(1970).
[6]T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Lett. 19, 859 (1967); J. S. Vaishya, Phys. Rev. 173, 1757 (1968). The results of these authors and those of V. S. Berezinskii, Yad. Fiz. 8, 1208 (1968) [Sov. J. Nucl. Phys. 8, 700 (1969)]; S. Brown, private communication; and S. Brown and G. West, Phys. Rev. 168, 1605 (1968), have been rescaled using $\tau_{\pi^0}=0.84\times10^{-16}$ sec.
[7]Berezinskii, Ref. 6.
[8]Brown, Ref. 6; Brown and West, Ref. 6.
[9]R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley, New York, 1969); Riazuddin and Fayyazuddin, Phys. Rev. 171, 1428 (1968).
[10]T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Lett. 19, 1067 (1967); S. G. Brown and G. B. West, Phys. Rev. Lett. 19, 812 (1967); D. A. Geffen, Phys. Rev. Lett. 19, 770 (1967); see also Ref. 8.
[11]J. H. Schnitzer and S. Weinberg, Phys. Rev. 164, 1828 (1967).
[12]Fayyazuddin and Riazuddin, Nuovo Cimento 54, 520 (1968).
[13]J. Gregor, Ph.D. thesis, Columbia University, 1974 (unpublished).
[14]W. Kummer and J. Kuti, Nuovo Cimento 45, 242 (1966).
[15]F. Schenk and A. Wullschleger, to be published.

Measurement of the $K_L{}^0\to\pi^+\pi^-\pi^0$ Dalitz Plot*

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A Dalitz plot of 509 000 $K_L{}^0\to\pi^+\pi^-\pi^0$ decays is analyzed, using $|M|^2\propto 1+aY+bY^2+cX+dX^2+\dots$, where X and Y are the Dalitz variables. We find no measurable X dependence; we do find that a significant X^2 and Y^2 dependence is present. In addition, the spectrum cannot be fit with a matrix element linear in Y. Our best fit yields $a=-0.917\pm0.013$, $b=0.149\pm0.013$, and $\alpha=0.055\pm0.010$. Comparison with τ decay indicates a definite $|\Delta I|=\frac{1}{2}$ violation in the nonsymmetric amplitudes.

We present herewith a high-statistics analysis of the Dalitz plot for the decay $K_L{}^0\to\pi^+\pi^-\pi^0$. Comparison of this decay distribution with those of other $K_{\pi3}$ modes provides information on the isospin structure of the strangeness-changing nonleptonic weak decays.

The experiment was performed using the Stanford Linear Accelerator Center $K_L{}^0$ spectrometer,[1] with a $K_L{}^0$ beam at 3° to a 16-GeV e^- beam incident on a 1.4-radiation-length Be target. The $K_L{}^0$ momentum (P_K) was determined by time of flight[2] (TOF), and the detected $K_L{}^0$ decay spectrum is shown in Fig. 1(a). The trigger required that at lease two charged tracks traverse the apparatus, while the muon counters were latched to enable us to study the question of pion decays and penetrations of the 7.7-interaction-length Pb muon filter. The data consisted of 5.2×10^6 triggers, of which 3.4×10^6 events had two full tracks having a vertex in the decay volume. These consisted of $K_{\pi3}{}^0$, $K_{\mu3}{}^0$, and $K_{e3}{}^0$ decays in approximately equal numbers. The number of $K_L{}^0$ and neutron interactions in the helium of the decay volume, as extrapolated from events with decay vertices in the region of the front trigger counters, was found to be negligible. Pion interactions in the apparatus were significant only for $P_\pi<800$ MeV/c. Events containing such pions were therefore cut from the data sample.

The detection efficiency and other characteristics of the apparatus were determined by Monte Carlo (MC) techniques. A total of 9×10^5 accepted $K_{\pi3}{}^0$ events were generated, together with appropriate numbers of background events due to $K_{\mu3}{}^0$ and $K_{e3}{}^0$ decays, and $K_{\pi3}{}^0$ and $K_L{}^0\to3\pi^0$ decays with subsequent Dalitz decays. The wire

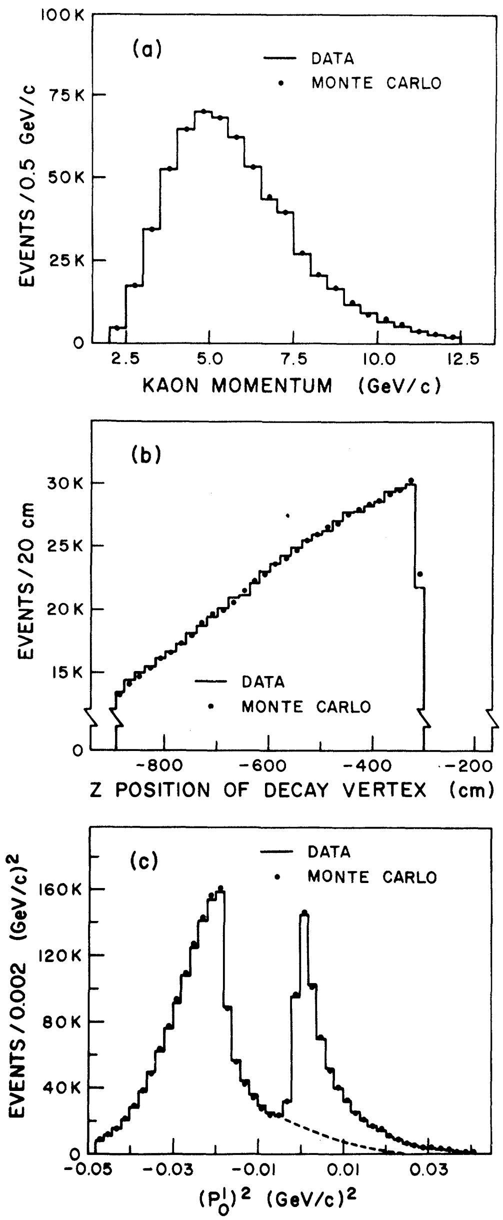

FIG. 1. Data and Monte Carlo predictions for the following distributions: (a) reconstructed P_K spectrum for $K_{\pi 3}{}^0$ decays, (b) decay vertex distribution for all decays passing $K_{\pi 3}{}^0$ cuts, (c) plot of the kinematical variable $P_0'^2$. The small $K_{\pi 3}{}^0/K_{l 3}{}^0$ kinematical overlap is clearly shown in this plot.

spacing, spark jitter, and TOF resolutions, as measured in a sample of regenerated $K_S{}^0 \to \pi^+\pi^-$ decays, were included in the MC analysis. Pion decays, $K_L{}^0$ scatters, and multiple scattering were also included. The MC data were generated in the same format as the experimental data, and both were processed through the same analysis programs. An iterative weighting procedure incorporated the final $K_{\pi 3}{}^0$ matrix element into the MC analysis yielding an overall $K_{\pi 3}{}^0$ detection efficiency of 37%.[3] Several distributions, which are rather sensitive to the MC analysis, are shown in Fig. 1.

In order to isolate $K_{\pi 3}{}^0$ decays, we removed all events from both the data and MC results in which either (1) there was an identifiable muon; (2) the transverse momentum of either charged track was >135 MeV/c, or the net transverse momentum was >130 MeV/c; (3) the invariant mass of the charged tracks was $m_{12} > 365$ MeV/c^2, assuming both were pions; (4) $P_0'^2 < -0.004$ (GeV/c)^2; or (5) the reconstructed $P_K > 7.0$ GeV/c. The $P_0'^2$ distribution[4] shown in Fig. 1(c) verifies that cut (4) strongly discriminates against $K_{l3}{}^0$ decays. The total number of $K_{\pi 3}{}^0$ decays lost by the above cuts was 4.6%, of which $\pi \to \mu\nu$ decays alone accounted for 3.5%. The background in the final sample was 4.6% $K_{\mu 3}{}^0$, 4.1% $K_{e3}{}^0$, and 0.2% Dalitz decay events. A detailed study indicates that the MC analysis reproduces the observed background to within 5%, or to within 0.5% of the total accepted data sample. We have also verified that the final results are only slightly sensitive to reasonable changes in the background normalization. This uncertainty is included in the quoted errors.

A total of 509 000 $K_{\pi 3}{}^0$ events remained after all cuts. The $K_L{}^0$ TOF was then used to resolve the quadratic ambiguity in placing the events on the Dalitz plot. Monte Carlo studies show that the correct choice was made in 85% of the cases. Since the number of incorrect assignments is highly dependent on P_K, we have grouped both the data and MC results into five subsets according to P_K in order to study any systematic biases. The highest and lowest P_K bins, which contain relatively few events, are most sensitive to incorrect assignments, while the central three P_K bins, which contain most of the data, are relatively insensitive. Since the results from each of these five bins are consistent with one another, we are confident that event reconstruction and fitting techniques are well understood. We have also compared many other features of the data and MC results, such as the distribution of vertex positions, and the acceptance of inbending versus outbending topologies. In all cases the MC technique was found to correctly simulate the detector.

Radiative corrections which took into account the resolution and efficiency of the apparatus were applied. The inner-bremsstrahlung contri-

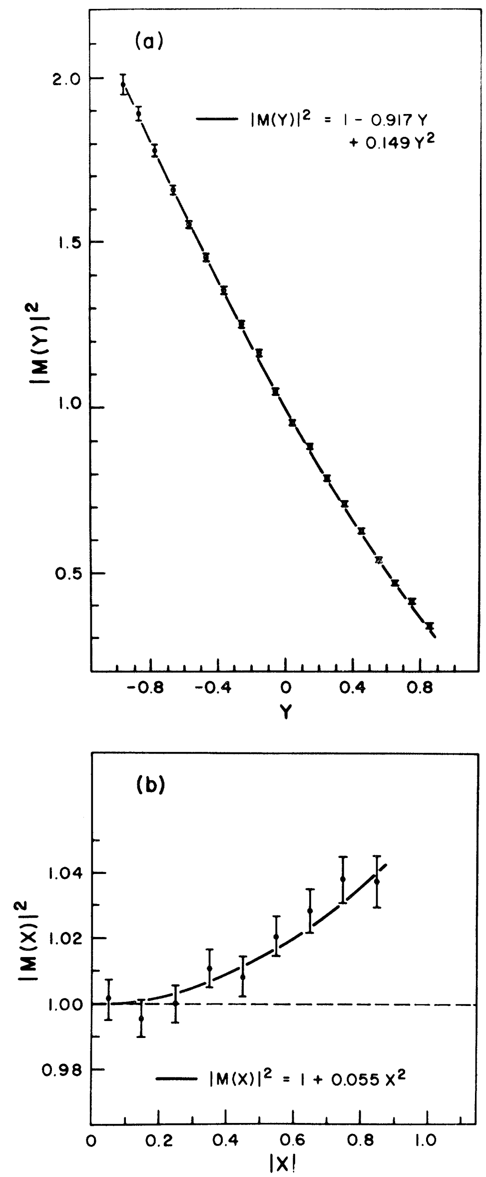

FIG. 2. (a) Y projection, (b) X projection of Dalitz plot (see footnote 6).

bution for which the phonon energy is >2 MeV was calculated by generating a sample of $K_L^0 \to \pi^+\pi^-\pi^0\gamma$ decays, analyzing them as $K_{\pi 3}{}^0$ decays, and treating them as another background subtraction. The overall electromagnetic correction to the Dalitz-plot coefficients due to radiative effects amounts to 2% of the extracted values.[5]

We have used the independent Dalitz-Fabri variables $X = \sqrt{3}(T_{\pi^+} - T_{\pi^-})/Q$ and $Y = (3T_{\pi^0} - Q)/Q$, where T is the c.m. kinetic energy and Q is the energy release in the decay, in fitting the decay spectrum to $|M|^2 \propto 1 + aY + bY^2 + cX + dX^2 + \ldots$. The Y and X projections[6] of the Dalitz-plot density are shown in Figs. 2(a) and 2(b), respectively. Previous experiments[7] have yielded inconsistent results for the linear coefficient a and have not been conclusive as to the existence of the quadratic coefficients b and d. Our results, given in Table I, clearly indicate the presence of both X^2 and Y^2 terms.[8] A similar conclusion was reached by Ford *et al.*[9] in their high-statistics study of τ decay, although they did not correct their data for inner bremsstrahlung. Since $b \neq a^2/4$, we conclude our data cannot be fitted with a matrix element linear in Y. We have also found no X dependence of $|M|^2$, our result being $c = (2 \pm 4) \times 10^{-3}$. The coefficient of X has been set equal to zero in subsequent fits. Note that we have assumed that the effect of final-state interactions is small, so that the expansion coefficients are relatively real. Relaxing this assumption results in a fit in even worse agreement with the assumption of a matrix element linear in Y.

Since in both this experiment and the Ford *et al.* experiment a significant X^2 dependence was observed we are able to test the $|\Delta I| = \frac{1}{2}$ rule predictions beyond the linear term. If the spectrum is parametrized as $|M|^2 \propto 1 + \alpha Y + (\frac{1}{4}\alpha^2 + \beta)Y^2 + \gamma X^2 + \ldots$, the $|\Delta I| = \frac{1}{2}$ rule requires $\alpha^{+-0} = \alpha^{00+} = -2\alpha^{++-}$, and $\beta^{+-0} = \beta^{00+} = (3\gamma^{++-} - \beta^{++-})/2.$[10] We find the $|\Delta I| = \frac{1}{2}$ rule to be badly violated in the linear terms ($\alpha^{+-0} = -0.917 \pm 0.013$, $-2\alpha^{++-} = -0.550 \pm 0.006$), but satisfied in the quadratic terms [$\beta^{+-0} = -0.061 \pm 0.014$, $(3\gamma^{++-} - \beta^{++-})/2 = -0.060 + 0.012$]. Since the quadratic coefficients contain symmetric amplitudes, it might be expected that the $|\Delta I| = \frac{3}{2}$ contributions to these

TABLE I. Results of least-squares fits to the Dalitz plot density using the functional form shown for data with $P_K = 2$–7 GeV/c. Both statistical and systematic errors are included.

		Fits to $\lvert M\rvert^2 \propto 1 + aY + bY^2 + dX^2 + eY^3 + fX^2Y$				
Fit	χ^2/ν	a	b	d	e	f
Linear	423/132	-0.844 ± 0.012				
Quadratic	130/130	-0.917 ± 0.013	0.149 ± 0.013	0.055 ± 0.010		
Cubic	121/128	-0.922 ± 0.018	0.144 ± 0.013	0.065 ± 0.013	0.033 ± 0.021	0.040 ± 0.020

terms would be of the same order as the contributions to the reduced rates, i.e., ~7%, a level at which current experiments are not sensitive. On the other hand, the disagreement in the linear coefficients clearly indicates a $|\Delta I| = \frac{1}{2}$ violation in the nonsymmetric amplitudes.

We thank R. Coombes, L. Birkwood, K. Hense, M. Mugge, D. Ouimette, C. Rasmussen, G. Schultz, and D. Porat for their various contributions to the success of this experiment. We also thank K. Mahanthappa and J. Smith for very helpful conversations. Finally, we acknowledge the support of the experimental facilities, accelerator operations, and computer operation groups of the Stanford Linear Accelerator Center.

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[1]R. Piccioni *et al.*, Phys. Rev. D 9, 2939 (1974).

[2]The K_L^0 time of flight was measured over a 62.5-m flight path with a resolution of ± 0.33 nsec. This allowed useful momentum determinations up to 7.5 GeV/c.

[3]The $K_{\pi 3}^0$ detection efficiency increases linearly from 12% for $P_K = 2$–4 GeV/c to 68% for $P_K = 7$–12 GeV/c, and is constant to within a factor of 1.5 across the Dalitz plot.

[4]$P_0'^2$ is the square of the K_L^0 momentum in the frame where the sum of the longitudinal momenta of the charged pair is zero, assuming a $K_{\pi 3}^0$ decay. For actual $K_{\pi 3}^0$ decays $P_0'^2 \geq 0$, while for K_{l3}^0 decays $P_0'^2$ is usually negative.

[5]J. Smith, private communication; A. Neveu and J. Scherk, Phys. Lett. 27B, 384 (1968); R. Ferrari and M. Rosa-Clot, Nuovo Cimento 56A, 582 (1968).

[6]Since $|M^2|$ depends on both Y and X^2 ($|M|^2 = 1 + aY + bY^2 + dX^2$), corrections must be applied to the direct projection of the data onto the Y or X axis before we can separate the two dependences and obtain the correct projected slopes. We introduce the following scheme (since we see no measurable X^2Y dependence): $|M(Y)|^2 = \sum_j (M_j^2 - dX_j^2)$ displayed in Fig. 2(a), and $|M(X)|^2 = \sum_i (M_i^2 - aY_i - bY_i^2)$ displayed in Fig. 2(b), where j is summed over all the X_j bins and i is summed over all the Y_i bins.

[7]C. Buchanan *et al.*, Phys. Lett. 33B, 623 (1970); R. Smith *et al.*, Phys. Lett. 32B, 133 (1970); M. Albrow *et al.*, Phys. Lett. 33B, 516 (1970).

[8]If we expand the matrix in terms of the invariants $S_i = (P_K - P_i)^2$, $|M|^2 = 1 + g/m_{\pi^+}{}^2(S_3 - S_0) + h/m_{\pi^+}{}^4(S_3 - S_0)^2 + k/m_{\pi^+}{}^4(S_2 - S_1)^2 + \ldots$ as described by T. A. Lasinski *et al.*, Rev. Mod. Phys. 45, S1 (1973), we obtain $g = 0.677 \pm 0.010$, $h = 0.079 \pm 0.007$, and $k = 0.0097 \pm 0.0018$.

[9]W. T. Ford, P. A. Piroué, R. S. Remmel, A. J. S. Smith, and P. A. Souder, Phys. Lett. 38B, 335 (1972).

[10]K. T. Mahanthappa, private communication; M. K. Gaillard, CERN Report No. TH-1963, 1973 (unpublished). If the $\pi^\pm \pi^0$ mass difference is ignored then a similar relationship exists between the expansion coefficients g, h, and k in the invariant expansion quoted in footnote 8.

COMMENTS

Yang's Gravitational Field Equations

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The gravitational field equations proposed by Yang are shown to be identical with those proposed by Kilmister in 1959. The static, spherically symmetric solution of the Kilmister-Yang field equations is found. It is shown that solar experiments cannot yet distinguish between these equations and Einstein's vacuum equations.

In a recent paper Yang[1] has proposed new gravitational field equations of the form

$$R_{ij;k} - R_{ik;j} = 0, \tag{1}$$

where R_{ij} is the Ricci tensor and the semicolon denotes covariant differentiation. These equations are identical to those proposed by Kilmister[2] in the form $R_{ijkl}{}^{;l}$, where R_{ijkl} is the Riemann-Christoffel tensor. Equations (1) will hereafter be referred to as the KY equations. While Yang has left discussions of the significance of these equations to a later paper, it is my purpose to find the solution of (1)