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Effect of Localized Electric Fields on the Evolution of the Velocity Distribution Function*

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The interaction between charged particles and sharply localized fields is investigated. The random particle scattering is described by a Fokker-Planck equation whose time-dependent solution exhibits the formation of a highly populated superthermal tail.

Recent theoretical,[1,2] experimental,[3] and computer simulation[4] studies have demonstrated that sharply localized electric fields of high intensity can be nonlinearly generated in a plasma by an external pump field whose frequency is close to the electron plasma frequency, ω_p. The underlying physical process which gives rise to these localized fields can be traced to the formation of density cavities by the ponderomotive force exerted by the total rf field in the plasma. Inside these cavities the field amplitude can build up to large levels as a result of both cavity-resonance and wave-trapping effects. These localized fields can be formed in nonuniform[2-4] as well as in uniform[1] plasmas, and can attain energy densities comparable with the mean particle kinetic energy density over a localized region of the order of a few Debye wavelengths (e.g., $\sim 10\lambda_D$). Such narrow and intense fields can accelerate certain electrons to very high velocities; hence they provide an efficient method of transforming external energy into plasma kinetic energy.

In this Letter we investigate the interaction between electrons and intense localized fields. In particular, we calculate the time evolution of the distribution function of a model plasma which is envisioned to be in a turbulent state consisting of random localized fields whose nature has been described above. Such turbulence is assumed to be driven and maintained by external agents (e.g., rf sources, lasers, relativistic beams). The formation and evolution of the turbulence itself is not investigated in the present work. This simplification permits the isolation of the fundamental particle-acceleration effects which should be contained in a more complete future theory of spiky turbulence.

The equation of motion for an electron subjected to a localized field of frequency ω, phase θ, and amplitude E_0 is

$$d^2x(t)/dt^2 = (e/m)E_0 g(x(t))\cos(\omega t+\theta), \qquad (1)$$

in which e and m are the charge and mass of the electron, and x is its position at time t. Equation (1) is difficult to solve analytically because the shape function g must be evaluated at the particle trajectory, which is not known *a priori*. However, there are two limiting cases in which Eq. (1) becomes manageable. For small velocities one can extract the ponderomotive-force effects by averaging over the fast rf oscillations. For fast particles one can evaluate g along the straight-line orbits and thus obtain a first-order trajectory given by

$$x^{(1)}(t)=vt-(e/m)E_0\int_{-\infty}^{t}dt'\int_{-\infty}^{t'}dt''\,g(vt'')\cos(\omega t''+\theta),$$
$$\Delta v^{(1)}(t)=-(e/m)E_0\int_{-\infty}^{t}dt'\,g(vt')\cos(\omega t'+\theta) \tag{2}$$

and a second-order velocity change

$$\Delta v^{(2)}(t)=-(e/m)E_0\int_{-\infty}^{t}dt'\,g(x^{(1)}(t'))\cos(\omega t'+\theta). \tag{3}$$

Equations (2) and (3) constitute the Born approximation often used in various branches of physics to handle scattering problems. In this investigation we use the latter technique to calculate the scattering of a single electron by a localized electric field. Clearly, this procedure would be in great error if it were used to describe the exact particle trajectory of a slow particle. However, that is not what is required here. What one needs is the phase-averaged velocity kick $\langle\Delta V\rangle$ imparted to an electron by the localized field. We have found that this averaged quantity can be well described, for all velocities, by the Born approximation, the reason being that phase averaging leads to strong phase mixing for those slow particles which spend several rf periods inside the localized field. Thus, the velocity scattering effects arising from those slow particles for which the Born approximation is not valid are negligible, as will be seen in the following. We have investigated the validity of this approximation procedure by comparing the numerical results obtained by a particle-pushing computer calculation with the analytical result predicted by the Born approximation. Figure 1 displays the scaled phase-averaged energy change $\langle\Delta E\rangle/(m\bar{v}^2/2)$ found by these two different methods for a Gaussian field of width $10\lambda_D$ and a scaled amplitude $a\equiv(E_0^2/4\pi n_0 T)^{1/2}(\omega/\omega_p)=1.0$, where n_0 is the plasma density, and T the electron temperature. Figure 1 clearly attests to the success of the Born-scattering description of this problem. Note that Fig. 1 shows that the contribution from the slow particles is indeed negligible, as was previously indicated.

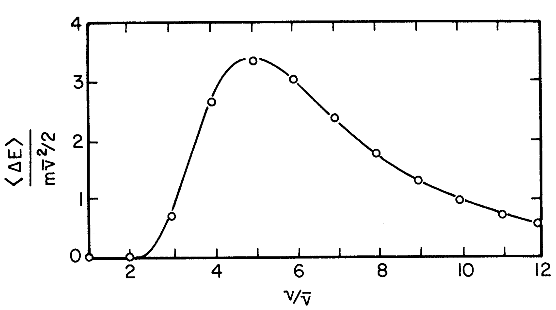

FIG. 1. Velocity dependence of the phase-averaged energy change. Solid curve is the Born-approximation result. Enlarged dots are the particle-pushing computer results.

Using the Born approximation, one can calculate the phase-averaged quantities $\langle\Delta u\rangle$ and $\langle(\Delta u)^2\rangle$, in which $u=v/\bar{v}$ and $\bar{v}$ is the thermal velocity. They are

$$\langle(\Delta u)^2\rangle=\tfrac{1}{4}a^2\left|\int_{-\infty}^{\infty}dy\,g(uy)\,e^{iy}\right|^2,$$
$$\langle(\Delta u)\rangle=\frac{1}{u}\left(1-\frac{1}{u}\frac{u}{du}\right)\langle(\Delta u)^2\rangle, \tag{4}$$

and they describe the average velocity changes produced by scattering off a localized field. We envision a turbulent state in which the electrons collide randomly with the localized fields, so that the evolution of their velocity distribution function f can be described by a Fokker-Planck equation of the form

$$\frac{\partial f}{\partial t}=-\frac{\partial}{\partial v}\left(\left[\left\langle\frac{\Delta V}{\Delta t}\right\rangle\right]f\right)+\frac{1}{2}\frac{\partial^2}{\partial v^2}\left(\left[\left\langle\frac{(\Delta V)^2}{\Delta t}\right\rangle\right]f\right), \tag{5}$$

in which the square brackets imply an ensemble average over the possible turbulent states and Δt refers to the mean time between collisions. In the spirit of the calculation $\Delta t=1/\tilde{n}v$, where $\tilde{n}$ is the linear density of localized fields. Combining Eqs. (4) and (5) yields

$$\frac{\partial f}{\partial t}=\frac{\partial}{\partial u}\left[\frac{a^2u\tilde{n}\bar{v}}{4}\left|\int_{-\infty}^{\infty}dy\,g(uy)\,e^{iy}\right|^2\right]\frac{\partial f}{\partial u} \tag{6}$$

which describes, in general, the evolution of f due to a localized electric field turbulence whose statistics enters through the ensemble average, which is yet to be performed. Unfortunately, at the present time one does not know enough about these spiky fields to be able to perform a rigorous average. Our present information is that these fields are roughly of the soliton shape, i.e., $g(uy)\simeq\operatorname{sech}(uy\lambda_D/d)$ with $d\sim5\lambda_D$, and $a\sim1$. We thus proceed to investigate Eq. (6) under the assumption that the ensemble average yields a

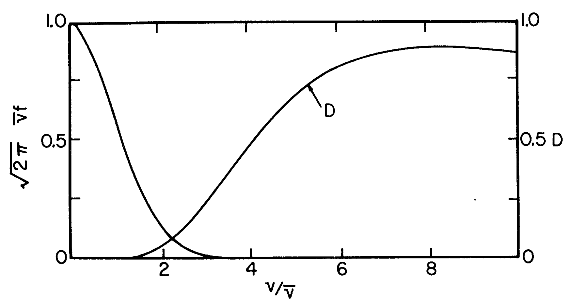

FIG. 2. Velocity dependence of the initial distribution function and of the normalized diffusion coefficient.

mean value for a^2 and d; i.e., we consider that diffusion coefficient that would be obtained as a result of scattering by the most probable spiky field. Defining the scaled width $w = d/\lambda_D$, and the scaled time $\tau = \pi w a^2 \bar{n} \bar{v} t$, yields

$$\frac{\partial f}{\partial \tau} = \frac{\partial}{\partial u}\left[D(u, w)\frac{\partial f}{\partial u}\right], \qquad (7)$$

with $D(u, w) = \pi (w/u)\,\mathrm{sech}^2(\pi w/2u)$.

We proceed to solve Eq. (7) numerically with a Maxwellian distribution as the initial condition. Figure 2 displays the velocity dependence of $f(\tau = 0)$ and D, corresponding to a field of full width $10\lambda_D$ at half-maximum. Figure 3 shows the distribution function at the times $\tau = 0$ and 25. It is evident from Fig. 3 that the localized fields accelerate some electrons very efficiently and thus produce a highly populated superthermal tail. The corresponding increase in the normalized kinetic energy of the system is shown in Fig. 4. This quantity exhibits an essentially linear time dependence, thus implying a constant heating rate given approximately by

$$(4\pi/25)\bar{n} d(\omega/\omega_p)^2(E_0^2/4\pi n_0 T)\omega_p^{-1}.$$

The development of superthermal tails has been observed in computer simulations.[5,6] Their formation has been previously attributed to quasilinear diffusion[6] by a broad wave spectrum, and to quasilinear diffusion by soliton-shaped wave packets.[7] It has been shown here that spiky turbulence can also give rise to this type of phenomenon.

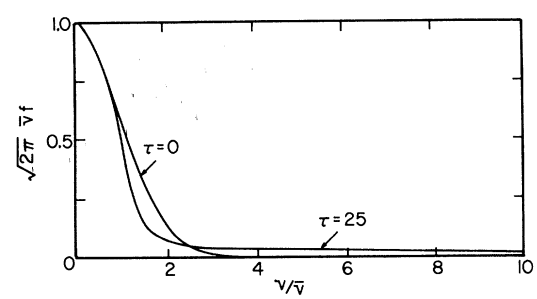

FIG. 3. Velocity distribution function at times $\tau = 0$ and 25.

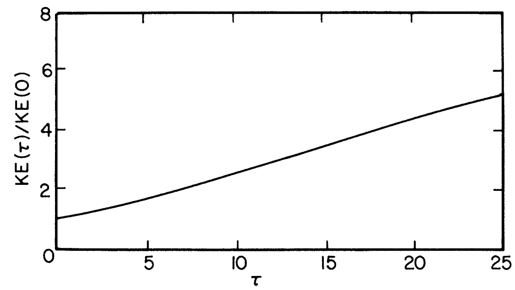

FIG. 4. Time evolution of the total kinetic energy.

An examination of Fig. 3 shows that the main bulk of the electron distribution function is not significantly altered by the localized fields. This is an important feature which is required by the internal consistency of the present model. The reason is that these background particles are the ones that participate in the formation of the density cavities which are required to support the localized fields. The response of these slow particles can be treated by a fluid approach, as is done in Refs. 1 and 2.

It has been found that the phase-averaged scattering of charged particles by localized electric fields is well described by the Born approximation. This technique permits the calculation of the diffusion coefficient that governs the evolution of the distribution function due to spiky turbulence. Such turbulence forms highly populated superthermal tails and thus provides an efficient method for increasing the kinetic energy of a plasma.

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Preferential Radiofrequency Plugging of Multi-Ion Species Plasmas in a Static Cusped Confinement System

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The rf preferential plugging of multi-ion species plasma is studied experimentally in a magnetic cusp: Desired ion species can be plugged and the others escape from the container. The conditions necessary to realize an impurity-free cusp reactor using this concept are discussed. It is shown that impurities, specifically krypton and argon, can be preferentially reduced in the reactor.

One of the recent problems of Tokamak plasma is the concentration of heavy impurity ions in the central region of the plasma column.[1] Such a problem can be solved more easily in an open-ended system than a closed system. It has been shown[2-4] that the end loss from the line cusp can be plugged effectively when an rf field near the ion-cyclotron frequency is applied at the line cusp. In this Letter, an experimental result of preferential rf plugging of the multi-ion species plasma is reported; that is, only the desired ions (resonance particles) are plugged preferentially. This technique may be used to achieve an impurity-free cusp reactor because the fusing particles are plugged and the impurities allowed to escape.

When the rf field is applied, the quasipotential φ_j, which acts on the particles, is given by the equation[5]

$$\varphi_j = \frac{(Z_j e)^2 E^2}{4 m_p A_j} \frac{1}{\omega^2 - \omega_{cj}^2} = \frac{e^2}{4 m_p} \frac{E^2}{\omega_{cr}^2} A_j \frac{(Z/A)_j^2}{a^2 - (Z/A)_j^2/(Z/A)_r^2}, \tag{1}$$

where m_p is the proton mass, e is the charge of the proton, ω_c is the cyclotron angular frequency, A is the atomic weight, and Z is the ionization multiplicity. The subscripts r and j refer to a resonance particle and an impurity. This expression was obtained by using the single-particle and cold-plasma approximations. When applying this equation to the plasma,[3] one should note that the rf field $E\exp(i\omega t)$ is not the external field but the net field in the plasma because the external field is altered by the self-field of the charged particles and is enhanced by the resonance of the plasma. Moreover, in the plasma, $(\omega^2 - \omega_{cj}^2)$ in Eq. (1) should be rewritten using the characteristic cyclotron oscillation frequency ω_{cr} of the plasma. For simplicity, we assume that $\omega = a\omega_{cr}$, where a is some constant $(1 \leq a < 2)^2$. Using Eq. (1), the potential, which acts on the impurity, can be calculated. If $\varphi_j < k_B T_j$, where k_B is Boltzmann's constant and T_j is the temperature of the impurity, more than half the impurities (i.e., that part of the Maxwellian energy distribution with energy $> \varphi_j$) are allowed to escape.

The experimental apparatus is shown in Fig. 1. The plasma consists of several ion species [for example, (He^+, He^{++}, N^+, and N_2^+) and (He^+, He^{++}, Ar^+, and Ar^{++})]. In order to produce these plasmas, argon or nitrogen gas is fed into the helium plasma which flows through the anode hole. The ratio of the density of Ar^+ and N^+ to that of He^+ is less than 0.2. The plasma density is typically 10^9 cm^{-3}. Ion and electron temperatures are equal to each other and are about 10 eV. A pair of ring-shaped rf electrodes 27 cm i.d., 37 cm o.d., and 5 cm in separation sandwich the plasma at the line cusp. Under the rf electrodes, the magnetic field strength is about 2.0 kG.

The ion loss from the line cusp is measured by a time-of-flight-type mass analyzer, which is set up outside of the line cusp. The detector signal is fed into an oscilloscope, so the brightness of the traces is proportional to the numbers of escaping ions. A typical experimental result is shown in Fig. 2, for the applied frequency equal to the resonance frequency of He^+. When the ap-