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Preferential Radiofrequency Plugging of Multi-Ion Species Plasmas in a Static Cusped Confinement System

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The rf preferential plugging of multi-ion species plasma is studied experimentally in a magnetic cusp: Desired ion species can be plugged and the others escape from the container. The conditions necessary to realize an impurity-free cusp reactor using this concept are discussed. It is shown that impurities, specifically krypton and argon, can be preferentially reduced in the reactor.

One of the recent problems of Tokamak plasma is the concentration of heavy impurity ions in the central region of the plasma column.[1] Such a problem can be solved more easily in an open-ended system than a closed system. It has been shown[2-4] that the end loss from the line cusp can be plugged effectively when an rf field near the ion-cyclotron frequency is applied at the line cusp. In this Letter, an experimental result of preferential rf plugging of the multi-ion species plasma is reported; that is, only the desired ions (resonance particles) are plugged preferentially. This technique may be used to achieve an impurity-free cusp reactor because the fusing particles are plugged and the impurities allowed to escape.

When the rf field is applied, the quasipotential φ_j, which acts on the particles, is given by the equation[5]

$$\varphi_j = \frac{(Z_j e)^2 E^2}{4m_p A_j} \frac{1}{\omega^2 - \omega_{cj}^2}$$
$$= \frac{e^2}{4m_p} \frac{E^2}{\omega_{cr}^2} A_j \frac{(Z/A)_j^2}{a^2 - (Z/A)_j^2/(Z/A)_r^2}, \qquad (1)$$

where m_p is the proton mass, e is the charge of the proton, ω_c is the cyclotron angular frequency, A is the atomic weight, and Z is the ionization multiplicity. The subscripts r and j refer to a resonance particle and an impurity. This expression was obtained by using the single-particle and cold-plasma approximations. When applying this equation to the plasma,[3] one should note that the rf field $E\exp(i\omega t)$ is not the external field but the net field in the plasma because the external field is altered by the self-field of the charged particles and is enhanced by the resonance of the plasma. Moreover, in the plasma, $(\omega^2 - \omega_{cj}^2)$ in Eq. (1) should be rewritten using the characteristic cyclotron oscillation frequency ω_{cr} of the plasma. For simplicity, we assume that $\omega = a\omega_{cr}$, where a is some constant $(1 \leq a < 2)^2$. Using Eq. (1), the potential, which acts on the impurity, can be calculated. If $\varphi_j < k_B T_j$, where k_B is Boltzmann's constant and T_j is the temperature of the impurity, more than half the impurities (i.e., that part of the Maxwellian energy distribution with energy $> \varphi_j$) are allowed to escape.

The experimental apparatus is shown in Fig. 1. The plasma consists of several ion species [for example, (He^+, He^{++}, N^+, and N_2^+) and (He^+, He^{++}, Ar^+, and Ar^{++})]. In order to produce these plasmas, argon or nitrogen gas is fed into the helium plasma which flows through the anode hole. The ratio of the density of Ar^+ and N^+ to that of He^+ is less than 0.2. The plasma density is typically 10^9 cm^{-3}. Ion and electron temperatures are equal to each other and are about 10 eV. A pair of ring-shaped rf electrodes 27 cm i.d., 37 cm o.d., and 5 cm in separation sandwich the plasma at the line cusp. Under the rf electrodes, the magnetic field strength is about 2.0 kG.

The ion loss from the line cusp is measured by a time-of-flight-type mass analyzer, which is set up outside of the line cusp. The detector signal is fed into an oscilloscope, so the brightness of the traces is proportional to the numbers of escaping ions. A typical experimental result is shown in Fig. 2, for the applied frequency equal to the resonance frequency of He^+. When the ap-

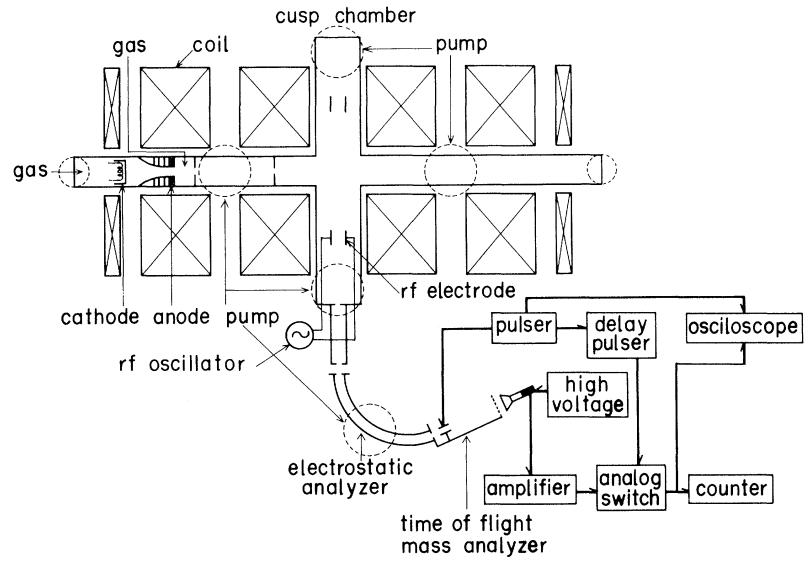

FIG. 1. Schematic of experimental apparatus and block diagram of measuring system.

plied rf field is increased, the brightness of the He^+ signal decreases but that of N^+ and N_2^+ remains unchanged. A similar result is also observed in the case of He-Ar plasma. To discuss the experimental results, we adopt the single-particle model ($a = 1.0$) because of low density. It is evident from Eq. (1) that the potential, $\varphi(He^+)$, which acts on the He^+ ion is extremely large (approaching infinity for no collisional damping), so the He^+ ion can be plugged in the container. The potentials which act on the other particles are also calculated: $\varphi(Ar^+) \sim 1.0$ eV, $\varphi(N_2^+) \sim 1.4$ eV, $\varphi(N^+) \sim 3$ eV, and $\varphi(Ar^{++}) \sim 4.1$ eV. These values are less than the energy of these particles, which is about 10 eV, so large fractions of the impurities escape from the container.

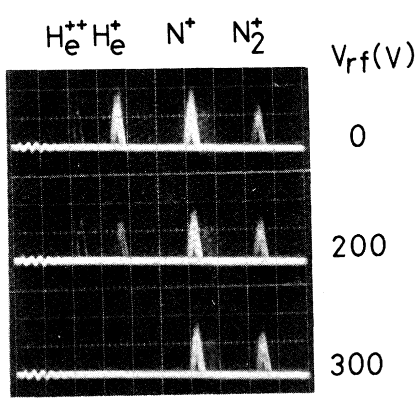

FIG. 2. The mass spectrum of escaping ions with the applied rf field as a parameter. The sweep time is 1 μsec/div.

Another experiment was carried out with a hydrogen plasma which consists of H^+, H_2^+, and H_3^+ ions. A counting method is adopted here to facilitate the quantitative discussion. The detector signal is gated by an analog switch and counted. The gate time can be adjusted so as to count only signals of desired ion species. The density ratio of each ion species is about 1 : 0.2 : 0.1. The typical result is shown in Fig. 3. The applied frequency is chosen as the resonance frequency of the H^+ ion. In the figure, α is defined as the ratio of the number of escaping ions when the rf field is applied to when it is not applied. For

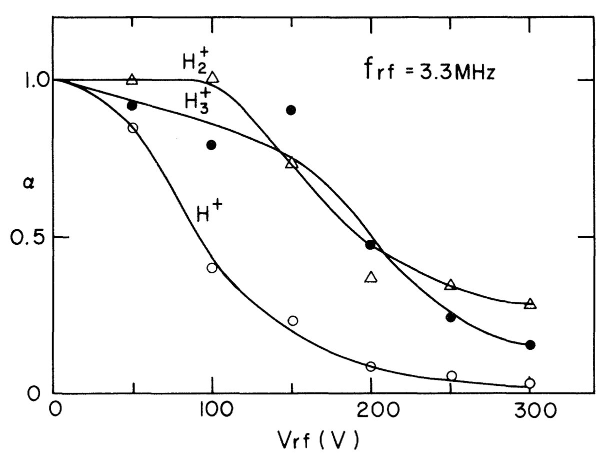

FIG. 3. Plot of loss factor of three ion species (H^+, H_2^+, and H_3^+) against the applied rf field. The rf frequency is adjusted to the resonance frequency of the H^+ ion.

TABLE I. Calculation of the conditions for krypton and xenon.

Ion species	Z	$\sigma \times 10^{22}$ (cm^2)	τ_f (msec)	τ_{eq} (msec)	τ_j (msec)	$(n\langle\sigma v\rangle)^{-1}$ (msec)	φ_j (keV)
Kr	9	2.0	0.47	3.0	3.5	1.5	17
	10		0.47	2.5	3.0		18
Xe	13	0.3	0.59	3.0	3.6	10	20

weak rf fields the loss of H^+ is reduced while the loss of the other ions remains unchanged. However for strong rf fields, all of the ion species are effectively plugged. Such an experimental result means that the particles whose cyclotron frequencies are near that of the resonance particle can be plugged at the same time; this result is also explained by Eq. (1). A plot of the negative slope $(-d\alpha/dV_{rf})$ versus V_{rf} would correspond approximately to the energy distribution of each species.

Applying this plugging method to a D-T fusion reactor, one can expect that the fuels can be plugged in the reactor while the impurities can be easily eliminated from the reactor. The conditions for impurities to be allowed to escape from the reactor are obtained as follows: (1) When the rf field necessary to plug the fuel is determined, the potential φ_j of each impurity can be calculated. So as to satisfy $\varphi_j < k_B T_j$ (corresponding to a loss $\gtrsim 0.5$), the critical values of A_j and $(Z/A)_j$ are determined. The density of impurity ions whose A_j and $(Z/A)_j$ are less than the above critical values can be preferentially reduced by $\gtrsim 50\%$ in the reactor. (2) From the above condition, the impurity must not be ionized more than the above critical ionization multiplicity in the interval of the confinement time of the impurity; that is,

$$\tau_j < (n\langle\sigma v\rangle)^{-1}, \tag{2}$$

where τ_j is the confinement time of the impurity and $(n\langle\sigma v\rangle)^{-1}$ is the time in which the impurity is ionized up to the above critical ionization multiplicity. We assume that the impurity is lost more easily from the end than to the wall across the static magnetic field, and is heated up to 20 keV in the time of equipartition of energy with the fusing particles. So the confinement time is the sum of the time of flight of impurity (τ_f) through the reactor and the time of equipartition (τ_{eq}), when the energy of impurity is equal to that of fuel.

We discuss concretely the above conditions in the case of krypton and xenon as impurities because the multiple ionization cross sections of these particles are known.[6] The calculated results are shown in Table I, where we adopt the following plasma parameters: The electron density is 4×10^{20} m^{-3}, the deuterium and tritium densities are equal to each other and are 2×10^{20} m^{-3}, the temperatures of the electrons and fusing particles are equal to each other and are 20 keV, the length of the reactor is 100 m, and the value (E/B) is 1.7×10^6 V m/Wb, $\sqrt{3}$ times less than that proposed by Hatori *et al.*[4] At such a value of E/B, the Lawson condition is not satisfied. However, it is not difficult to change the plasma parameters so as to satisfy the Lawson condition.

As is evident from Table I, Xe^{+13} can be allowed to escape (i.e., loss rate >0.5) from the reactor while Kr^{+9} cannot be allowed to escape (i.e., loss rate <0.5) because $\tau_j \gtrsim (n\langle\sigma v\rangle)^{-1}$. However, Kr^{+10} may be allowed to escape from the reactor because the ionization cross section of Kr^{+10} is less than that of Kr^{+9}.

Since there are insufficient data on the multiple ionization cross section of heavy metals which consist of the first wall of the reactor, we cannot discuss in detail whether the above conditions are satisfied or not. In the case of niobium, the critical ionization multiplicity becomes $Z_j = 10$. To satisfy condition (2), the ionization cross section must be less than 2×10^{-22} cm^2. If $\sigma < 2\times10^{-22}$ cm^2, niobium can be reduced by more than 50% in the reactor.

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Renormalization-Group Approach to the Critical Behavior of Random-Spin Models*

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A renormalization-group technique is used to study the critical behavior of spin models in which each interaction has a small independent random width about its average value. The cluster approximation of Niemeyer and Van Leeuwen indicates that the two-dimensional Ising model has the same critical behavior as the homogeneous system. The ϵ expansion for n-component continuous spins shows that this behavior holds to first order in ϵ for $n>4$. For $n<4$, there is a new stable fixed point with $2\nu = 1 + [3n/16(n-1)]\epsilon$.

The critical behavior of randomly diluted magnetic systems has been the object of some interest for many years.[1-3] Until recently, except for special models,[2] phase transitions in such systems could only be studied via series expansions.[1,2] This technique is not entirely satisfactory since it does not indicate whether the phase transition remains sharp with possibly different exponents or whether it washes out upon randomization of the interactions. A simple heuristic argument[4] indicates that the former behavior occurs when the specific heat exponent α is negative and the latter when α is positive. However, the validity of this argument is uncertain, and in any event, it gives no prediction for two-dimensional Ising models where $\alpha = 0$.

In this paper, we will discuss the application of the renormalization group,[5] which has been used so successfully in the calculation of critical exponents in pure systems, to phase transition in random systems. We introduce additional variables describing the randomization of the potential. These variables can be either irrelevant or relevant[6] in the vicinity of the pure-system fixed point. In the former case, we argue that the transition will be sharp with expoments of the pure system. In the latter case, either the system goes to a new stable fixed point, indicating a sharp phase transition with new critical exponents, or the renormalized randomization variables become infinite and the transition is probably smeared. We have applied two versions of the renormalization group to the random problem: the Niemeyer–Van Leeuwen (NL) cluster expansion for the two-dimensional Ising model, and the Wilson-Fisher ϵ expansion for n-component continuous spins with lattice dimensionality $d = 4 - \epsilon$. In the first case, we find for the small clusters we treat here that the randomization variables are irrelevant. In the second case, we find the same to be true for $n>4$ to first order in ϵ. For $n<4$, we find a new stable fixed point (i.e., a sharp transition) in which fluctuations in the local transition temperature have a nonvanishing value. Since space is limited, we will present in detail the calculations for the two-dimensional Ising model and only outline the results from the ϵ expansion. A detailed presentation of the ϵ expansion will follow shortly.

We consider Ising models described by the Hamiltonian

$$-\beta\mathcal{H} = \sum_{\langle ij\rangle} J_{ij} s_i s_j . \qquad (1)$$

Here $\beta = 1/kT$, the sum is over nearest neighboring pairs of sites, $s_i = \pm 1$, and each J is an independent random variable governed by a probabil-

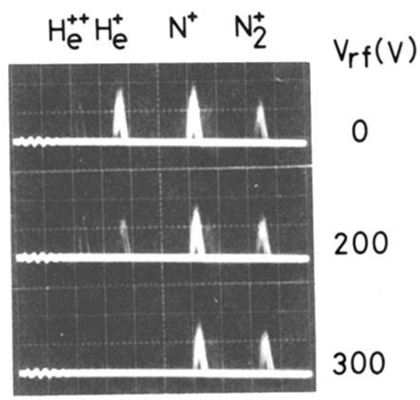

FIG. 2. The mass spectrum of escaping ions with the applied rf field as a parameter. The sweep time is 1 μsec/div.