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Determination of the Axial-Vector Form Factor in the Radiative Decay of the Pion*

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The branching ratio for the decay $\pi\rightarrow e\nu\gamma$ has been measured in a counter experiment in which the e^+ was detected in a magnetic spectrometer and the γ ray in a lead-glass hodoscope. From the measured branching ratio we determine γ, the ratio of the axial-vector form factor to the vector form factor. The latter is computed by using conserved-vector-current theory and τ_{π^0}, the π^0 lifetime. Adopting a best value 0.86×10^{-16} sec, we obtain $\gamma=0.15\pm0.11$ or $\gamma=-2.07\pm0.11$. A comparison between the measured values of γ and various theories is made.

Recent theoretical developments in quark models and current algebra have made it interesting to make a more accurate measurement of the axial-vector form factor of the pion radiative decay, $\pi\rightarrow e\nu\gamma$, first measured at CERN over ten years ago.[1] The general form for the radiative decay amplitude has been calculated by several authors.[2] The so-called inner-bremsstrahlung term (IB) arises from diagrams in which a photon is radiated from one of the charged, external lines of the ordinary decay $\pi\rightarrow e\nu$, and can be calculated from the observed rate of the decay $\pi\rightarrow e\nu$ by standard methods of quantum electrodynamics:

$$\frac{d^2W_{\rm IB}}{dx\,dy}=\frac{\alpha W_{e\nu}}{2\pi}\left(\frac{1-y}{x^2}\right)\left(\frac{(x-1)^2+1}{x+y-1}\right). \quad (1)$$

In Eq. (1), $\alpha=1/137$, $W_{e\nu}$ is the rate of $\pi\rightarrow e\nu$, $x=2P_\gamma/m_\pi$, $y=2P_e/m_\pi$, and the rest mass of the electron has been set equal to zero.

The interesting effect is a structure-dependent (SD) process involving intermediate states generated by the strong interaction. These intermediate states are described by vector and axial-vector form factors, $a(q^2)$ and $b(q^2)$, which may be treated as constants because the momentum transfer in the decay is small. The equation for the SD rate is customarily written in terms of the vector form factor $a(0)$ and $\gamma\equiv b(0)/a(0)$:

$$\frac{d^2W_{\rm SD}}{dx\,dy}=\frac{(G\cos\theta)^2\alpha m_\pi^{\,7}|a(0)|^2}{64\pi^2}\times[D(1+\gamma)^2+E(1-\gamma)^2]. \quad (2)$$

Here G is the weak coupling constant, θ is the Cabibbo angle, $D=(1-x)(x+y-1)$, and $E=(1-x)\times(1-y)^2$. The SD-IB interference term is small and is neglected.

The experimental layout is shown in Fig. 1. With the low-energy achromatic pion beam at the Berkeley 184-in. cyclotron, about 2×10^5 π^+/sec were stopped in a counter hodoscope, which was slanted to increase the stopping material and to minimize the positron energy loss. The positron momentum was measured in the magnet–spark-chamber spectrometer system with a resolution of about 2 MeV. Momentum normalization and resolution were determined by fitting the end point in the momentum spectrum of positrons from μ decay and by triggering the system occasionally on the monoenergetic positrons from $\pi^+\rightarrow e^+\nu$.

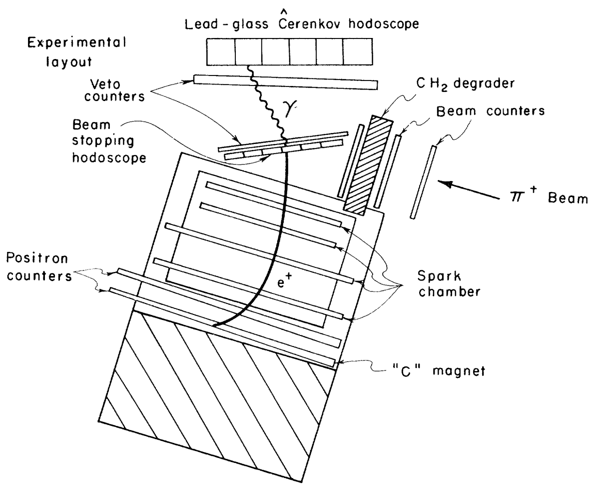

FIG. 1. Experimental layout.

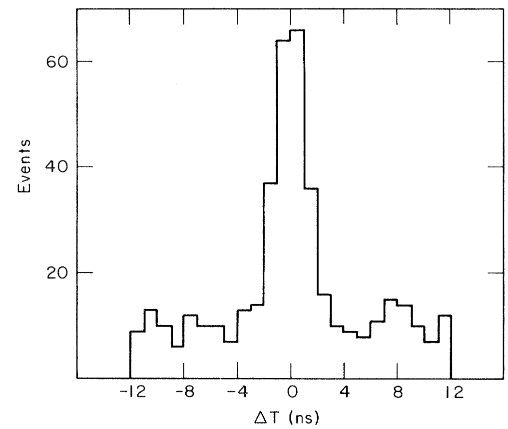

FIG. 2. Spectrum of time differences, ΔT, between positron and photon signals.

The photon was detected in a Cherenkov hodoscope consisting of 24 6-in. cubes of lead glass, each with its own 5-in. photomultiplier, which determined the position of the photon to within $\pm 8°$. The apparatus was designed to accept events with a large opening angle between the positron and photon, thus covering a region of phase space where the SD part of the amplitude is at its largest and the IB portion is small. For radiative decay events the fractional acceptance of the apparatus was 0.0185, as evaluated by a Monte Carlo calculation, and the event rate was 0.3/h.

Candidates for radiative decay events were required to have a prompt coincidence between the positron trigger counters and the Cherenkov counter within 100 nsec of a pion stop. Only those events with positron momentum greater than 58 MeV were analyzed further. In this way we avoided the overwhelming positron background from muon decay. The distribution of events in ΔT, the time difference between the positron and photon signals, is shown in Fig. 2. Radiative decay events stand out as a sharp peak at $\Delta T = 0$ above a flat background. After the subtraction of an appropriately normalized (P, θ) distribution of background events chosen from the out-of-time region of the ΔT spectrum, we are left with 170 $\pm$ 15 events.

In analyzing our data the theoretical distributions given by Eqs. (1) and (2) were folded with the experimental resolution and acceptance; then a maximum-likelihood technique was used to fit the resulting expression to the binned data as a function of momentum and angle simultaneously. Because Eq. (2) is quadratic one obtains two values of γ which give good fits to the data.

Figure 3 shows the relation between the axial-vector form factor and the vector form factor determined from our data. The dashed curves give the 1-standard-deviation error in $b(0)$ due to sources of error internal to this experiment. Approximately equal contributions to this error come from the uncertainty in the photon detection efficiency and from the statistical uncertainty given by the width of the likelihood function.

In order to evaluate the axial-vector form factor, or γ, a value for the vector form factor must be taken from some other source. In the framework of the conserved-vector-current (CVC) theory one may calculate the vector form factor from the π^0 lifetime,[3]

$$|a(0)| = (2/\pi m_\pi^3 \tau_{\pi^0})^{1/2} = (0.0259 \pm 0.0015) m_\pi^{-1}.$$

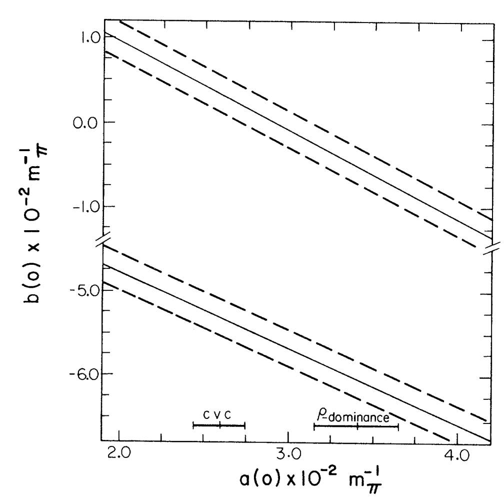

FIG. 3. Relationship between $a(0)$ and $b(0)$ as determined by our data. The portion of the graph which is symmetric upon the exchange $b \rightarrow -b$, $a \rightarrow -a$ is not shown.

Here τ_{π^0} is the lifetime for $\pi^0 \to \gamma\gamma$, for which we have used the value $\tau_{\pi^0} = (0.84 \pm 0.10) \times 10^{-16}$ sec.[4] This value for τ_{π^0} must be used cautiously since even the most recent measurements do not form a consistent set. With the above value for $|a(0)|$ we find $\gamma = 0.15 \pm 0.11$ or -2.07 ± 0.11. A value for the vector form factor, independent of the π^0 lifetime, may also be obtained from a vector-dominance model in which an unsubtracted dispersion relation is saturated by the ρ meson.[5] This method yields $|a(0)| = (0.035 \pm 0.0025) m_\pi^{-1}$. With this value we find $\gamma = -0.18 \pm 0.09$ or -1.74 ± 0.09. Note that although we obtain both the magnitude and sign of γ (for each solution), the sign of $b(0)$ is not determined because the sign of $a(0)$ is unknown.

To compare our experiment with that of Depommier *et al.*,[1] we chose the value of $a(0)$ used in their analysis to calculate γ from our data: $\gamma = 0.263 \pm 0.10$ or -2.18 ± 0.10 (internal errors only). On the basis of 143 events they obtained $\gamma = 0.4$ or -2.2 (no errors quoted), in agreement with our results.

The predictions made within the framework of current algebra[6-8] range from $|\gamma| = 0.12$ to $|\gamma| = 2.14$. Despite the wide range of these predictions, the calculations are essentially identical except in their treatment of the matrix element $\langle \pi | A_\mu | \gamma \rangle$, where A_μ is the axial-vector current. If it is assumed that this vertex function satisfies an unsubtracted dispersion relation dominated by the A_1 pole, then $b(0) = f_\pi / m_\rho^2$ (f_π is the ordinary pion decay coupling constant) and $|\gamma| = 0.59$.[9] It is generally believed, however, that this matrix element must have a subtraction since $\langle \pi^0 | A_\mu | \rho^- \rangle$ and $\langle \pi^0 | V_\mu | A_1^- \rangle$, which are related to it through vector dominance, are known to require subtractions.[10] Schnitzer and Weinberg[11] have used current algebra to compute the various matrix elements; the parameter δ appearing in their formalism relates the unknown subtraction constant to the rates for ρ and A_1 decay, to the π^+-π^0 mass difference, and to the electromagnetic form factor of the pion. This is summarized in Table I. $\delta = -1$ corresponds to no subtraction in the pion form factor, $\delta = -\frac{1}{2}$ comes from a fit to the experimental widths of the ρ and A_1. The last choice, $\delta = 0$, has the merit that, in the soft-pion limit, the logarithmic divergence from the A_1 contribution to the mass difference cancels out.[12]

It is clear that theory requires a value of γ smaller in magnitude than our large negative solutions. (An exception is the prediction $|\gamma| = 2.14$,[7] which is, however, based on a very uncertain determination of the pion charge radius.) Thus even with the uncertainty in the value of $a(0)$ we can conclude from this experiment that γ must be close to zero and that, at least within the context of the Schnitzer-Weinberg formalism, a subtraction is required in the dispersion relations for both the pion form factor and the $\langle \pi | A_\mu | \rho \rangle$ matrix element. We should point out that recent work using substantially different current-algebra techniques predicts $\gamma \sim 0.7$.[13]

TABLE I. Predicted values of several observables for various choices of δ.

δ	$\|\gamma\|$	$\Gamma(A_1 \to \rho\pi)$ (MeV)	$\Gamma(\rho \to \pi\pi)$ (MeV)	Pion charge radius (fm)
-1	0.59	61	140	0.63
$-\frac{1}{2}$	0.35	116	107	0.59
0	0	190	79	0.55

Finally, we would like to draw attention to the prediction $\gamma = 0$ of the static quark model[14] which is in agreement with our results. However, this model involves assumptions which are quite incompatible with the current-algebra point of view.[15]

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Measurement of the $K_L{}^0\to\pi^+\pi^-\pi^0$ Dalitz Plot*

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A Dalitz plot of 509 000 $K_L{}^0\to\pi^+\pi^-\pi^0$ decays is analyzed, using $|M|^2\propto 1+aY+bY^2+cX+dX^2+\dots$, where X and Y are the Dalitz variables. We find no measurable X dependence; we do find that a significant X^2 and Y^2 dependence is present. In addition, the spectrum cannot be fit with a matrix element linear in Y. Our best fit yields $a=-0.917\pm0.013$, $b=0.149\pm0.013$, and $\alpha=0.055\pm0.010$. Comparison with τ decay indicates a definite $|\Delta I|=\frac{1}{2}$ violation in the nonsymmetric amplitudes.

We present herewith a high-statistics analysis of the Dalitz plot for the decay $K_L{}^0\to\pi^+\pi^-\pi^0$. Comparison of this decay distribution with those of other $K_{\pi3}$ modes provides information on the isospin structure of the strangeness-changing nonleptonic weak decays.

The experiment was performed using the Stanford Linear Accelerator Center $K_L{}^0$ spectrometer,[1] with a $K_L{}^0$ beam at 3° to a 16-GeV e^- beam incident on a 1.4-radiation-length Be target. The $K_L{}^0$ momentum (P_K) was determined by time of flight[2] (TOF), and the detected $K_L{}^0$ decay spectrum is shown in Fig. 1(a). The trigger required that at lease two charged tracks traverse the apparatus, while the muon counters were latched to enable us to study the question of pion decays and penetrations of the 7.7-interaction-length Pb muon filter. The data consisted of 5.2×10^6 triggers, of which 3.4×10^6 events had two full tracks having a vertex in the decay volume. These consisted of $K_{\pi3}{}^0$, $K_{\mu3}{}^0$, and $K_{e3}{}^0$ decays in approximately equal numbers. The number of $K_L{}^0$ and neutron interactions in the helium of the decay volume, as extrapolated from events with decay vertices in the region of the front trigger counters, was found to be negligible. Pion interactions in the apparatus were significant only for $P_\pi<800$ MeV/c. Events containing such pions were therefore cut from the data sample.

The detection efficiency and other characteristics of the apparatus were determined by Monte Carlo (MC) techniques. A total of 9×10^5 accepted $K_{\pi3}{}^0$ events were generated, together with appropriate numbers of background events due to $K_{\mu3}{}^0$ and $K_{e3}{}^0$ decays, and $K_{\pi3}{}^0$ and $K_L{}^0\to3\pi^0$ decays with subsequent Dalitz decays. The wire