Solar Gravitational Deflection of Radio Waves Measured by Very-Long-Baseline Interferometry

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Utilizing a four-antenna technique, we observed simultaneously, at each end of an 845-km baseline, the radio sources 3C279 and 3C273B which are 10° apart in the sky. Differences in interferometric phases at 3.7-cm wavelength monitored near the time of the 1972 occultation of 3C279 by the sun, yielded a gravitational deflection of 0.99 ± 0.03 times the value predicted by general relativity, corresponding to $\gamma = 0.98 \pm 0.06$ (standard error).

A resurgence of interest in the measurement of the gravitational deflection of light rays by the sun followed the realization in 1967 that radio interferometry could be gainfully employed for the purpose.[1] Here we describe the first accurate measurement of the deflection by very-long-baseline interferometry.

Our experiment, performed in September and October 1972 at a radio frequency of 8105 MHz ($\lambda \simeq 3.7$ cm), utilized the 120-ft-diam "Haystack" and 60-ft-diam "Westford" antennas of the Haystack Observatory in Westford, Massachusetts, and two of the 85-ft-diam antennas of the National Radio Astronomy Observatory (NRAO) in Green Bank, West Virginia, some 845 km to the southwest. "Haystack" and one NRAO antenna, forming a long-baseline interferometer, observed the compact extragalactic radio source 3C279 which was occulted by the sun on 8 October. "Westford" and the other NRAO antenna formed another long-baseline interferometer and were directed at a similar source, 3C273B, located about 10° to the northwest of 3C279. A single hydrogen-maser frequency standard governed the heterodyning and the recording of the signals received by both of the antennas at a given site. By taking as the basic observable the difference $\Delta\varphi$ between the interferometric fringe phases of 3C279 and 3C273B, we effectively prevented the introduction of errors due to differences between the independent standards used at the two sites, and also reduced the effects of the neutral atmosphere and ionosphere.

Gravitational deflection changes noticeably the apparent positions of the two sources: For example, the difference $\Delta\alpha$ in their apparent right ascensions on 3 and 11 October is predicted to be different by about $(1+\gamma)100$ arc msec. Here γ is the Eddington-Robertson parameter whose value is 1 according to general relativity. Our observable, $\Delta\varphi$, is, in fact, affected most importantly by $\Delta\alpha$ since

$$\Delta\varphi \simeq (2\pi B/\lambda)\Delta\alpha \cos H + \Delta\varphi_0, \quad (1)$$

where B is the length of the equatorial projection of the long-baseline vector, H is the hour angle at the midpoint of the baseline of a point midway between the sources, and $\Delta\varphi_0$ is a constant which includes the unknown constant instrumental phase, and which also depends on the declinations of the baseline and the sources. Since $\Delta\varphi_0$ is unknown, it is necessary in order to determine $\Delta\alpha$ to observe $\Delta\varphi$ continuously through a range of H during which $\cos H$ varies significantly, preferably near the times of rise ($H \approx -6^{\mathrm{h}}$) or set ($H \approx +6^{\mathrm{h}}$), despite atmospheric effects on $\Delta\varphi$ being most severe at these times. If the change of $\Delta\varphi$ between the time of rise (or set) and transit can be measured with an uncertainty of $\sigma(\Delta\varphi)$, the uncertainty of the determination of $\Delta\alpha$ is approximately

$$\sigma(\Delta\alpha) \approx (\lambda/2\pi B)\sigma(\Delta\varphi), \quad (2)$$

or ~3 arc msec for $\sigma(\Delta\varphi) \approx 100°$, i.e., about 1.5% of the predicted change in $\Delta\alpha$ for $\gamma \approx 1$.

Because phase is intrinsically an ambiguous observable, no large gaps in the determination of $\Delta\varphi$ as a function of time can be tolerated. For a gap to be "acceptable," one must be able to connect the measurements of $\Delta\varphi$ before and after the gap without the introduction of a 2π ambiguity. In other words, the constant $\Delta\varphi_0$ must be the same for all measurements throughout a day's, or a half-day's, observations. During observations when the ray path to 3C279 passes within a few degrees of the sun, a gap of only a few seconds may be unacceptable, because the solar corona introduces rapid and unpredictable variations in the fringe phase. On the other hand, when the separation of sun and source is $10°$ or more, even a gap of 30 min may be successfully bridged.

The signals received from each source were converted from microwave to low frequencies, then clipped, sampled, and recorded digitally at a rate of 720 000 bits/sec on magnetic tape. On each tape alternate records, of duration 0.2 sec, were used for a given source, so that the recorded signals from the two sources were interleaved. Each tape contained 3 min of data; all told 5000 tapes were recorded.

Pairs of tapes which had been recorded simultaneously at the two sites were cross correlated to obtain the interferometric fringe amplitude and phase for each 0.2-sec record; these data were then averaged coherently over longer intervals. Usually we formed 10-sec averages and, from these, the $\Delta\varphi$ observable. A simple computer program was used to connect the sequence of values of $\Delta\varphi$ without the introduction of spurious 2π changes. However, because of the subtlety of the process, we examined every single phase connection graphically to insure its validity; in every doubtful case, we re-examined the connection with successively smaller averaging intervals for $\Delta\varphi$ until either the reliability of the connection could be assured or the statistical uncertainty in the determination of the fringe phase became too high to allow a reliable connection. This latter stage was reached for an averaging interval of about 1 sec. In such cases we assumed the connection to be broken and we introduced a new unknown constant [see Eq. (1)] at the appropriate epoch into the theoretical model for $\Delta\varphi$.

The amount of phase fluctuation because of coronal turbulence was exceedingly time variable; for example, $\Delta\varphi$ was very smooth for the first few hours of observation on 3 October and then, within less than 5 min, $\Delta\varphi$ became impossible to follow with 1-sec averaging. Such severe coronal fluctuations caused us to eliminate as worthless some segments of the data.

After completion of this phase-connection and editing process, the $\Delta\varphi$ data were smoothed by straight-line fitting over 3-min intervals prior to the final analysis to determine the deflection.

In this analysis, γ and the undeflected position of 3C279 relative to that of 3C273B were estimated simultaneously with a large set of other parameters by means of iterative, weighted-least-squares adjustment. The parameters included the $\Delta\varphi_0$'s and the zenith atmospheric phase delay at each site for each day of observation, with the minor exceptions indicated below.

The undeflected position of 3C273B and the locations of the antennas were fixed in accord with prior determinations. The rotations of the baseline vectors with respect to the inertial frame formed by the sources were calculated from standard formulas that included corrections for precession, nutation, solid-earth tides, polar motion, and universal time. Simple models were used for the ionosphere, atmosphere, and solar corona. For the corona, the most important and difficult part to model, we assumed[2] an electron density of $5\times10^5 r^{-2}$ cm^{-3}, where r is given in solar radii.

Our least-squares analysis yielded $\gamma = 0.98 \pm 0.01$ with the formal standard error being based on the root-mean-square value of $100°$ for the postfit residuals of $\Delta\varphi$ and on the assumption that the data points, 3 min apart, have statistically independent errors. The more reliable assumption (see Fig. 1) that 2 points per hour are independent leads to a formal error of approximately 0.03. But no estimate of uncertainty derived solely from the properties of postfit residuals can be trusted: The most significant, long-term trends of the measurement errors may have been absorbed in the model-fitting process in such a way that they contributed to the errors of the estimated parameters, but were not revealed in the residuals. Furthermore, errors in the assumed values of parameters not estimated might seriously affect the solution but have no perceptible effect on the residuals. To evaluate the uncertainties due to both kinds of error, we performed two sets of computer experiments.

First, we took the $\Delta\varphi$ data from each day separately, for the six days when the sun was farthest from both radio sources, and we made independent solutions for $\Delta\alpha$, keeping both declinations fixed and γ equal to 1. The rms scatter of the re-

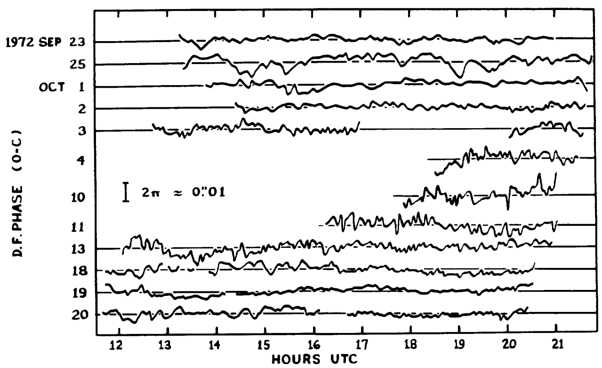

FIG. 1. Postfit residuals (observed minus computed values) for the difference fringe phase. A change of 2π in the difference fringe phase corresponds to an apparent change in relative source direction of about 10 arc msec, as indicated. On 29 September, the ray path to 3C273B passed within about 20 solar radii of the sun and at 0 h UT of 8 October, 3C279 was occulted by the sun with the apparent separation increasing by about 4 solar radii per day. Coronal turbulence caused the omissions of data on parts of 3, 4, 10, and 11 October; the gaps on 18, 19, and 20 October reflect missing recordings.

sults about the mean was 3.7 arc msec compared to 1 arc msec, the appropriate average of the formal standard errors, confirming that the formal standard error for γ should be multiplied by a factor of between 3 and 4 to reflect the correlation of measurement errors over times greater than 3 min.

To estimate the uncertainty specifically due to errors in the values assumed for fixed parameters, we re-solved for γ repeatedly, each time changing one of these parameters by no less, and often by grossly more, than our estimate of its true uncertainty. Included were all relevant antenna and source coordinates (changed by 10 m and 1 arc sec, respectively), variations of universal time (2 msec over 5 days) and polar motion (1 m over 5 days) placed both symmetrically and antisymmetrically around the date of occultation, atmospheric zenith delays on the three days nearest occultation (0.2 nsec), amplitudes of earth tides at one and at both sites (100%), and mean electron densities of the ionosphere and solar corona (each 100%). The maximum change in γ due to each was under 0.01 in all cases except that of the solar corona, for which $\Delta\gamma_{\max} = 0.016$. In regard to the latter, we note that the deflection of a ray for the model corona is only 1% of the gravitational deflection for $\gamma \sim 1$ and for an impact parameter of 10 solar radii, the smallest for our observations. This percentage decreases inversely with increase in impact parameter which leads to the insensitivity of our result to a gross change in the coronal model.

The combination of independent errors in all of these parameters, each with a standard deviation equal to the change used, would yield a 1-σ uncertainty in the estimate of γ of 0.024. If, conservatively, we consider the effects of these errors to be independent of, and in addition to, whatever effects caused the scatter of our postfit residuals and our six solutions for $\Delta\alpha$, we obtain a combined uncertainty of 0.042.

Finally, we considered the effects of spurious 2π changes (despite our elaborate precautions!) and of coronal fluctuations. An extensive sensitivity study in which 2π errors were deliberately inserted on various days at times of maximum effect and in which the data from various single days, and combinations of days, were omitted from the analysis led us to conclude that these sources contribute no more than 0.04 to the 1-σ uncertainty in γ. Combining all sources of errors, as if independent, leads to

$$\gamma = 0.98 \pm 0.06, \tag{3}$$

or, equivalently, to a result 0.99 ± 0.03 times the value predicted by general relativity and to 1.04 ± 0.03 times the latest prediction[3] based on the Brans-Dicke theory of gravitation.

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[3] R. H. Dicke, Science 184, 419 (1974).