(3). The configuration-space potentials V_s and V_l are positive. Exact color symmetry corresponds to $V_s = V_l$.

If the $\mu\nu$ state is in the representation r of AB SU(2), then

$$\sum_{i=1}^{3} J_i^{\mu} J_i^{\nu} = \tfrac{1}{2}[C_2(r) - C_2(\mu) - C_2(\nu)],$$

where $C_2(x)$ is the eigenvalue of the quadratic Casimir operator of AB SU(2) for the representation x. It follows from this expression that the short-range potential is attractive only between the A and B quarks, and only if they are in the singlet of AB SU(2), i.e., the color wave function is antisymmetric in AB exchange. We assume that the wave function is in the ground state of the strong short-range V_s potential, so that there are no excitations of the ρ variable. The quarks obey Fermi statistics, so the wave function may be written

$$\sqrt{6}\,\psi_{jk} = \sum_P \tau_P \alpha_A \beta_B \gamma_C U_j(\alpha\beta\gamma) R_g(\vec{\rho}_{\alpha\beta}) L_k(\vec{\lambda}_{\alpha\beta}) = \sum_P \tau_P \alpha_A \beta_B \gamma_C U_j(ABC) R_g(\vec{\rho}_{AB}) L_k(\vec{\lambda}_{AB}), \tag{3}$$

where U_j is the SU(6) wave function, R and L are orbital functions, and the subscript g denotes the ground state. The sum is over the six permutations of α, β, and γ, and τ is 1 and -1 for even and odd permutations, respectively. Since the color wave function is antisymmetric in the transposition (AB), and since $(AB)\vec{\lambda}_{AB} = \vec{\lambda}_{AB}$, U_j must be symmetric in AB exchange. Therefore U_j is either the symmetric ($\underline{56}$) representation U_s or the λ component U_λ of the mixed ($\underline{70}$) representation.

The potential does not depend on the SU(6) representation. Since the diquark internal wave function is the same for all states, both in color and in configuration space, the relative energies of states depend on the potential between the quark and diquark. Since the diquark is in the singlet of AB SU(2), the term

$$\sum_{i=1}^{3} J_i^{\mu} J_i^{\nu}$$

does not contribute to the quark-diquark potential. Thus, only the long-range part of V contributes to quark-diquark binding, but the 1, 2, and 3 gluons may be included in the sum. The color operator of a two-quark V_l term then becomes

$$\sum_{i=1}^{8} J_i^{\mu} J_i^{\nu} = \tfrac{1}{2}[C_3 - 2C_3(q)], \tag{4}$$

where C_3 is the quadratic Casimir operator of SU(3) and $C(q)$ is its eigenvalue for the quark representation.

The states are mixtures of color octets and singlets. It two states are of the same oscillator level, that with the larger singlet component will be favored energetically. This is because each pair of quarks in the singlet are in an antisymmetric color state, thus minimizing the contribution of C_3 in Eq. (4).

In order to make the mathematical procedure clear, we review some properties of mixed representations. The permutation properties of a function of A, B, and C may be specified completely from the behavior of the function under general products of the two transpositions (AB) and (AC). The behavior of the two mixed-symmetry components λ and ρ is[2]

$$(AB)\lambda = \lambda, \quad (AC)\lambda = -\tfrac{1}{2}\lambda - \tfrac{1}{2}\sqrt{3}\,\rho, \tag{5a}$$

$$(AB)\rho = -\rho, \quad (AC)\rho = \tfrac{1}{2}\rho - \tfrac{1}{2}\sqrt{3}\,\lambda. \tag{5b}$$

The two variables $\vec{\lambda}_{AB}$ and $\vec{\rho}_{AB}$ of Eqs. (1a) and (1b) are such mixed-symmetry components; the subscript AB will be suppressed in the rest of the paper.

For each orbital wave function RL of Eq. (3), and for both U_s and U_λ, we will compute the magnitudes of the components of URL that are completely symmetric, and of λ mixed symmetry. These correspond to a color singlet and a color octet, respectively.

We consider first the orbital wave function $\varphi_k = RL_k$. Since φ is symmetric under (AB), it is a linear combination of a symmetric and a λ mixed state, i.e.,

$$\varphi = a\varphi_s + (1-a^2)^{1/2}\varphi_\lambda, \tag{6}$$

where φ, φ_s, and φ_λ are normalized. We define the matrix element M_k for the k state by

$$M_k = \langle \varphi_k, (AC)\varphi_k\rangle. \tag{7}$$

It follows from Eq. (5a) and the orthogonality of φ_s, φ_λ, and φ_ρ that

$$M = a^2 - \tfrac{1}{2}(1-a^2). \tag{8}$$

Next we consider the products of SU(6) and orbital functions $\chi = U\varphi$. If $\chi = U_\lambda\varphi_\lambda$, evaluation of $\langle \chi, (AC)\chi\rangle$ from Eq. (5a) leads to the value $\frac{1}{4}$. Using an equation analogous to Eq. (8), one can write $U_\lambda\varphi_\lambda = (1/\sqrt{2})(\chi_s + \chi_\lambda)$, where U_λ, φ_λ, χ_s, and χ_λ are normalized. Therefore, if φ is in the form of Eq. (6), the probabilities that $U_s\varphi$ and $U_\lambda\varphi$ are symmetric (and consequently in a color singlet) are a^2 and $\frac{1}{2}(1-a^2)$, respectively. It follows from Eq. (8) that the symmetric SU(6) representation $\underline{56}$ is favored over the $\underline{70}$ if and only if M is positive.

We will evaluate M for harmonic oscillator wave functions, using the usual dimensionless variables, determined from the λ (long-range) force constant. The orbital wave function is

$$\varphi_k = N_k H_k(\lambda)\exp(-\tfrac{1}{2}\lambda^2)\exp(-\tfrac{1}{2}D_\rho^{\ 2}), \tag{9}$$

where N_k is a normalization constant, H_k is a three-dimensional Hermite polynomial, and $D > 1$, since the ρ wave function is of comparatively short range. For convenience we define M_k' by $M_k = M_k' f_k(D)$, where $f_k(1) = 1$. We first evaluate M' by setting $D = 1$, in which case the exponent $-\frac{1}{2}(\lambda^2 + \rho^2)$ is invariant to the permutation (AC). The Hermite polynomial H may be written as a sum of polynomials h of the form

$$h = N'\lambda_x^{\ n_x}\lambda_y^{\ n_y}\lambda_z^{\ n_z} + O(l),$$

where N' is a constant, $n = n_x + n_y + n_z$ is the order of the energy level, and $O(l)$ is a lower-order polynomial in λ_i.

We consider the quantity $(AC)h$. Since (AC) leaves $\lambda^2 + \rho^2$ invariant, this transposition leaves the quantum number $n(\lambda) + n(\rho)$ unchanged when $D = 1$. Therefore, when (AC) is applied to h, the lower-order term $(AC)O(l)$ is determined from the leading term of the polynomial. From these facts and Eq. (5a), $(AC)\varphi_k = [(-\frac{1}{2})^n\varphi_k + \text{orthogonal states}]$, when $D = 1$. Hence,

$$\langle \varphi_k, (AC)\varphi_k\rangle = M = (-\tfrac{1}{2})^n f_k(D). \tag{10}$$

Since the sign of M determines the preferred SU(6) representation, we see that if $f_k(D)$ is positive, the representations $\underline{56}$ and $\underline{70}$ are favored for even and odd oscillator levels, respectively.

The evaluation of $f_k(D)$ for a specific oscillator state is straightforward, but tedious. It is convenient to define a function $\Delta_k(D)$ by the formula

$$f_k(D) = \left[\frac{16D}{(3D+1)(D+3)}\right]^{(3/2+n)}[1 + \Delta_k(D)].$$

A calculation shows that for every energy level n, the function $\Delta(D)$ is zero when $l = n$, where l is the orbital angular momentum. In the cases $n=2$, $l=0$ and $n=3$, $l=1$, Δ may be written $\Delta = \kappa(D^2-1)^2/D^2$, where the constant κ is 9/128 and 99/640 in the (2, 0) and (3, 1) cases, respectively.

Since $f_k(D)$ is positive, its inclusion does not change the predicted relative ordering of the $\underline{56}$ and $\underline{70}$ representations. Furthermore, if D is on the order of 2 or so, and n is not large, $f(D)$ is appreciable, so the expected difference between the $\underline{56}$ and $\underline{70}$ energies may be large enough so that the unfavored state does not appear physically. On the other hand, $|M|$ is small when n is large, primarily because of the $(-\frac{1}{2})^n$ factor in Eq. (10). Consequently, we predict that for sufficiently high quark-model level, the $\underline{56}$ and $\underline{70}$ should both appear.

Since the sign of M depends primarily on the parity of the wave function $L(\lambda)$, the use of oscillator wave functions is not crucial to the result. A more detailed treatment of this color-symmetry breaking effect, including the case when the symmetry breaking is relatively small, will be published elsewhere.

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[1]P. J. Litchfield, in Rapporteur's Talk at Proceedings of the Seventeenth International Conference on High Energy Physics, London, England, 2-10 July 1974 (to be published).

[2]G. Karl and E. Obryk, Nucl. Phys. B8, 609 (1968).

[3]D. B. Lichtenberg, Phys. Rev. 178, 2197 (1968).

[4]S. Ono, Prog. Theor. Phys. 48, 964 (1972).

[5]If color symmetry is exact, this mechanism leads to color singlets as the lightest states as shown by Y. Nambu, in *Preludes in Theoretical Physics*, edited by A. de-Shalit, H. Feshbach, and L. Van Hove (North-Holland, Amsterdam, 1966), p. 133.

Infrared Behavior of Yang-Mills Theories

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The ultraviolet freedom of non-Abelian gauge theories is used to suggest the presence of a new symmetry of the exact theory which is not present in finite orders of perturbation theory. The infrared behavior of these theories is determined as a consequence of this symmetry.

There is a distinct possibility that the strong interactions may be described by an unbroken non-Abelian (generalized Yang-Mills[1]) gauge theory.[2] The gauge group is presumably colored SU(3) with the gauge fields A_μ^a in the adjoint representation ($a=1-8$) and the quark fields ψ in the (triplet) fundamental representation. The quark fields should also be ordinary SU(3) triplets or simple (charmed) generalizations. Such a theory is asymptotically free and so has a computable ultraviolet behavior.[3] More importantly, the theory is therefore *not* infrared free and so offers a unique possibility of providing for color confinement.[2]

Because the theory is not infrared free, it has not been possible to perform any reliable calculations which could determine what actually happens in the infrared region.[4] The hoped for confinement cannot be investigated with conventional perturbation or renormalization-group[5] techniques.[6] In this note, we shall attempt to overcome this impasse and argue that, precisely because of the lack of infrared freedom (or the corresponding presence of ultraviolet freedom), exact statements *can* be made about the infrared behavior of the theory. Unlike conventional zero-momentum theorems, our statements, which are consequences of renormalization, should be valid only in the exact theory and not classically or in finite orders of perturbation theory. Our present results do not answer the confinement question, but, because they constitute hopefully exact statements about the infrared behavior of non-infrared-free theories, they should provide new tools for investigating this and related problems.

Our approach is as follows. We consider the renormalized gauge field equations in the form

$$\partial^\nu[\partial_\mu A_\nu^a(x) - \partial_\nu A_\mu^a(x)] = J_\mu^a(x), \qquad (1)$$

where $J_\mu^a(x)$ contains the usual Yang-Mills self-couplings and couplings to quark fields and ghost fields, gauge-fixing terms (we work in the generalized-Lorentz-gauge formalism), and all necessary counterterms.[7] Classically, or order by order in perturbation theory, (1) possesses no interesting symmetry besides the usual Poincaré and non-Abelian gauge invariance. Asymptotic freedom, however, enables us to make precise