terms would be of the same order as the contributions to the reduced rates, i.e., ~7%, a level at which current experiments are not sensitive. On the other hand, the disagreement in the linear coefficients clearly indicates a $|\Delta I| = \frac{1}{2}$ violation in the nonsymmetric amplitudes.

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†A. P. Sloan Foundation Fellow.

[1]R. Piccioni *et al.*, Phys. Rev. D 9, 2939 (1974).

[2]The $K_L{}^0$ time of flight was measured over a 62.5-m flight path with a resolution of ± 0.33 nsec. This allowed useful momentum determinations up to 7.5 GeV/c.

[3]The $K_{\pi 3}{}^0$ detection efficiency increases linearly from 12% for $P_K = 2-4$ GeV/c to 68% for $P_K = 7-12$ GeV/c, and is constant to within a factor of 1.5 across the Dalitz plot.

[4]$P_0'^2$ is the square of the $K_L{}^0$ momentum in the frame where the sum of the longitudinal momenta of the charged pair is zero, assuming a $K_{\pi 3}{}^0$ decay. For actual $K_{\pi 3}{}^0$ decays $P_0'^2 \geq 0$, while for $K_{l3}{}^0$ decays $P_0'^2$ is usually negative.

[5]J. Smith, private communication; A. Neveu and J. Scherk, Phys. Lett. 27B, 384 (1968); R. Ferrari and M. Rosa-Clot, Nuovo Cimento 56A, 582 (1968).

[6]Since $|M^2|$ depends on both Y and X^2 ($|M|^2 = 1 + aY + bY^2 + dX^2$), corrections must be applied to the direct projection of the data onto the Y or X axis before we can separate the two dependences and obtain the correct projected slopes. We introduce the following scheme (since we see no measurable X^2Y dependence): $|M(Y)|^2 = \sum_j (M_j{}^2 - dX_j{}^2)$ displayed in Fig. 2(a), and $|M(X)|^2 = \sum_i (M_i{}^2 - aY_i - bY_i{}^2)$ displayed in Fig. 2(b), where j is summed over all the X_j bins and i is summed over all the Y_i bins.

[7]C. Buchanan *et al.*, Phys. Lett. 33B, 623 (1970); R. Smith *et al.*, Phys. Lett. 32B, 133 (1970); M. Albrow *et al.*, Phys. Lett. 33B, 516 (1970).

[8]If we expand the matrix in terms of the invariants $S_i = (P_K - P_i)^2$, $|M|^2 = 1 + g/m_\pi{}^2(S_3 - S_0) + h/m_\pi{}^4(S_3 - S_0)^2 + k/m_\pi{}^4(S_2 - S_1)^2 + \ldots$ as described by T. A. Lasinski *et al.*, Rev. Mod. Phys. 45, S1 (1973), we obtain $g = 0.677 \pm 0.010$, $h = 0.079 \pm 0.007$, and $k = 0.0097 \pm 0.0018$.

[9]W. T. Ford, P. A. Piroué, R. S. Remmel, A. J. S. Smith, and P. A. Souder, Phys. Lett. 38B, 335 (1972).

[10]K. T. Mahanthappa, private communication; M. K. Gaillard, CERN Report No. TH-1963, 1973 (unpublished). If the $\pi^\pm \pi^0$ mass difference is ignored then a similar relationship exists between the expansion coefficients g, h, and k in the invariant expansion quoted in footnote 8.

COMMENTS

Yang's Gravitational Field Equations

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The gravitational field equations proposed by Yang are shown to be identical with those proposed by Kilmister in 1959. The static, spherically symmetric solution of the Kilmister-Yang field equations is found. It is shown that solar experiments cannot yet distinguish between these equations and Einstein's vacuum equations.

In a recent paper Yang[1] has proposed new gravitational field equations of the form

$$R_{ij;k} - R_{ik;j} = 0, \qquad (1)$$

where R_{ij} is the Ricci tensor and the semicolon denotes covariant differentiation. These equations are identical to those proposed by Kilmister[2] in the form $R_{ijkl}{}^{;l}$, where R_{ijkl} is the Riemann-Christoffel tensor. Equations (1) will hereafter be referred to as the KY equations. While Yang has left discussions of the significance of these equations to a later paper, it is my purpose to find the solution of (1) which is the analog of Schwarzschild's solution for general relativity and to examine its physical consequences.[3]

I consider the spherically symmetric static coordinate system

$$ds^2 = -A(r)\,dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) + D(r)\,dt^2, \tag{2}$$

and shall seek an inverse-power-series solution for which A and D are asymptotically flat. For the metric (2) I find only two independent coupled differential equations which, from (1), are

$$\begin{aligned} &r^2[2AA''D^2 + AA'DD' - 4(A')^2D^2 + A^2(D')^2] + 4A^2D^2(1-A) = 0,\\ &r^2[2A^2D^2D''' - 4A^2D''D'D - 3AA'D^2D'' + 2A^2(D')^3 + 2AA'D(D')^2 - AA''D^2D' + 2(A')^2D^2D']\\ &\qquad + 2ArD[2ADD'' - A(D')^2 - A'DD'] - 4A^2D^2D' = 0, \end{aligned} \tag{3}$$

where the prime denotes differentiation with respect to r. From the form of the KY equations it is clear that all solutions of Einstein's vacuum equations $R_{ij} = 0$ will satisfy (1). Thus Schwarzschild's solution as well as its transformations will satisfy (3). However, one wishes to know whether there are more general solutions which are physically meaningful. I take the following expansions for A and D:

$$A = 1 + \sum_{i=1}^{\infty} \alpha_i / r^i, \quad D = 1 + \sum_{i=1}^{\infty} \beta_i / r^i, \tag{4}$$

where α_i and β_i are constants. Substitution of (4) into (3) results in the solution[4]

$$A = \frac{1}{1-\alpha_1/r} + \frac{1}{r^2}(\alpha_1 + \beta_1)\sum_{i=0}^{\infty}\frac{\gamma_i}{r^i}, \quad D = 1 + \frac{\beta_1}{r} + \frac{1}{r^2}(\alpha_1+\beta_1)\sum_{i=0}^{\infty}\frac{\rho_i}{r^i}. \tag{5}$$

Here γ_i and ρ_i are complicated functions of the integration constants α_1 and β_1. Note that (5) would degenerate to the Schwarzschild solution if one could demonstrate the equality of α_1 and $-\beta_1$. In fact this cannot be done since α_1 and β_1 can be determined only by experiment. However, the difference between these numbers is sufficiently small so that the physical predictions of the KY equations and Einstein's are essentially the same with respect to solar experiments. To see this we have $\beta_1 = -2m$ from Newtonian correspondence, where m is the gravitational mass. From the light-deflection experiment we may conclude that $\alpha_1 = 2m$ to within 10% accuracy.[5] However, we can do much better by considering Weinberg's analysis of the gravitation experiments and the determination of the expansion parameters.[6] From the advance of the perihelion of Mercury we are able to conclude that

$$\alpha_1 = 2m(1.000 \pm 0.001). \tag{6}$$

This means that (5) may be written as

$$A = \frac{1}{1-2m/r} \pm \frac{m}{500r} + O(m^2), \quad D = 1 - \frac{2m}{r} \mp \frac{3m^2}{2000r^2} + O(m^3). \tag{7}$$

The terms which cause this solution to differ from Schwarzschild's are well beyond the state of observation at this time. Accordingly, I conclude that solar experiments are presently unable to distinguish between the KY equations and Einstein's vacuum equations.

However, (7) is not identical to the Schwarzschild solution and as an immediate consequence (7) does not possess a Schwarzschild-type singularity at $r = 2m$. In the vicinity of $r = 2m$ this solution differs markedly from Schwarzschild's solution. It is therefore possible that observations of collapsed bodies could lead to a distinction between the KY and Einstein equations.

As another remark I note that setting $A = 1/D$ in (3) results in differential equations which are easily integrated. For this case I find the general solution to be

$$D = A^{-1} = -2m/r + \lambda r^2, \tag{8}$$

which will be recognized as Kottler's solution[7] for Einstein's equations with a cosmological constant. It is therefore found that a Birkhoff theorem does not apply to the KY equations since it is not possible to find a coordinate transformation which carries the Schwarzschild solution into the Kottler solution in four dimensions. This result could have been anticipated since (1) is satisfied by $G_{ij}=\lambda g_{ij}$, where λ is constant.

While I have taken a noncommittal attitude in this note it must be pointed out that many objections to the KY equations have been raised.[8] As an example, Thompson found a static solution with axial symmetry which seems to represent two unsupported mass points. Such unphysical situations seem to be readily connected to the KY equations.

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[1]C. N. Yang, Phys. Rev. Lett. 33, 445 (1974).

[2]C. W. Kilmister, *Les Theories Relativistes de la Gravitation* (Centre National de la Recherche Scientifique, Paris, 1962).

[3]Many aspects of Kilmister's equations were considered by A. H. Thompson in his Ph. D. thesis, University of London, 1962 (unpublished). From a personal communication I find that Thompson has not considered the analysis I present.

[4]The first few terms of (5) are given by

$$\gamma_0=-\beta_1/8,\quad \gamma_1=-\beta_1(23\alpha_1-3\beta_1)/80,\quad \gamma_2=-\beta_1(449\alpha_1^2-113\alpha_1\beta_1-6\beta_1^2)/960;$$

$$\rho_0=3\beta_1/8,\quad \rho_1=\beta_1(17\alpha_1+3\beta_1)/80,\quad \rho_2=\beta_1(136\alpha_1^2+23\alpha_1\beta_1-9\beta_1^2)/960.$$

[5]S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972), Chap. 8.

[6]Weinberg, Ref. 5, (8.3.5) and page 198.

[7]F. Kottler, Ann. Phys. (Leipzig) 56, 401 (1918).

[8]Thompson, Ref. 3; A. H. Thompson and C. W. Kilmister, U. S. Air Force Technical Report, Contract No. AF 61(052)-457, 1963 (unpublished).

ERRATUM

POPULATION OF EXCITED ELECTRONIC LEVELS BY BETA DECAY, AND ITS INFLUENCE ON MÖSSBAUER SOURCE SPECTRA. L. L. Hirst, J. Stöhr, G. K. Shenoy, and G. M. Kalvius [Phys. Rev. Lett. 33, 198 (1974)].

The quantitative solution for Mössbauer source spectra with rearrangement effects given in our paper contains an error. A correct solution has been derived and will be published elsewhere. Our qualitative discussion remains correct, except that the condition for slow rearrangement must be obtained by comparing the electronic rearrangement rate $w_{e\to g}$ with the reciprocal nuclear lifetime, $1/t_{\rm nuc}$, rather than with $w_{\rm hf}$.

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