Application of Current Algebra Techniques to Neutral-Current–Induced Threshold Pion Production

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I apply current-algebra techniques to study threshold pion production induced by the weak neutral current. In addition to specific predictions for the Weinberg-Salam–model current, I find upper bounds on the magnitude of threshold pion production for an isoscalar neutral current and for a general hadronic neutral current formed from the usual vector and axial-vector nonets. Violation of these bounds would suggest the presence of new coupling types in the neutral semileptonic interaction.

The initial experiments discovering weak neutral currents in high-energy deep-inelastic neutrino reactions[1] have now been supplemented with the observation of neutral-current effects in low-energy neutrino pion production.[2,3] Obtainable invariant-mass resolutions will permit the study of πN production in the threshold region below the (3, 3) resonance, and in fact preliminary Argonne data[2] (without final corrections for neutron background) raise the possibility that the threshold cross section for $\pi^- p$ production by the neutral current may be appreciable. In this Letter we study threshold pion-production processes by using current-algebra, soft-pion techniques. I briefly describe the methods used in making such an analysis, and summarize the results obtained.

I begin by giving a simple analytic treatment of threshold pion production, which, although somewhat naive, illustrates the basic ideas which we exploit in our more careful numerical calculations. According to standard soft-pion lore,[4] the amplitude for the pion emission process $\mathcal{J}+\alpha \to \pi^j+\beta$, with α and β hadronic states and $\mathcal{J}$ an external current, is given as the sum of two terms. The first consists of a sum of external line insertions in which the pion π^j is emitted from the external hadronic lines of the pionless process $\mathcal{J}+\alpha \to \beta$, while the second is an equal-time commutator term proportional to the amplitude for the reaction $\mathcal{J}'+\alpha\to\beta$, with $\mathcal{J}'$ the modified current obtained from the commutator $\mathcal{J}'=[F_j^5,\mathcal{J}]$. In the case of neutral-current weak pion production, the current $\mathcal{J}$ is, of course, the hadronic weak neutral current and the states α and β are each a single free nucleon. For simplicity, let us restrict ourselves for the moment to cases in which the equal-time commutator term vanishes, as occurs, for example, if the current $\mathcal{J}$ is an isoscalar $V-A$ structure containing an arbitrary linear combination of $\mathfrak{F}_0^{\lambda}$, $\mathfrak{F}_8^{\lambda}$, $\mathfrak{F}_0^{5\lambda}$, $\mathfrak{F}_8^{5\lambda}$.[5] The pion emission amplitude then consists entirely of the external line insertion terms. Evaluating these terms at threshold (where the insertion on the outgoing nucleon line vanishes) and neglecting the pion mass in all kinematics, we find the following relation between threshold pion production and neutrino proton elastic scattering:

$$\frac{1}{|\vec{q}|}\frac{d\sigma(\nu+N\to\nu+N+\pi^j)}{d(k^2)dW}\bigg|_{\text{threshold}}=\frac{a^2}{4\pi^2M_\pi^2}\left(\frac{g_rM_\pi}{2M_N}\right)^2\frac{k^2}{M_N^2}\left(1+\frac{k^2}{4M_N^2}\right)\left(1+\frac{k^2}{2M_N^2}\right)^{-2}\frac{d\sigma(\nu+p\to\nu+p)}{d(k^2)}. \tag{1}$$

Here M_N, M_π are the nucleon and pion mass, W is the mass of the final π^jN isobar, $|\vec{q}|$ is the pion momentum in the isobaric rest frame, k^2 is the leptonic squared four-momentum transfer (spacelike, $k^2>0$), $g_r\approx 13.5$ is the pion-nucleon coupling constant, and the isospin matrix element a takes the values $|a|=\sqrt{2}$ for $\pi^j=\pi^\pm$ and $|a|=1$ for $\pi^j=\pi^0$. *The significance of Eq. (1) is that it allows one to translate an upper bound on the cross section for $\nu_\mu+p\to\nu_\mu+p$ into an upper bound on the strength of threshold pion production by the weak neutral current.*

As I have already suggested, the above derivation is too naive in a number of respects. First of all, the external line insertion terms are rapidly varying pole terms, and so the kinematic approximation of neglecting M_π in calculating them is dangerous. Secondly, by considering only cases in which the equal-time commutator term $\mathcal{J}'$ vanishes, we exclude from consideration such processes as π^- production in the SU(2) $\otimes$ U(1) gauge model. And finally, it is important to estimate the leading $O(q)$ corrections to the soft-pion approximation, and to calculate the effects in the threshold region of the tail of the

(3,3) resonance. We deal with these problems by using an extended version of a model for weak pion production which has been described in detail elsewhere.[6] In its original form, the model included the rapidly varying pole terms and the resonant (3,3) multipoles, with no kinematic approximations. The extensions consist of adding subtraction constants (in the dispersion-theory sense) to the non-Born terms of the model, which guarantee that it satisfies the relevant soft-pion theorems and which include the leading corrections (of first order in the pion four-momentum q and zeroth order in the lepton four-momentum transfer k) to the soft-pion limit. These latter corrections are calculated by the method of Low[7] and Adler and Dothan[7]; for the vector current amplitude they vanish, while for the isovector axial-vector amplitude they are related by partial conservation of axial-vector current to momentum derivatives of the pion-nucleon scattering amplitude at the crossing-symmetric point. For an isoscalar axial-vector current the order-q corrections cannot be precisely calculated, but a heuristic resonance-dominance argument suggests that they should be much smaller than in the isovector axial-vector case, and so we neglect them.

I give now the results of numerical calculations using the extended model in various cases, focusing attention on the reaction[8] $\nu_\mu+n\to\nu_\mu+\pi^-+p$.

(1) *Isoscalar neutral current.*—For the vector and axial-vector form factors in this case we take, for definiteness, a dipole formula with characteristic mass M_N,

$$F_1^S(k^2)=\lambda_1(1+k^2/M_N^2)^{-2},\quad 2M_NF_2^S(k^2)=\lambda_2(1+k^2/M_N^2)^{-2},\quad g_A^S(k^2)=\lambda_3(1+k^2/M_N^2)^{-2}, \tag{2}$$

with λ_1, λ_2, and λ_3 free parameters. Assuming the 95% confidence bound[2]

$$\sigma(\nu_\mu+p\to\nu_\mu+p)\leq 0.32\sigma(\nu_\mu+n\to\mu^-+p), \tag{3}$$

we find that the cross section for $\nu_\mu+n\to\nu_\mu+\pi^-+p$, with π^-p invariant mass W between[8] 1080 and 1120 MeV, is bounded by[9]

$$\sigma(\nu_\mu+n\to\nu_\mu+\pi^-+p)\leq 0.32\sigma(\nu_\mu+n\to\mu^-+p)[\sigma(\nu_\mu+n\to\nu_\mu+\pi^-+p)/\sigma(\nu_\mu+p\to\nu_\mu+p)], \tag{4a}$$

$$\leq 1.0\times10^{-41}\text{ cm}^2. \tag{4b}$$

The inequality in Eq. (4b) is obtained by maximizing the ratio in square brackets with respect to variation of λ_1, λ_2, and λ_3. We find in this case that the naive form of the low-energy theorem in Eq. (1) is reasonably good, predicting a bound about one-third as large as that of Eq. (4).[10]

(2) *Weinberg-Salam SU(2)⊗U(1) model.*—In the simplest, one-parameter version of this model, the neutral current has the form

$$\mathcal{J}_N{}^\lambda=\mathfrak{F}_3{}^\lambda-\mathfrak{F}_3{}^{5\lambda}-2x(\mathfrak{F}_3{}^\lambda+3^{-1/2}\mathfrak{F}_8{}^\lambda)+\Delta\mathcal{J}^\lambda,\quad x\equiv\sin^2\theta_W, \tag{5}$$

with $\Delta\mathcal{J}^\lambda$ an isoscalar, $V-A$, strangeness- and "charm"-current contribution which is conventionally assumed to couple only weakly to nonstrange low-mass hadrons. Neglecting $\Delta\mathcal{J}^\lambda$ for the moment, we can make an absolute calculation of the cross section for $\nu_\mu+n\to\nu_\mu+\pi^-+p$. We find, for π^-p invariant mass W between 1080 and 1120 MeV, a predicted cross section of 0.75 $\times10^{-41}$ cm^2. To assess the reliability of our calculations, Fig. 1 gives a comparison of our model with the Argonne National Laboratory results for the charged-current reaction $\nu_\mu+p\to\mu^-+\pi^++p$. The predicted cross section for π^+p invariant mass W between 1080 and 1120 MeV is 6.9 $\times10^{-41}$ cm^2, in satisfactory agreement with the observed cross section of $(9.3\pm4.7)\times10^{-41}$ cm^2.

In certain extensions of the original Weinberg-Salam model, the neutral current has the general form of Eq. (5), but with an adjustable strength parameter κ in front. A useful upper bound on the magnitude of κ is provided by deep-inelastic neutrino-scattering neutral-current data. In terms of the standard ratios $R_{\nu,\bar\nu}\equiv\sigma(\nu,\bar\nu+N\to\nu,\bar\nu+\Gamma)/\sigma(\nu,\bar\nu+N\to\mu^-,\mu^++\Gamma)$, we find[11] the 95% confidence limit[1]

$$1.5\geq 3R_\nu+R_{\bar\nu}\geq\kappa^2[1+(1-2x)^2]. \tag{6}$$

Continuing for the moment to neglect the isoscalar addition $\Delta\mathcal{J}^\lambda$, we can combine the bound of Eq. (6) with the extended model to predict that the cross section for neutral-current π^- production, with π^-p invariant mass W between 1080 and 1120 MeV, is bounded by 1.5×10^{-41} cm^2 for all allowed values[12] of κ and x. Finally, we can include the isoscalar addition $\Delta\mathcal{J}^\lambda$ by parametriz-

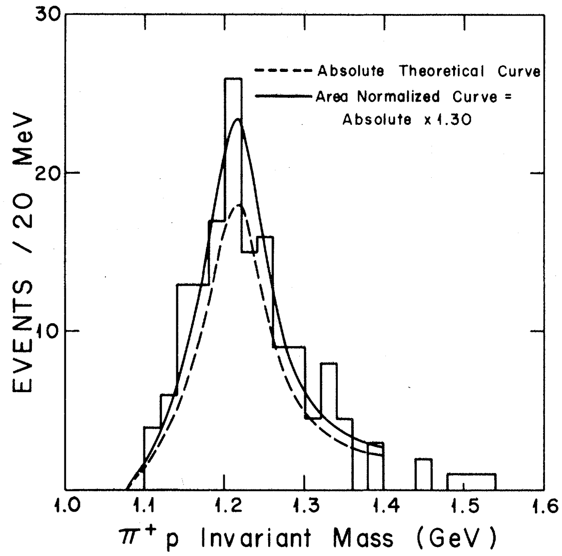

FIG. 1. Comparison of the extended pion production model with the Argonne National Laboratory charged-current data. Each event represents an Argonne flux-averaged cross section of 2.3×10^{-41} cm^2.

ing the total isoscalar contribution to $\mathcal{J}_N{}^\lambda$ as in Eq. (2), giving a cross section dependent on the five parameters κ, x, λ_1, λ_2, and λ_3. Combining the bounds of Eqs. (6) and (4a) with the extended model and maximizing over the five-parameter space,[12] we find that the cross section for $\nu_\mu+n\to\nu_\mu+\pi^-+p$, with W between 1080 and 1120 MeV, is bounded by 4.4×10^{-41} cm^2, for a general hadronic neutral current formed from the usual vector and axial-vector nonets.[13]

Experimental violation of this general bound, or the observation of evidence for an isoscalar neutral current together with violation of the bound of Eq. (4b), would suggest that the neutral current involves unusual types of coupling, in addition to or in place of the usually assumed $V-A$ structure. One possible source of violations could be an interaction of the $V-A$ type involving currents outside the usual quark-model vector and axial-vector nonets. An alternative source of violations could be the presence of S-, P-, and T-type neutral-current couplings.[14] If we define S, P, and T hadronic "currents" $\mathfrak{F}_j$, $\mathfrak{F}_j{}^5$, $\mathfrak{F}_j{}^{\lambda\sigma}$ and abstract their commutation relations from the quark-model forms

$$\mathfrak{F}_j=\bar{q}\,\tfrac{1}{2}\lambda_j q,\quad \mathfrak{F}_j{}^5=\bar{q}\,\tfrac{1}{2}\lambda_j\gamma_5 q,$$
$$\mathfrak{F}_j{}^{\lambda\eta}=\bar{q}\,\tfrac{1}{2}\lambda_j\sigma^{\lambda\eta}q, \tag{7}$$

then the commutator term $\mathcal{J}'$ appearing in the soft-pion analysis above will have SU(3) D- rather than F-type structure. This will substantially alter the structure of the low-energy theorems; for instance, the commutator term will no longer vanish for an isoscalar neutral current. The effect of this altered structure on the bounds given above is presently under study.

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[1]F. J. Hasert *et al.*, Phys Lett. 46B, 138 (1973); A. Benvenuti *et al.*, Phys. Rev. Lett. 32, 800 (1974).

[2]P. A. Schreiner, in Proceedings of the Seventeenth International Conference on High Energy Physics, London, England, July 1974 (to be published).

[3]Columbia-Rockefeller-Illinois Collaboration, in Proceedings of the Seventeenth International Conference on High Energy Physics, London, England, July 1974 (to be published).

[4]S. L. Adler and R. F. Dashen, *Current Algebras* (Benjamin, New York, 1968).

[5]Vanishing of the equal-time commutator in this case was noted by J. J. Sakurai, in Proceedings of the Fourth International Conference on Neutrino Physics and Astrophysics, Philadelphia, Pennsylvania, April 1974 (to be published).

[6]S. L. Adler, Ann. Phys. (New York) 50, 189 (1968). [See also S. L. Adler, Phys. Rev. D 9, 229 (1974).] The extended model is obtained by adding as subtraction constants Eq. (5A.21) for $\bar{A}_2{}^{(-)}|_0$, $\bar{A}_4{}^{(-)}|_0$, and $\bar{A}_7{}^{(+)}|_0$; Eq. (5A.22) for $\bar{V}_1{}^{(+)}|_0$, $\bar{V}_1{}^{(0)}|_0$, and $\bar{V}_6{}^{(-)}|_0$; Eq. (5A.9) for $\bar{A}_3{}^{(+)}|_0$; and Eq. (5A.30) for $\bar{A}_1{}^{(-)}|_0$. The order-q terms $\bar{A}_3{}^{(+)}|_0$ and $\bar{A}_1{}^{(-)}|_0$ were assumed to have k^2 dependence $(1+k^2/M_N{}^2)^{-2}$; variation of this assumed dependence produced only small changes in the results. We took the axial-vector form-factor mass as $M_A=0.9$ GeV.

[7]F. E. Low, Phys. Rev. 110, 974 (1958); S. L. Adler and Y. Dothan, Phys. Rev. 151, 1267 (1966).

[8]Analogous bounds can be given for other pion-production channels and for larger invariant-mass intervals than the one considered here.

[9]The quoted bounds are not corrected for possible differences in the k^2 distributions of the reactions $\nu_\mu+p\to\nu_\mu+p$ and $\nu_\mu+n\to\mu^-+p$. For neutral-current form factors which decrease much more slowly than the charged-current form factors, the effect of such corrections would be to decrease the bounds.

[10]In the case of $\nu_\mu+N\to\nu_\mu+\pi^0+N$ in the Weinberg-

Salam model, where Eq. (1) should formally hold, we find that the order-q corrections increase the (greatly suppressed) threshold pion production by an order of magnitude. As a result, the threshold π^0 production becomes comparable to that in $\nu_\mu + n \to \nu_\mu + \pi^- + p$ (where the order-q corrections have only an ~20% effect).

[11]Equation (6) assumes scaling, and also uses the fact that $\sigma(\bar{\nu}_\mu + N \to \mu^+ + \Gamma)/\sigma(\nu_\mu + N \to \mu^- + \Gamma) \approx \frac{1}{3}$. See A. Pais and S. B. Treiman, Phys. Rev. D 6, 2700 (1972).

[12]We search over all real values of x, even though only the range $0 \leq x \leq 1$ is physically meaningful in the SU(2)$\otimes$U(1) model.

[13]This bound would be reduced if Eq. (6) were strengthened to include the isoscalar current contributions on the right-hand side.

[14]Tests for such couplings in the neutral current have been discussed by B. Kayser, G. T. Garvey, E. Fischbach, and S. P. Rosen (to be published) and by R. L. Kingsley, F. Wilczek, and A. Zee (to be published).