

pressed in angstroms and energy in inverse centimeters.

<sup>11</sup>We have calculated the intensity from the semiclassical formula [Eq. (3) in Ref. 5] over the range 2.5–3.7 Å, and found some disagreement with the curve in Fig. 3(a) of Ref. 5. We attribute this difference to errors in the determination of  $dr/d(\epsilon_2 - \epsilon_1)$ . We have evaluated this derivative numerically from the RKR data of Ref. 8, extrapolated as discussed in the text, whereas Callender *et al.* (R. W. Leigh, private communication) utilized a less accurate method which involved approximating the RKR potentials with Morse potentials. A difference of up to a factor of 4 was found between the

quantum curve calculated from our Eq. (3) and the semiclassical intensity curve determined with the RKR data.

<sup>12</sup>Recently Callender *et al.* [R. H. Callender, J. I. Gersten, R. W. Leigh, and J. L. Yang, Phys. Rev. Lett. **33**, 1311(C) (1974)] incorporated angular momentum in their semiclassical theory and obtained a result in qualitative agreement with our quantum calculation.

<sup>13</sup>P. A. Fraser, Can. J. Phys. **32**, 515 (1954).

<sup>14</sup>For  $R$  expressed in angstroms, the values used for  $A$ ,  $B$ , and  $C$  are 24.64, -142.52, and 207.02, respectively.

## Comments on a New Scaling Hypothesis in High-Energy Collisions\*

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Dao *et al.* have demonstrated that single-particle distributions in inclusive reactions scale in a new variable. We comment on the nature of the scaling relation satisfied and note that their parametrization of the single-particle distributions requires only two parameters rather than the six they used.

A recent paper by Dao *et al.*<sup>1</sup> begins with the sentences, "The scaling behavior of a distribution  $f(x_1, x_2)$  with independent variables  $x_1$  and  $x_2$  can be defined in terms of a homogeneous equation,

$$f(\lambda_1 x_1, \lambda_2 x_2) = \lambda f(x_1, x_2). \quad (1)$$

The distribution  $f$  scales if there exist scaling parameters  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda$  for which Eq. (1) is satisfied."<sup>2</sup> The authors then make the hypothesis that single-particle distributions in inclusive reactions satisfy Eq. (1). It becomes apparent on further reading of the paper that the authors do not intend  $f(\lambda_1 x_1, \lambda_2 x_2)$  to be the same function of its argument as  $f(x_1, x_2)$  is of its argument. Rather, the authors mean that a given function of three variables  $f(x_1, x_2, s)$  scales if there exists a function of two variables  $g(x_1, x_2)$  such that

$$g(\lambda_1 x_1, \lambda_2 x_2) = \lambda f(x_1, x_2, s), \quad (2)$$

where  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda$  are functions of  $s$  alone. It is the presence of the third variable  $s$  which makes Eq. (2) nontrivial.

The authors give as examples the single-particle distributions in transverse ( $p_T$ ) and longitudinal ( $p_L$ ) momentum variables, with

$$\begin{aligned} \lambda(s) &= \langle p_T \rangle \langle p_L \rangle, & \lambda_1(s) &= 1/\langle p_T \rangle, \\ & & \lambda_2(s) &= 1/\langle p_L \rangle, \end{aligned} \quad (3)$$

and  $s$  standing for the square of the energy in the

c.m. system. The authors test Eq. (2) after a single integration over  $p_T$  or  $p_L$ , obtaining the following relations for the inclusive cross sections  $d\sigma/dp_T$  and  $d\sigma/dp_L$ :

$$\langle \langle p_T \rangle / \sigma \rangle d\sigma/dp_T = \varphi_T(p_T/\langle p_T \rangle), \quad (4)$$

$$\langle \langle p_L \rangle / \sigma \rangle d\sigma/dp_L = \varphi_L(p_L/\langle p_L \rangle). \quad (5)$$

The functions appearing in Eqs. (4) and (5) are related to the  $f$  and  $g$  of Eq. (2) by<sup>3</sup>

$$\sigma^{-1} d\sigma/dp_i = \int f(p_T, p_L, s) dp_j, \quad (6)$$

$$\varphi_i \left( \frac{p_i}{\langle p_i \rangle} \right) = \int g \left( \frac{p_T}{\langle p_T \rangle}, \frac{p_L}{\langle p_L \rangle} \right) \frac{dp_j}{\langle p_j \rangle}, \quad (7)$$

where the subscripts  $i, j$  stand for either  $T$  or  $L$  and  $i \neq j$ . Equations (4) and (5) are conventional scaling equations in the new variables  $p_i/\langle p_i \rangle$ .

The authors have shown the important result that, in a rather large energy range, the existing data on inclusive cross sections satisfy Eqs. (4) and (5) to a good approximation. They have also shown that the functions  $\varphi_i$  can be well approximated by the simple forms

$$\varphi_T \left( \frac{p_T}{\langle p_T \rangle} \right) = a \left( \frac{p_T}{\langle p_T \rangle} \right)^c \exp \left[ -b \left( \frac{p_T}{\langle p_T \rangle} \right) \right], \quad (8)$$

$$\varphi_L \left( \frac{p_L}{\langle p_L \rangle} \right) = d \exp \left[ -e \frac{p_L}{\langle p_L \rangle} - f \left( \frac{p_L}{\langle p_L \rangle} \right)^2 \right], \quad (9)$$

where  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$ , and  $f$  are parameters which

the authors adjusted to obtain best agreement with experiment. However, only two of these six parameters are independent, as we shall now show.

The average values  $\langle p_T \rangle$  and  $\langle p_L \rangle$  are conventionally defined by the equations

$$\langle p_i \rangle = \int \varphi_i \left( \frac{p_i}{\langle p_i \rangle} \right) p_i dp_i \left[ \int \varphi_i \left( \frac{p_i}{\langle p_i \rangle} \right) dp_i \right]^{-1}. \quad (10)$$

Also, the total cross section  $\sigma$  is defined by

$$\sigma = \int (d\sigma/dp_i) dp_i. \quad (11)$$

If we substitute Eq. (11) into Eqs. (4) and (5) and integrate, we obtain that the functions  $\varphi_i$  are normalized as follows:

$$\int \varphi_i \left( \frac{p_i}{\langle p_i \rangle} \right) \frac{dp_i}{\langle p_i \rangle} = 1. \quad (12)$$

Equations (10) and (12) each constitute two constraints on the six parameters, and so only two of the parameters are independent, one each for  $\varphi_T$  and  $\varphi_L$ . The four relations among the parameters are

$$b = c + 1, \quad a = b^b / \Gamma(b),$$

$$\sqrt{\pi} \left( \sqrt{f} + \frac{e}{2\sqrt{f}} \right) \exp \left( \frac{e^2}{4f} \right) \operatorname{erfc} \left( \frac{e}{2\sqrt{f}} \right) = 1,$$

$$d = e + 2f.$$

The values of the parameters found by the authors satisfy these four equations within the stated errors. The fact that the relations are not all exactly satisfied by the central values of the parameters means that a slightly better fit to the distributions can be obtained by a small relaxation of the constraints. The functions  $\varphi_T$  and  $\varphi_L$  then yield values of the total cross section and/or  $\langle p_i \rangle$  which differ somewhat from the input values, but presumably the differences are within the errors.

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<sup>1</sup>F. T. Dao *et al.*, Phys. Rev. Lett. **33**, 389 (1974).

<sup>2</sup>Dao *et al.* obviously mean that there should exist parameters  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda$  not all equal to unity, because if they are all unity, Eq. (1) is an identity.

<sup>3</sup>We take the limits of integration for  $p_T$  and  $p_L$  to be between 0 and  $\infty$ , although of course at any given incident energy the upper limit is finite. We assume that the energies are large enough so that this assumption leads to a negligible error.

## Comment on "Self-Consistent Solution for an Axisymmetric Pulsar Model"

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The numerical results of Kuo-Petavic, Petavic, and Roberts, in which magnetic field lines are found to close beyond the light cylinder, are reconciled with the canonical theoretical view that these field lines must be open. We argue that a small region of trapped plasma on closed field lines relatively near the star (but nevertheless outside of it) expands until the light cylinder is reached. The remaining field lines are forced open and do not close beyond the light cylinder.

Kuo-Petavic, Petavic, and Roberts<sup>1</sup> have numerically integrated for the self-consistent particle and field structure about a rotating star having an aligned magnetic dipole moment. This problem is of considerable astrophysical interest in that pulsars are generally assumed to be rapidly rotating magnetized neutron stars, and analysis of the aligned dipole moment case is viewed as a first step in understanding the physics of such objects. Goldreich and Julian<sup>2</sup> first

discussed the aligned dipole case. Their view was that the induction electric field (which would be quadrupolar in a vacuum, hence  $\vec{E} \cdot \vec{B} \neq 0$ ) acts to draw charged particles from the surface. These particles establish a space charge about the star such that  $\vec{E} \cdot \vec{B} \approx 0$  and consequently corotate rigidly with the star if they are on closed field lines. Such rigid corotation is physically impossible beyond the "light cylinder," i.e., the axial distance  $R_L = c/\omega$ , where  $\omega$  is the rotation