

PHYSICAL REVIEW LETTERS

VOLUME 33

30 DECEMBER 1974

NUMBER 27

Transverse NMR Measurements on Superfluid ^3He

A. I. Ahonen, M. T. Haikala, M. Krusius, and O. V. Lounasmaa
Low Temperature Laboratory, Helsinki University of Technology, SF-02150 Otaniemi, Finland
(Received 30 October 1974)

Transverse NMR measurements on liquid ^3He are reported down to 0.7 mK at several pressures below 27 bar. In the *A* liquid the quadratic NMR frequency shift is proportional to $T_C^2(1 - T/T_C)$. In the *B* phase the integrated dynamic susceptibility falls to a limiting value which is 25% of the Fermi-liquid susceptibility. NMR data on the *A* and *B* liquids confined in the voids of fine powder are also presented.

Ever since the work of Osheroff *et al.*¹ NMR measurements on liquid ^3He have provided important data for testing different pairing models proposed to explain the nature of the superfluid phases. It is now believed that the *A* liquid is in an axial triplet state of equal-spin pairing and, with some uncertainty, that the *B* liquid is in an isotropic triplet state.

In this Letter we shall first briefly describe our experimental technique. We next present our results on the *A* liquid and then proceed to the *B* phase; our data are the first transverse NMR results on bulk ^3He off the melting plane. Finally we shall describe the resonance when the liquid is intermixed with platinum powder.

With the exception of our earlier experiments,² Pomeranchuk cooling techniques have been employed in all conventional NMR work on ^3He so far.³ We have used nuclear demagnetization of copper for cooling; in this way we could select the liquid pressures freely and were able to reach 0.7 mK. Some details of our apparatus have been described elsewhere.⁴ Two cylindrical

pickup coils are located on top of each other inside the ^3He cell. The coil formers have an inner diameter of 4.5 mm and a length of 14 mm; the windings extend only over 7 mm. The upper coil is filled with platinum powder; it is used, besides for investigating ^3He , for measuring the liquid temperature from the pulsed NMR signal of ^{195}Pt . The lower coil contains only ^3He . A superconducting magnet is employed for generating the static field \vec{B}_0 perpendicular to the axis of the pickup coils. The homogeneity of the field, $\Delta B/B_0$, is 6×10^{-4} . Most of our measurements have been performed at $B_0 = 32.0$ mT.

Although magnetization measurements⁵ in the *A* phase show that the static susceptibility χ_A is constant, the transverse NMR signal in small fields decreases rapidly towards lower temperatures⁶ because of broadening of the resonance line. The most prominent feature of the NMR signal in the *A* liquid is, however, the shift between the observed transverse frequency ω_T and the Larmor frequency γB_0 .¹ Leggett's⁷ expression for the shift is

$$\omega_T^2 - (\gamma B_0)^2 = C(1 + \frac{1}{4}Z_0)(dn/d\epsilon)_{\epsilon_F}(kT_C)^2[\ln(1.14\epsilon_0/kT_C)]^2(1 - T/T_C) \sim T_C^2(1 - T/T_C), \quad (1)$$

where the pressure dependence comes from the Landau parameter Z_0 , from the density of states at the Fermi surface $(dn/d\epsilon)_{\epsilon_F}$, and especially from the variation of the transition temperature T_C with pressure; C is a constant, ϵ_0 is the cutoff energy of the ^3He quasiparticle pairing interaction, and k

is Boltzmann's constant.

In our measurements at 32 mT broadening of the signal was observed but the integrated NMR absorption remained constant. At the boundary between the *A* and Fermi liquids a narrowing of the line was seen within a few per cent of T_C . This phenomenon was particularly clear near 22 bar pressure where it caused an increase of roughly 5% in the maximum resonance absorption.

We measured the resonance shift at three different pressures to well below T_{AB} taking advantage of the rather substantial supercooling of the *A* → *B* transition. During warming a reproducible transition was observed and the shift agreed with that obtained upon cooling except for a puzzling parallel displacement of 10–20 μ K towards higher temperatures. Our results are shown in Fig. 1. The data at different pressures have been normalized to the melting curve at $P_m \cong 34$ bar by means of Eq. (1); values of Fermi-liquid parameters were obtained from Wheatley.⁹ All points fall nicely, within their precision, on the solid curve which represents the results of Osheroff and Brinkman⁸ from measurements along the melting surface. We have thus verified Leggett's relationship, Eq. (1). In plotting this curve the temperature dependence of P_m was assumed to be that obtained recently by Halperin *et al.*¹⁰ with T_C on the melting curve at $T_A = 2.75$ mK and $dP_m/dT = -35.6$ bar/K. The dash-dotted curve above our

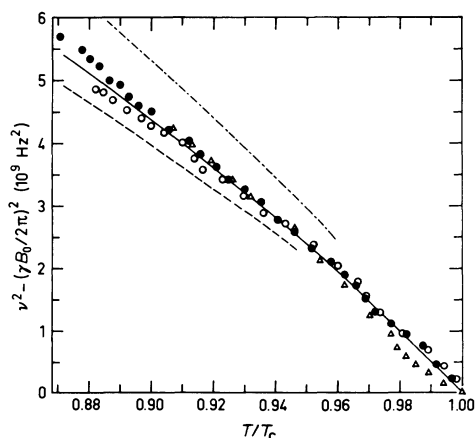


FIG. 1. Transverse resonance shifts in the *A* phase plotted as a function of the reduced temperature; all data have been scaled according to Eq. (1) to the shift measured on the melting surface. Solid curve, melting-plane data of Ref. 8 with $T_A = 2.75$ mK; dashed curve, melting-plane data of Ref. 8 with $T_A = 2.60$ mK; dash-dotted curve, melting-plane data of Ref. 5 with $T_A = 2.75$ mK. Present results with T_C from Ref. 4: ○, 27.2 bar; ●, 22.4 bar; △, 19.8 bar.

data represents the resonance shift as measured by Bozler *et al.*⁵ making use of the same temperature dependence of P_m . The parallel "ringing" experiments of Webb, Kleinberg, and Wheatley¹¹ agree with the solid curve in Fig. 1.

Our result, i.e., the universal curve of Fig. 1, was to be expected on the basis of the good quantitative agreement between Leggett's⁷ theory and earlier NMR measurements, especially those on the longitudinal resonance frequency $\omega_L^2 = \omega_T^2 - (\gamma B_0)^2$.^{5,8} However, Fig. 1 can also be regarded as evidence for agreement between our platinum NMR temperatures and the thermodynamic scale of Halperin *et al.*¹⁰ As a comparison, the dashed curve in Fig. 1 illustrates the sensitivity of the frequency shift to changes in the temperature scale; this curve shows the NMR shift obtained by Osheroff and Brinkman⁸ with $T_A = 2.60$ mK and $dP_m/dT = -30.0$ bar/K.

Resonance measurements in the *B* liquid are troubled by a severe line broadening at lower temperatures which has been explained,¹² in the case of an isotropic triplet state, as an orientational anisotropy created by an interplay between the applied magnetic field and a solid surface. Typically the axis of orientation is bent with a continuous distribution of directions, ranging from parallel to \vec{B}_0 far from a surface to perpendicular to the wall near a surface. Within such a texture the NMR signal is shifted towards higher frequencies in those parts of the liquid where the axis of orientation is bent away from \vec{B}_0 . The characteristic length determining the scale of such bending has been estimated,¹² in the low-temperature limit, as $10/B_0$ mm (B_0 in mT) which is comparable with the diameter of the pickup coil in low-field NMR.

Our measurements on the *B* liquid at different pressures show that the amplitude of the transverse NMR signal remains constant at $T/T_C < 0.4$; both the line broadening and the drop in χ_B have thus largely occurred above this temperature. Figure 2 illustrates our result at 21 bar and at $T/T_C < 0.3$ as a function of B_0 . In the lower part of the figure $\chi_B''(\nu)/\chi_F''$, the peak value of the normalized absorption, is shown. Line broadening is obvious; when B_0 is increased, the volume of the liquid dominated by the field energy grows inside the pickup coil whereby the NMR signal becomes narrower and its amplitude increases.

In the upper part of the figure χ_B/χ_F , the resonance absorption integrated over a frequency interval of 15 kHz, is shown. It appears to stay

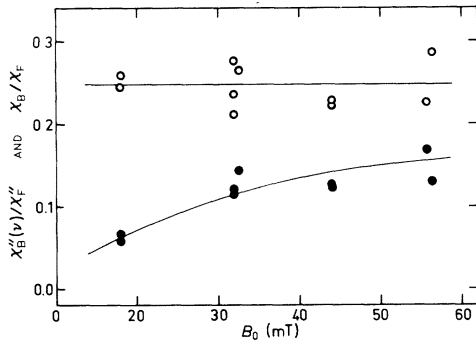


FIG. 2. Limiting low-temperature properties of the B -liquid transverse NMR absorption, normalized to the respective Fermi-liquid values and plotted as a function of the external magnetic field: \circ , total integrated absorption, χ_B/χ_F ; \bullet , maximum absorption, $\chi_B''(\nu)/\chi_F''$. The data were measured at 21 bar and at $T/T_C < 0.3$.

constant between 18 and 56 mT. This probably means that the main part of the high-frequency tail was included by our integration process and that no sizable fraction of the resonance absorption was lost in a low-frequency branch.

Figure 2 shows that the limiting low-temperature value of χ_B/χ_F is 0.25 ± 0.05 . For the isotropic triplet state this ratio is $\frac{2}{3}$; when Fermi-liquid corrections with $Z_0 = -2.9$ are included, the ratio drops to 0.35.¹³ To explain our experimental result one would have to assume, for instance, that $Z_2 = 12$ which is a larger number than has been anticipated so far.

In order to study the behavior of superfluid ^3He in narrow geometries we made transverse NMR measurements on liquid intermixed with fine platinum powder. The grains have the form of compact lumps with an average diameter of $8 \mu\text{m}$; they occupy 35% of the total volume within the upper pickup coil. We can approximate our geometry with one in which the major part of the liquid is in spherical cavities of $12 \mu\text{m}$ diameter.

In the A -phase region both a shifted and an unshifted resonance line are simultaneously observed. The amplitude of the former is strongly dependent on B_0 . At 6 mT the shifted line is 5% of the Fermi-liquid signal but at 32 mT it has increased considerably; precise estimates are not possible because the line also shows an irreproducible temperature dependence. The frequency shift is roughly the same as in the bulk A liquid and it extrapolates to zero at the same T_C . Con-

sequently, we interpret the shifted signal as originating from ordinary A liquid.

The unshifted line appears to be more dependent on temperature than on the magnetic field. At T_C the intensity of the signal begins to decrease rapidly and irreproducibly when the temperature is lowered; the linewidth remains roughly equal to that in the Fermi liquid. During cooling the unshifted signal disappears abruptly, usually well below T_{AB} , while the shifted line can still be seen. Similarly, upon warming the unshifted signal reappears, normally well above T_{AB} , only after the shifted line is already present.

In the B -liquid region an unshifted resonance line is observed. It has roughly the same temperature dependence as the signal in bulk ^3He ; T_C , in particular, remains unchanged. However, the B -liquid signal in the presence of platinum powder is very strongly dependent on the external field. Below 10 mT no signal was detected whereas at 32 mT the integrated resonance absorption was found to be the same as that of the bulk B liquid with the same liquid volume. This behavior is explained by the formation of domains of uniformly oriented liquid. Such a structure arises because it is energetically unfavorable to bend the axis of orientation within a single void. Therefore, at low fields the orientation of each domain is determined by the surface of the void. From such a random ensemble no NMR signal is observed because of the large variation in resonance frequencies. However, within a fairly narrow field region all domains are expected to enter a field-dominated regime with their orientation parallel to \vec{B}_0 .

In the low-field measurements² it was observed that upon cooling into the B phase the resonance signal decayed exponentially to zero between $1 > T/T_C > 0.95$. According to Brinkman *et al.*¹² this is quantitatively explained by domain formation. Since the characteristic distance within which the axis of orientation may bend is $10(1 - T/T_C)^{1/2}/B_0$ mm (B_0 in mT) this length is expected to surpass the dimensions of the void close below T_C . At lower temperatures no bending may occur; domains are formed instead and because of different orientations the NMR signal is lost.

We wish to thank T. A. Alvesalo, D. S. Fisher, M. S. Grönstrand, U. A. Lähteenmäki, and E. Turtiainen for help with the experiments and in analyzing the data. We are grateful to W. F. Brinkman and M. Vuorio for instructive discus-

sions.

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Renormalization-Group Approach to the Solution of General Ising Models*

M. Nauenberg† and B. Nienhuis

Institute for Theoretical Physics, University of Utrecht, Utrecht, Holland

(Received 15 May 1974)

It is shown how the renormalization-group ideas of Wilson can be applied to obtain the complete thermodynamic functions for general Ising models. The most singular term near the critical transition is given explicitly in terms of the renormalization-group transformations. As an example an approximate numerical solution is given to the original Ising model.

The fundamental renormalization-group approach of Wilson¹ based on Kadanoff's intuitive deduction² of Widom's scaling law³ has led to considerable progress in understanding critical transitions. In particular the well-known scaling behavior and the universality of critical exponents is obtained from the existence of a fixed point and the analyticity of the renormalization transformations. The values of these exponents were first calculated by Wilson and Fisher⁴ in the ϵ expansion which exhibited the role of lattice dimensionality, and have been obtained also by other methods. Recently Niemeijer and van Leeuwen⁵ have shown how to implement more directly Kadanoff's ideas using the Wilson approach to evaluate the critical exponents and the transition temperature for Ising models. In this note we want to show how to extend this method to obtain the complete thermodynamic function for the general Ising model in terms of the renormalization-group transformations. The essential point is to use the fact that the partition function of the Ising-spin system is form invariant under the renormalization-group transformation only up to a factor $\exp[Ng_N(K)]$ [see Eq. (2)], where $g_N(K)$ is

a function of the spin interaction parameter K for N spins and is completely determined by this transformation. We will show that the partition function can be evaluated in terms of $g_N(K)$ for any desired range of values of K for which the nonlinear renormalization-group mappings are determined, including the singular behavior on the critical surface. Near the critical surface we obtain the well-known power-law or logarithmic singularities, but including explicitly an expression for the coefficient of these singularities. As an example, we have solved numerically our equations for the case of a square lattice in a four-cell approximation, and obtained the free energy, the energy, and the specific heat as a function of the nearest-neighbor spin coupling.

The renormalization-group transformation for N spins

$$K_{\alpha}' = F_{\alpha}(K) \quad (1)$$

in the parameter space K of the generalized Ising-spin lattice and the function $g_N(K)$ are determined by the invariance requirement

$$\exp[H_{N/L}(K', S') + Ng_N(K)] = \sum_{\{S\}} \exp[H_N(K, S)], \quad (2)$$